Satellite Laser Ranging Precision Ultimate Limit

Lukas Kral, Ivan Prochazka, Josef Blazej, Karel Hamal
Czech Technical University in Prague, Brehova 7, 115 19 Prague 1, Czech Republic

ABSTRACT

We have estimated the contribution of atmospheric turbulence effects to the satellite laser ranging precision. This work was motivated by the observed discrepancy between the precision of laser ranging to short baseline ground targets and space born targets. The contribution of the atmosphere is expected to be the limiting factor to the satellite laser ranging precision on millimeter level. Two different atmospheric optical models were investigated. The geometry approach showed that at some situations the turbulence-induced random ranging error could reach the millimeter level, as observed in laser ranging experiment. This effect significantly decreases with the station's altitude above sea level and satellite altitude above horizon. The results depend on the value of the atmospheric outer scale parameter; its value is only approximate due to hardly predictable nature of the turbulence strength height profile. A novel experiment with high repetition rate satellite laser ranging is introduced, which should prove the turbulence contribution to the satellite laser ranging precision.

Keywords: laser ranging, atmospheric fluctuations, precision

1. INTRODUCTION

The ultimate goal of the Satellite Laser Ranging (SLR) is the millimeter precision and accuracy. To achieve this goal, all the individual contributors to the ranging error budget must be well below millimeter level. The best existing ground based ranging systems are capable to achieve millimeter ranging precision when ranging to short distance terrestrial targets. However, if ranging to Earth-orbiting satellites, the best precision obtained is typically 3 times worse, about 3 millimeters RMS. As this value is obtained even for satellite targets not spreading the echo signal, there is a speculation, that the remaining contribution to the random error budget is contributed from the atmosphere. The ground targets ranging experiments based on streak camera technology demonstrated an increase of the ranging jitter by 0.9 ps for 100 meters atmospheric path[1], thus supporting this idea.

The suspect effect is atmospheric turbulence – mixing of air of different temperatures, which causes random and rapidly changing fluctuations of air refractive index and hence unpredictable fluctuations from standard models of atmospheric range correction. We tried to estimate the atmospheric contribution to the ranging jitter using 1) an existing numerical modeling code (physical optics approach) and 2) an analytical model developed by C. S. Gardner (geometric optics approach).

2. PHYSICAL OPTICS APPROACH – NUMERICAL MODEL

We used the commercial version of the GLAD code[2,3,4]. GLAD is an extensive program for modeling of diffractive propagation of light through various media and optical devices. The light is considered to be monochromatic and coherent (or partially coherent). The electromagnetic field in GLAD is described by its two-dimensional transversal distribution. Two arrays of complex numbers (one for each polarization state) represent the intensity and phase at each point in x and y axis. The propagation is done by the angular spectrum method. That means the field distribution is decomposed into a summation of plane waves, these plane waves are propagated individually and then resumed into resulting distribution. A user specifies a starting distribution at first and then applies aberrations, apertures, etc., and finally performs diffractive propagation of the distribution to some distance. At the end, the resulting distribution can be analyzed.

I.P.: fax: +420 221912252; prochazk@cesnet.cz

Using GLAD, we developed a model of atmospheric light propagation according to recommendations in GLAD Theoretical Description[3]. It consists of alternating steps of random aberration and diffractive propagation applied to the initial plane wave (see Figure 2).

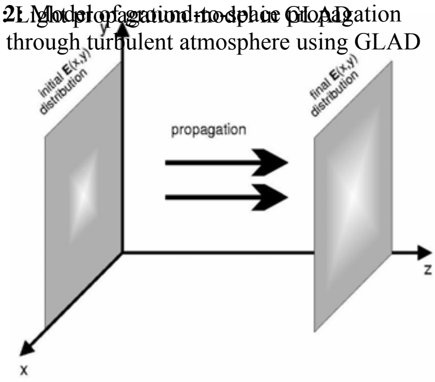


Figure 1: Light propagation model in GLAD

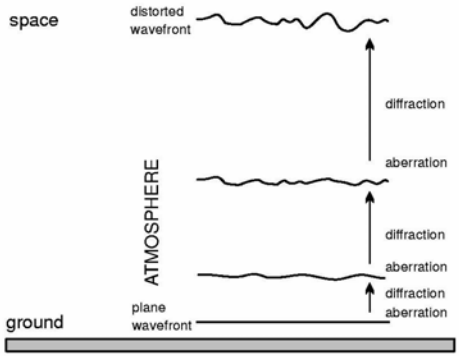


Figure 2: Model of ground-to-space propagation of light through turbulent atmosphere using GLAD

GLAD contains a command, which computes the random aberration caused by atmospheric turbulence. It is based on Lutomirski-Yura model of power spectrum of the aberrated wavefront:

$$W^2(\rho) = \frac{0.023 \cdot e^{-\rho^2 L_i^2}}{r_0^{5/3} \left(\rho^2 + \frac{1}{L_o^2}\right)^{11/6}} \qquad (1)$$

where ρ is spatial frequency, L_i inner scale and L_o outer scale size and r_0 is so-called Fried's parameter (to be explained later on).

Every step of application of atmospheric aberration to the propagated wavefront is characterized by the three parameters L_i, L_o and r_0. Their correct estimation is therefore a very important task. The random pathlength fluctuations are predominantly influenced by refractive index perturbations of low spatial frequencies[5] ($\rho \leq 2\pi/L_o$), therefore the results are practically very weakly dependent on L_i, which is the size of the smallest turbulent eddies (we used L_i = 1 mm). On the other hand, the value of L_o should impact the results strongly. Unfortunately, the values of L_o in higher parts of atmosphere are still not known well, thus we tried several different values from L_o = 10 m to L_o = 500 m and then compared the results.

The parameter r_0 is a measure of overall turbulence strength and is related with seeing. Its value for an atmospheric path between points x_0 and x_1 is given by[3]

$$r_0 = \left[\frac{16.5}{\lambda^2} \int_{x_0}^{x_1} C_n^2(x)dx\right]^{-3/5} \qquad (2)$$

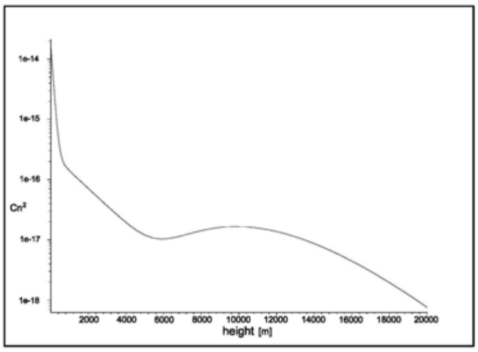


Figure 3: Hufnagel-Valley model of C_n^2 (the units at the vertical axis are $m^{-2/3}$)

where λ is wavelength and C_n^2 is refractive index structure constant, which is a local measure of turbulence strength. C_n^2 varies mainly with height above sea level, but also with time (different values during day and night) and is also influenced by weather and type of the terrain, but there still exist several models of its average height profile. For paths starting near the sea level, *Hufnagel-Valley model* should be the most suitable[6]:

$$C_n^2(h) = 3.59 \cdot 10^{-3} (h \cdot 10^{-5})^{10} \exp(-h/1000) + 2.7 \cdot 10^{-16} \exp(-h/1500) +$$

where h is height above sea level and A is commonly set to $1.7 \cdot 10^{-14}$.

At the end of the GLAD computation, we obtained a set of randomly aberrated wavefronts (arrays of relative phase shifts) corresponding to individual passes through the atmosphere. We performed statistical analysis by an external program to get a histogram and RMS of these relative phase shifts $\Delta\varphi$, which were at first converted to pathlength fluctuations by $\Delta s = (\Delta\varphi / 2\pi)\cdot\lambda$.

After many attempts with different input parameters (r_0, L_o), it seems, that this model gives always pathlength RMS only several micrometers, i.e. negligible. What is even more surprising, the computed pathlength RMS does not significantly increase with L_o, as was expected from theory, although the wavefront size was always selected large enough ($10\cdot L_o$) to model even the lowest-frequency aberrations. Therefore *we have found this model not well describing the satellite laser ranging signal delay although the far field intensity profile has been modeled correctly.* The origin of the problem has not been identified. The GLAD atmospheric model and its results correspond well to the "adaptive optics problem"; the corrections applied in adaptive optics are of the order of micrometers, just the values predicted by the model.

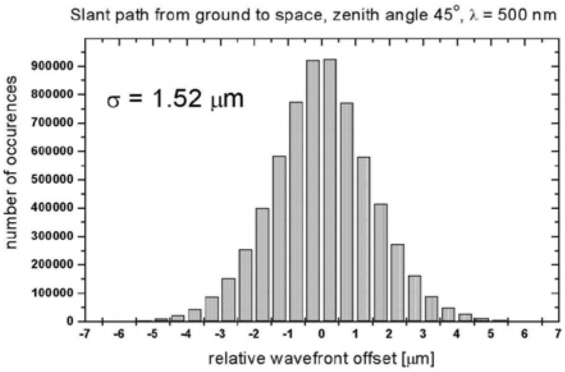


Figure 4: Example of result of the GLAD model for a slant ground-to-space propagation path

3. GEOMETRIC OPTICS APPROACH – ANALYTICAL MODEL

In 1976, Gardner[5] derived analytical formulae that allow us to predict the turbulence-induced random pathlength fluctuations, directly for the case of satellite laser ranging. He also computed some concrete results and predicted that the RMS path deviations could reach millimeters, and at some extreme situations even several centimeters. However, Gardner used a very rough model of C_n^2 height dependence, which resulted in larger values of C_n^2 than are recently observed.

We evaluated the Gardner's formulae using the recent model of C_n^2 height profile. Gardner derives the formulae for the mean-square path deviation using geometric optics approach. He considers von Karman or Greenwood-Tarazano model of the turbulence spatial spectrum and he also assumes that the air refractive index perturbations are completely statistically independent for individual laser shots (i.e. low repetition rate or rapidly changing atmosphere). For the Greenwood-Tarazano spectral model, which should be more realistic[5], the resulting Gardner's formula leads to:

$$< \Delta L^2 > = 26.31\, C_n^2(0)\, L_o^{5/3}\, L_e \qquad (4)$$

where $< \Delta L^2 >$ is the mean square pathlength deviation caused by atmospheric turbulence, $C_n^2(0)$ is refractive index structure constant (local turbulence strength) at initial point of the path (at the laser station), L_o is outer scale and L_e is effective pathlength, given by

$$L_e = \frac{1}{C_n^2(0)} \int_0^L C_n^2(\xi) d\xi \qquad (5)$$

where L is distance from the ranging site to the target.

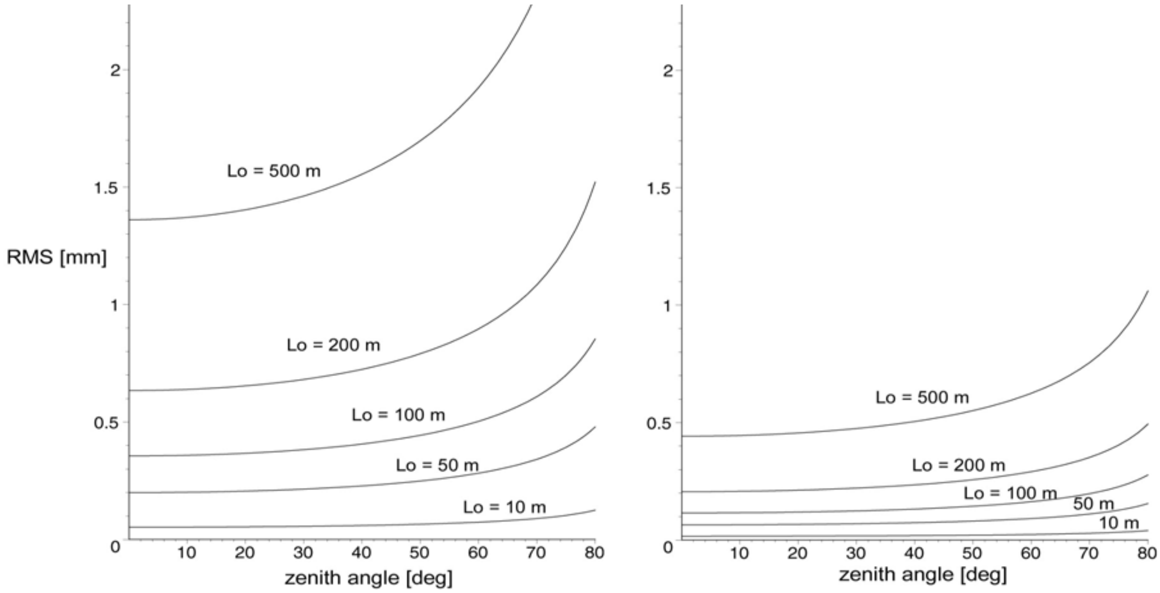
To obtain the ranging jitter RMS, we have simply to compute the square root from $< \Delta L^2 >$ from Eq. 4. For horizontal paths to a ground target, $C_n^2(\xi) = C_n^2(0)$ is constant in Eq. 5, then $L_e = L$. It means that for horizontal paths the turbulence-induced ranging jitter should be proportional to square root of the target distance. Considering typical near-ground values $L_o = 10$ m and $C_n^2 = 1\cdot10^{-13}$ $m^{-2/3}$, we obtain ranging jitter of 0.1 mm RMS (~ 0.7 ps) for 100 meters of atmospheric path, in perfect agreement with experimental results by Hamal et al.[1] Another experiment was done by Hamal et al. at SLR station Graz[7]. Overall ranging machine random error (laser, timing system, start and stop devices) was estimated to 7 ps RMS, and measurement to a 6-km distant ground target yielded 10–14 ps RMS. For this situation (and previously used values of L_o and C_n^2), Eq. 4 gives 0.9 mm (~6 ps) RMS of atmospheric contribution.

Summarization of these two independent sources of deviations (sqrt($7^2 + 6^2$) = 9 ps) results in a value close to the measured one.

For ground-to-space paths, we have to select some model of C_n^2 height dependence and compute the effective pathlength from Eq. 5 by its integration. For SLR stations located near the sea level, we used the Hufnagel-Valley model (Eq. 3). Figure 5 shows the computed results for two different altitudes above sea level and for various values of L_o. It can be seen, that in the case of low-located station, for large zenith angles and for large values of L_o, pathlength RMS reaches the order of millimeters.

In the case of laser station located 2000 m above sea level (Figure 5), we should consider, if the Hufnagel-Valley (H-V) model of turbulence can be used, as well. The problem is that the H-V model assumes that the Earth's surface is located at the sea level, which is of course not true in the case of a station located at mountains. Using the H-V model in such situation means omitting the surface boundary layer of strong turbulence, which can normally contribute about 30% to the value of the integral in Eq. 5 [8]. Therefore we can expect that the pathlength RMS computed using the H-V model in this case will be about 30% less than the real value. We also made an independent check of H-V model applicability for mountain observatories: we know an average value of r_0 measured 2332 m high at Canarian Islands, which is 15 cm. Integration of the H-V model (by Eq. 2) for this case gives r_0 = 19 cm as expected, the turbulence predicted by the H-V model is slightly weaker than the real one, but the difference is not significant. The conclusion is, that in the case of SLR station located high in mountains the Hufnagel-Valley model can be still used, but we should note, that the computed RMS will be slightly (~30%) smaller than the real one.

Figure 5: Turbulence-induced random ranging error as function of satellite zenith angle – results of the Gardner's / H-V model for the laser station located at the sea level and at the elevation of 2000 meters above the sea level



4. PROPOSED SATELLITE LASER RANGING EXPERIMENT

A new satellite laser ranging experiments are planned to be carried out at the laser station in Graz, Austria[9] in autumn 2003. The laser ranging calibration to the terrestrial targets at the distances of 1 and 6000 meters, horizontal path, will be completed under various atmospheric and seeing conditions. It is expected that the validity of the Equation 4 will be verified with the precision and accuracy on the picosecond level. Additionally, the returned optical signal direction fluctuations will be estimated, what will enable the determination the correlation between the local seeing conditions and atmospheric turbulence contribution to the laser ranging jitter.

The satellite laser ranging will be performed at a high repetition rate 100–400 Hz, more than 1 order higher that repetition rates used till now. Ranging at high repetition rate, the satellite displacement between the consequent laser shots will be minimal and also the atmospheric turbulent structures should not change significantly from shot to shot. It has been measured for the purposes of adaptive optics that the coherence time of seeing is several milliseconds[10]. Hence, if measuring at repetition rate close to 500 Hz, it might be observed, that the turbulence-induced fluctuations of the measured range are not statistically independent from shot to shot. That means some sort of "waves" might appear at the graphs of the measured range versus time[11]. Our numerical calculations following the Gardner approach are predicting significant correlation for the consequent laser ranging shots for both low and high repetition rate systems. Till now, no such correlation has been observed for the repetition rates 1-10 Hz. However, the model available is based on a static model of atmospheric inhomogenities, the only path length difference is induced by the angular motion of the satellite. If SLR data correlation will be observed using a high repetition rate system, the time evolution of the atmospheric turbulence might be estimated.

5. CONCLUSION

The physical optics model using GLAD code gives rather low values, several micrometers only, of the turbulence-induced pathlength RMS, independently on the input parameters. We have found this model not well describing the satellite laser ranging signal delay in particular. Promising results were obtained from Gardner's analytical model (geometric optics) with Hufnagel-Valley model of turbulence height profile implemented: the atmospheric contribution to the SLR jitter can be of the order of millimeters at some situations according to this model. The turbulence-induced random ranging error should significantly decrease with the laser station's altitude above sea level. However, these results are strongly dependent on the value of atmospheric outer scale L_o (unlike the problems of adaptive optics etc., only weakly dependent on L_o), which is still not known precisely. Additionally, the substituted height profile of turbulence strength is only average – in reality it can vary with time, terrain type, season etc. As a result, the computed values are only a rough estimate of average values. The proposed experiments with high precision and high repetition rate SLR might verify the applicability of the geometric optics approach and consequently verify that the contribution of the atmosphere is the limiting factor to the satellite laser ranging precision on millimeter level.

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