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THE PHASE COHERENCE OF LIGHT FROM EXTRAGALACTIC SOURCES: DIRECT EVIDENCE AGAINST FIRST-ORDER PLANCK-SCALE FLUCTUATIONS IN TIME AND SPACE

RICHARD LIEU AND LLOYD W. HILLMAN
Department of Physics, University of Alabama in Huntsville, Huntsville, AL 35899
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ABSTRACT

We present a method of directly testing whether time continues to have its usual meaning on scales of $\lesssim t_P = (\hbar G/c^5)^{1/2} \approx 5.4 \times 10^{-44}$ s, the Planck time. According to quantum gravity, the time t of an event cannot be determined more accurately than a standard deviation of the form $\sigma_t/t = a_0(t_P/t)^\alpha$, where a_0 and α are positive constants ∼1; likewise, distances are subject to an ultimate uncertainty $c\sigma_t$, where c is the speed of light. As a consequence, the period and wavelength of light cannot be specified precisely; rather, they are independently subject to the same intrinsic limitations in our knowledge of time and space, so that even the most monochromatic plane wave must in reality be a superposition of waves with varying ω and $\boldsymbol{k}$, each having a different phase velocity ω/k. For the entire accessible range of the electromagnetic spectrum this effect is extremely small, but it can cumulatively lead to a complete loss of phase information if the emitted radiation propagated a sufficiently large distance. Since, at optical frequencies, the phase coherence of light from a distant point source is a necessary condition for the presence of diffraction patterns when the source is viewed through a telescope, such observations offer by far the most sensitive and uncontroversial test. We show that the *Hubble Space Telescope* detection of Airy rings from the active galaxy PKS 1413+135, located at a distance of 1.2 Gpc, excludes all first-order ($\alpha = 1$) quantum gravity fluctuations with an amplitude $a_0 > 0.003$. The same result may be used to deduce that the speed of light *in vacuo* is exact to a few parts in 10^{32}.

Subject headings: distance scale — early universe — gravitation — radiation mechanisms: general — techniques: interferometric — time

1. INTRODUCTION: THE PLANCK SCALE

It is widely believed that time ceases to be well defined at intervals $\lesssim t_P$, where quantum fluctuations in the vacuum metric tensor render general relativity an inadequate theory. Both t_P and its corresponding distance scale $l_P = ct_P$, the Planck length, play a vital role in the majority of theoretical models (including superstrings) that constitute innumerable papers attempting to explain how the universe was born and how it evolved during infancy (see, e.g., Silk 2001 and references therein). Given this background, we desperately lack experimental data that reveal even the slightest anomaly in the behavior of time and space at such small scales. Although the recent efforts in utilizing gravitational wave interferometry and the observation of ultra–high-energy quanta carry potential (Amelino-Camelia 2001; Ng et al. 2001; Lieu 2002), they are still some way from delivering a verdict, because the conclusions are invariably subject to interpretational issues. Here we wish to describe how an entirely different yet well-established technique has hitherto been overlooked: not only would it enable direct tests for Planck-scale fluctuations (and reveal the detailed properties of any such effects), but also the measurements performed to date could already be used to eliminate prominent theories.

Owing to the variety of proposed models, we begin by describing the common feature that defines the phenomenon being searched: if a time t is so small that $t \to t_P$, even the best clock ever made will only be able to determine it with an uncertainty $\delta t \gtrsim t$. To express this mathematically, we may write the intrinsic standard deviation of time as $\sigma_t/t = f(t_P/t)$, where $f \ll 1$ for $t \gg t_P$ and $f \gtrsim 1$ for $t \lesssim t_P$. Over the range $t \gg t_P$, the (hitherto unknown) function f can be expanded as follows:

$$f(x) = x^\alpha(a_0 + a_1 x + a_2 x^2 + \ldots) \approx a_0 x^\alpha \text{ for } x \ll 1, \quad (1)$$

where $x = t_P/t$ and both a_0 and α are positive constants (∼1 for all reasonable scenarios). Since for the rest of this Letter we shall be concerned only with times $t \gg t_P$, we may take an approximate form of equation (1) as

$$\frac{\sigma_t}{t} \approx a_0 \left(\frac{t_P}{t}\right)^\alpha . \quad (2)$$

Our appreciation of how equation (2) may affect measurements of angular frequencies and wavevectors (ω, $\boldsymbol{k}$) arises from the realization that if a quantity $\omega > \omega_P = 2\pi/t_P$ can be determined accurately, such a calibration will lead to a "superclock" that keeps time to within $\delta t < t_P$. Thus, as $\omega \to \omega_P$, ω should fluctuate randomly such that $\delta\omega/\omega \to 1$. Indeed, for the case of $\sigma_t \approx t_P$ (i.e., eq. [2] with $\alpha = 1$), the following equation was shown by Lieu (2002) to be an immediate consequence:

$$\frac{\sigma_\omega}{\omega} \approx a_0 \frac{\omega}{\omega_P}, \text{ or } \frac{\sigma_E}{E} \approx a_0 \frac{E}{E_P}, \quad (3)$$

where $E = \hbar\omega$ and $E_P = \hbar\omega_P = h/t_P \approx 8.1 \times 10^{28}$ eV. Furthermore, for any value of α it can be proved (see Ng & van Dam 2000) that equation (2) leads to

$$\frac{\sigma_\omega}{\omega} \approx a_0 \left(\frac{\omega}{\omega_P}\right)^\alpha, \text{ or } \frac{\sigma_E}{E} \approx a_0 \left(\frac{E}{E_P}\right)^\alpha . \quad (4)$$

The same reasoning also applies to the intrinsic uncertainty in data on the wavevector $\boldsymbol{k}$ (note also that for measurements directly taken by an observer, $\delta\omega$ and $\delta\boldsymbol{k}$, like δt and $\delta\boldsymbol{r}$, are uncorrelated errors), for if any component of $\boldsymbol{k}$ could be known to high accuracy even in the limit of large k, we would be able to surpass the Planck-length limitation in distance determination for that direction. Thus, a similar equation may then be formulated as

$$\frac{\sigma_k}{k} \approx a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha}, \quad (5)$$

where $c = 1$ hereafter, k is the magnitude of $\boldsymbol{k}$ and the right-hand side is identical to the previous equation because $\omega = k$ for photons. Note, indeed, that equations (4) and (5) hold good for ultrarelativistic particles as well.

About the value of α, the straightforward choice is $\alpha = 1$, which by equation (2) implies $\sigma_t \sim t_P$; i.e., the most precise clock has uncertainty $\sim t_P$. Indeed, $\alpha = 1$ is just the first-order term in a power series expansion of quantum loop gravity. However, the quantum nature of time at scales $\lesssim t_P$ may be manifested in other (more contrived) ways. In particular, for random walk models of spacetime, where each step has size t_P, $\alpha = \frac{1}{2}$ (Amelino-Camelia 2000). On the other hand, it was shown (Ng 2002) that as a consequence of the holographic principle (which states that the maximum degrees of freedom allowed within a region of space is given by the volume of the region in Planck units; see Wheeler 1982; Bekenstein 1973; Hawking 1975; t'Hooft 1993; Susskind 1995) σ_t takes the form $\sigma_t/t \sim (t_P/t)^{2/3}$, leading to $\alpha = \frac{2}{3}$ in equations (3) and (4). Such an undertaking also has the desirable property (Ng 2002) that it readily implies a finite lifetime τ for black holes, viz., $\tau \sim G^2m^3/\hbar c^4$, in agreement with the earlier calculations of Hawking.

Although the choice of α is not unique, the fact that it appears as an exponent means different values can lead to wildly varying predictions. Specifically, even by taking $E = 10^{20}$ eV [i.e., the highest energy particles known, where Planck-scale effects are still only $\sim(E/E_P)^{\alpha} \approx 10^{-9\alpha}$ in significance], an increment of α by 0.5 would demand a detection sensitivity 4.5 orders of magnitude higher. The situation gets much worse as E becomes lower. Thus, if an experiment fails to offer confirmation at a given α, one can always raise the value of α, and the search may never end. Fortunately, however, it turns out that all of the three scenarios $\alpha = \frac{1}{2}, \frac{2}{3}$, and 1 may be clinched by the rapid advances in observational astronomy.

2. THE PROPAGATION OF LIGHT IN FREE SPACE

How do equations (4) and (5) modify our perception of the radiation dispersion relation? By writing the relation as

$$\omega^2 - k^2 = 0, \quad (6)$$

the answer becomes clear—one simply needs to calculate the uncertainty in $\omega^2 - k^2$ due to the intrinsic fluctuations in the measurements of ω and $\boldsymbol{k}$, viz., $\delta(\omega^2 - k^2) = 2\omega\delta\omega - 2k\delta k$, bearing in mind that $\delta\omega$ and δk are independent variations, as already discussed. This allows us to obtain the standard deviation

$$\sigma_{\omega^2-k^2} = 2\sqrt{2}\omega^2 a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha}. \quad (7)$$

Thus, typically equation (6) will be replaced by

$$\omega^2 - k^2 \approx \pm 2\sqrt{2}\omega^2 a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha}. \quad (8)$$

In the case of a positive fluctuation on the right-hand term of equation (6) by unit σ, the phase and group velocities of propagation will read, for $E/E_P \ll 1$, as

$$v_p = \frac{\omega}{k} \approx 1 + \sqrt{2}a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha},$$
$$v_g = \frac{d\omega}{dk} \approx 1 + \sqrt{2}(1+\alpha)a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha}. \quad (9)$$

The results differ from that of a particle—here $\omega^2 - k^2$ is a function of ω and not a constant, so that both v_p and v_g are greater than 1, i.e., greater than the speed of light $v = 1$. On the other hand, if the right-hand side of equation (6) fluctuates negatively, the two wave velocities will read like

$$v_p = \frac{\omega}{k} \approx 1 - \sqrt{2}a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha},$$
$$v_g = \frac{d\omega}{dk} \approx 1 - \sqrt{2}(1+\alpha)a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha}, \quad (10)$$

and will both be less than 1.

Is it possible to force a reinterpretation of equation (8) in another (more conventional) way; viz., for a particular off-shell mode, $\omega^2 - k^2$ typically assumes a *constant* value different from zero by about the unit σ of equation (7)? The point, however, is that even in this (highly artificial) approach, the quantities $v_p = \omega/k$ and $v_g = d\omega/dk = k/\omega$ will still disagree with each other randomly by an amount $\sim(\omega/\omega_P)^{\alpha}$, so that the chief outcome of equations (9) and (10) is robust.

3. STELLAR INTERFEROMETRY AS AN ACCURATE TEST

But is such an effect observable? Although an obvious approach is to employ the highest energy radiation, so as to maximize ω/ω_P, such are difficult to detect. More familiar types of radiation, e.g., optical light, have much smaller values of ω/ω_P, yet the advantage is that we can measure their properties with great accuracy. Specifically, we consider the phase behavior of 1 eV light received from a celestial optical source located at a distance L away. During the propagation time $\Delta t = L/v_g$, the phase has advanced from its initial value ϕ (which we assume to be well defined) by an amount

$$\Delta\phi = 2\pi \frac{v_P \Delta t}{\lambda} = 2\pi \frac{v_P}{v_g}\frac{L}{\lambda}.$$

According to equations (9) and (10), $\Delta\phi$ should then randomly fluctuate in the following manner:

$$\Delta\phi = 2\pi \frac{L}{\lambda}\left[1 \pm \sqrt{2}\alpha a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha}\right]. \quad (11)$$

In the limit when

$$\sqrt{2}\alpha a_0 \left(\frac{\omega}{\omega_P}\right)^{\alpha} \frac{L}{\lambda} \geq 1, \text{ or } \frac{\sqrt{2}\alpha a_0}{h} E^{1+\alpha}E_P^{-\alpha}L \geq 1, \quad (12)$$

the phase of the wave will have appreciable probability of assuming any value between 0 and 2π upon arrival, irrespective of how sharp the initial phase at the source may be. Since

a_0 and α are free parameters, equations (11) and (12) are a common consequence of many quantum gravity models—both equations can be derived in a variety of ways—although the approach presented here may be taken as representative.

From the preceding paragraph, a way toward testing the behavior of time to the limit has become apparent. In stellar interferometry (see, e.g., Baldwin & Haniff 2002 for a review), light waves from an astronomical source are incident upon two reflectors (within a terrestrial telescope) and are subsequently converged to form Young's interference fringes. By equation (11), however, we see that if time ceases to be exact at the Planck scale, the phase of light from a sufficiently distant source will appear random—when L is large enough to satisfy equation (12), the fringes will disappear. In fact, the value of L at which equation (12) holds may readily be calculated for the case of $\alpha = \frac{2}{3}$ and $\alpha = 1$, with the results

$$L_{\alpha=2/3} \geq 2.47 \times 10^{15} a_0^{-1} (E/1 \text{ eV})^{-5/3} \text{ cm},$$
$$L_{\alpha=1} \geq 7.07 \times 10^{24} a_0^{-1} (E/1 \text{ eV})^{-2} \text{ cm}. \quad (13)$$

For $a_0 = 1$ and $E = 1$ eV, these distances correspond, respectively, to 165 AU (or 8×10^{-4} pc) and 2.3 Mpc.

4. EXAMINING THE TEST IN MORE DETAIL

Since the subject of our search is no small affair we provide here an alternative (and closer) view of the proposed experiment.

In a classical approach to the "untilted" configuration of Young's interferometry, the phase of a plane wave (from a distant source) at the position $\boldsymbol{r}$ of the double-slit system may be written as $\omega t - \boldsymbol{k} \cdot \boldsymbol{r}$, where t is the arrival time of the wavefront. The electric fields $\mathcal{E}_1$ and $\mathcal{E}_2$ of the waves at some point P behind the slits where they subsequently meet may then assume the form

$$\mathcal{E}_1 = \boldsymbol{\epsilon}_1 |\mathcal{E}_1| e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r} + \phi_1)}, \; \mathcal{E}_2 = \boldsymbol{\epsilon}_2 |\mathcal{E}_2| e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r} + \phi_2)}, \quad (14)$$

where $|\mathcal{E}|$ denotes the modulus of the (complex) magnitude of vector $\mathcal{E}$, ϵ_1 and ϵ_2 are unit vectors, and ϕ_1, ϕ_2 are the advances in the wave phase during the transits between each of the two slits and P. The intensity of light at P is proportional to $|\mathcal{E}_1 + \mathcal{E}_2|^2$, which contains the term essential to the formation of fringes, viz., $2|\mathcal{E}_1||\mathcal{E}_2| \cos\phi$, where $\phi = \phi_1 - \phi_2$ and $\boldsymbol{\epsilon}_1$, $\boldsymbol{\epsilon}_2$ are taken to be parallel (as is commonly the case).

If, however, there exist intrinsic and independent uncertainties in one's knowledge of the period and wavelength of light on scales t_P and ct_P, respectively, the most monochromatic plane wave will have to be a superposition of many waves, each having slightly varying ω and $\boldsymbol{k}$, and the phase velocity $v_p = \omega/k$ will fluctuate according to equations (9) and (10). For optical frequencies, the only measurable effect is the phase separation between these waves after traveling a large distance at different speeds; i.e., equation (14) will be replaced by

$$\mathcal{E}_1 = \boldsymbol{\epsilon}_1 |\mathcal{E}_1| \sum_j a_j e^{i(\omega_j t - \boldsymbol{k}_j\cdot\boldsymbol{r} + \phi_1 + \theta_j)} \approx \boldsymbol{\epsilon}_1 |\mathcal{E}_1| e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r} + \phi_1)} \sum_j a_j e^{i\theta_j},$$

$$\mathcal{E}_2 = \boldsymbol{\epsilon}_2 |\mathcal{E}_2| \sum_l a_l e^{i(\omega_l t - \boldsymbol{k}_l\cdot\boldsymbol{r} + \phi_2 + \theta_l)} \approx \boldsymbol{\epsilon}_2 |\mathcal{E}_2| e^{i(\omega t - \boldsymbol{k}\cdot\boldsymbol{r} + \phi_2)} \sum_l a_l e^{i\theta_l},$$

where $\{a_i\}$ are real coefficients (not to be confused with the a_0 of eq. [1]) normalized such that a_i^2 equals the occurrence probability of the ith wave, which of course is governed by how far θ_i differs from its zero mean value when compared with the standard deviation in equation (11) [viz., $2\sqrt{2}\pi\omega L/(\omega_P \lambda)$ when $a_0 = \alpha = 1$]. Note that this time the intensity at P depends on many "cross" terms, each of the form $2a_j a_l |\mathcal{E}_1||\mathcal{E}_2| \cos\phi$, where now $\phi = (\phi_1 - \phi_2) + (\theta_j - \theta_l)$. If the propagation length L is large enough to satisfy equation (12), θ_j and θ_l, hence $\theta_j - \theta_l$, will spread over one phase cycle, so that the original term $2|\mathcal{E}_1||\mathcal{E}_2| \cos(\phi_1 - \phi_2)$ will no longer be characteristic of the point P. This is the mathematical demonstration of why no appreciable fringe contrast across the detector can be expected. Obviously, the argument can readily be generalized to conclude that if equation (12) is fulfilled, interference effects from *multiple slits* (or a single large slit as limiting case) will also disappear.

5. THE DIFFRACTION OF LIGHT FROM EXTRAGALACTIC POINT SOURCES: ABSENCE OF ANOMALOUS BEHAVIOR IN TIME AND SPACE AT THE PLANCK SCALE

Let us now consider the observations to date. The Young's type of interference effects were clearly seen at $\lambda = 2.2$ μm ($E \approx 0.56$ eV) light from a source at 1.012 kpc distance, viz., the star S Ser, using the Infrared Optical Telescope Array, which enabled a radius determination of the star (van Belle, Thompson, & Creech-Eakman 2002). When comparing with equation (13), we see that this result can already be used to completely exclude the $\alpha = \frac{2}{3}$ model, because for such a value of α and for all reasonable values of a_0, $\Delta\phi$ carries uncertainties $\gg 2\pi$, and the light waves would not have interfered. It is also evident from equation (13), however, that no statement about $\alpha = 1$ can be made with the S Ser findings.

Within the context of § 4's development, it turns out that the well-recognized presence of diffraction pattern in the image of extragalactic point sources when they are viewed through the finite aperture of a telescope provides even more stringent constraints on α. Such patterns have been observed from sources located at distances much larger than 1 kpc, implying, as before, that phase coherence of light is maintained at the aperture entrance despite the contrary prediction of quantum gravity. In particular, to clinch the first-order prediction $\alpha = 1$, we note that Airy rings (circular diffraction) were clearly visible at both the zeroth and first maxima in an observation of the active galaxy PKS 1413+135 ($L = 1.216$ Gpc) by the *Hubble Space Telescope* at 1.6 μm wavelength (Perlman et al. 2002). Referring back to equation (13), this means exclusion of all $\alpha = 1$ quantum gravity fluctuations that occur at an amplitude $a_0 \geq 3.14 \times 10^{-3}$ (moreover, the speed of light does not fluctuate fractionally by more than λ/L, or several parts in 10^{32}). To facilitate those who wish to explore the implications in full, we offer the following inequality, derivable directly from equation (12), which the reader can use to readily find for any value of α the range of a_0 still permitted by the PKS 1413 result:

$$a_0 < 3.14 \times 10^{-3} \frac{(10^{29})^{\alpha-1}}{\alpha}. \quad (15)$$

That equation (15) is highly sensitive to α has already been discussed. Two consequences immediately emerge from equation (15): (1) for any of the $\alpha < 1$ models to survive, they must involve ridiculously small values of a_0, and (2) $\alpha > 1$ remains essentially unconstrained.

Further investigation of sources that lie beyond PKS 1413 will scrutinize the $\alpha \leq 1$ scenarios more tightly than what is already a very stringent current limit. More sophisticated ways of pursuing stellar interferometry are necessary to test the cases of $\alpha > 1$ or systematic effects where the dispersion relation of equation (6) is modified by definite rather than randomly varying terms.

Nevertheless, the obvious test bed for quantum gravity has indeed been provided by this gigaparsec distance source; the outcome is negative. No doubt one anticipates interesting propositions on how time and space may have contrived to leave behind not a trace of their quanta. Thus, from Michelson-Morley to extragalactic interferometry there remains no direct experimental evidence of any sort that compels us to abandon the structureless, etherless spacetime advocated by Einstein. These points, together with in-depth discussions on how Planck-scale phenomenology affects the appearance of the extragalactic sky, will be the subject matter of a paper by Ragazzoni, Turatto, & Gaessler (2003).

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Note added in proof.—Since the time of writing this Letter, R. L. thanks Professor Gary Gibbons for in-depth discussions, from which the following point emerged. The possibility of time and space being subject to Planck-scale fluctuations may remain, but only with the requirement that these ordinarily separate dimensions are correlated microscopically. Upon close examination, however, it is difficult to see how the idea may be implemented. Any fluctuation in time that changes ω must occur in coordination with the fluctuation in space at *all* points within the telescope aperture, so as to ensure that across an entire wave front k is changed by the corresponding amount to preserve the exactness of $c = \omega/k$. Thus, for the scenario to work, correlations between Planck-scale effects have to occur over a macroscopic spacetime domain that coincides with that of the experiment.