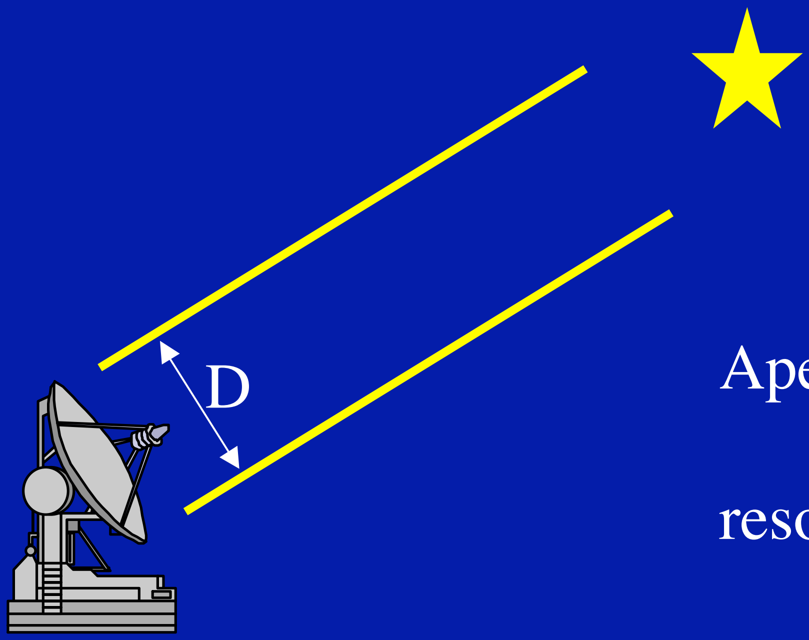
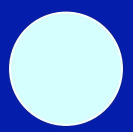
Optical interferometry

Astrometry and image formation with optical interferometers

The basics - I

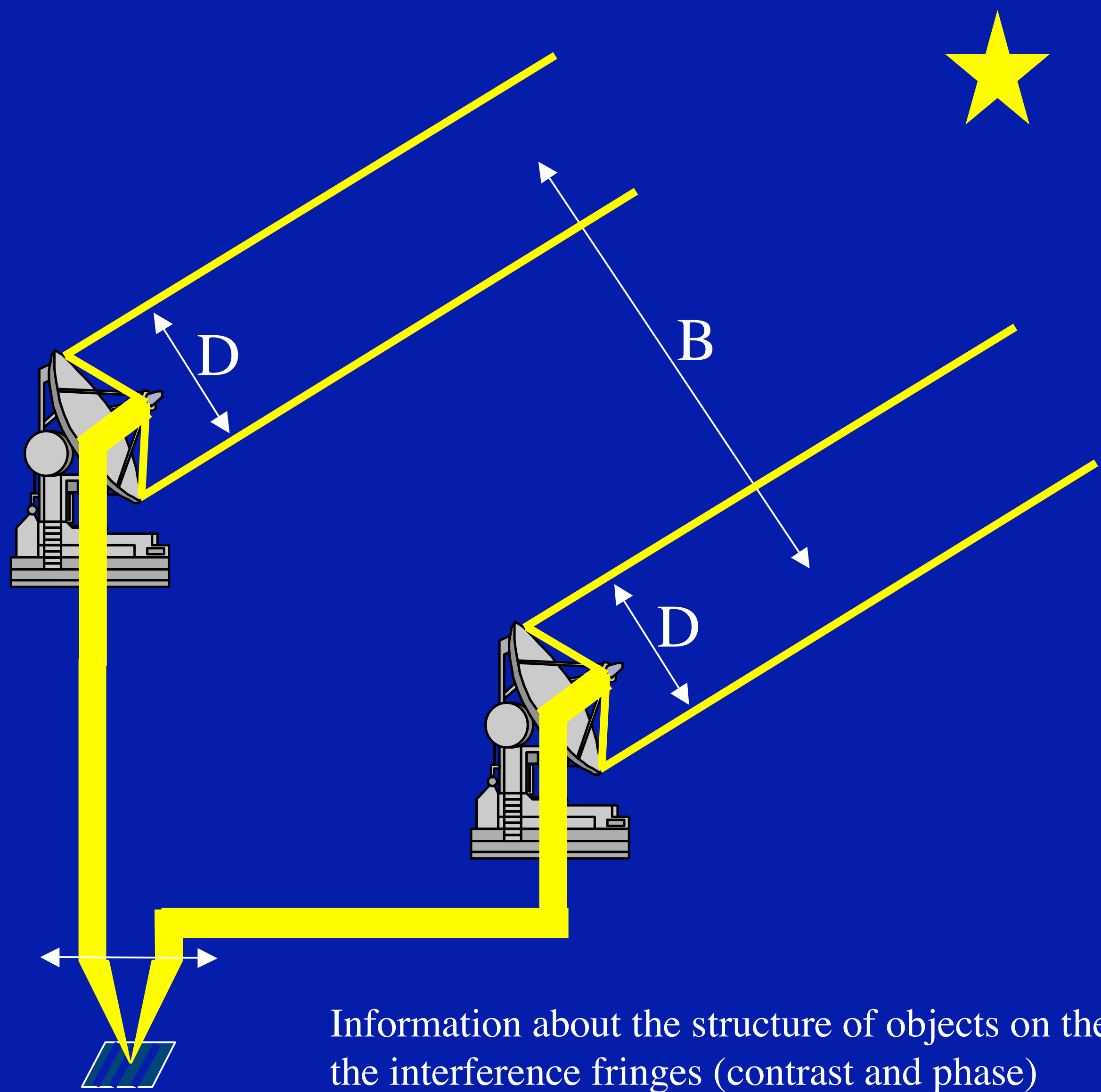


Aperture: 

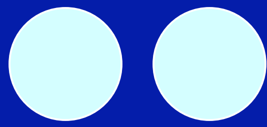
resolution : λ/D

sensitivity : $\sim D^2$

The basics - II



Aperture:

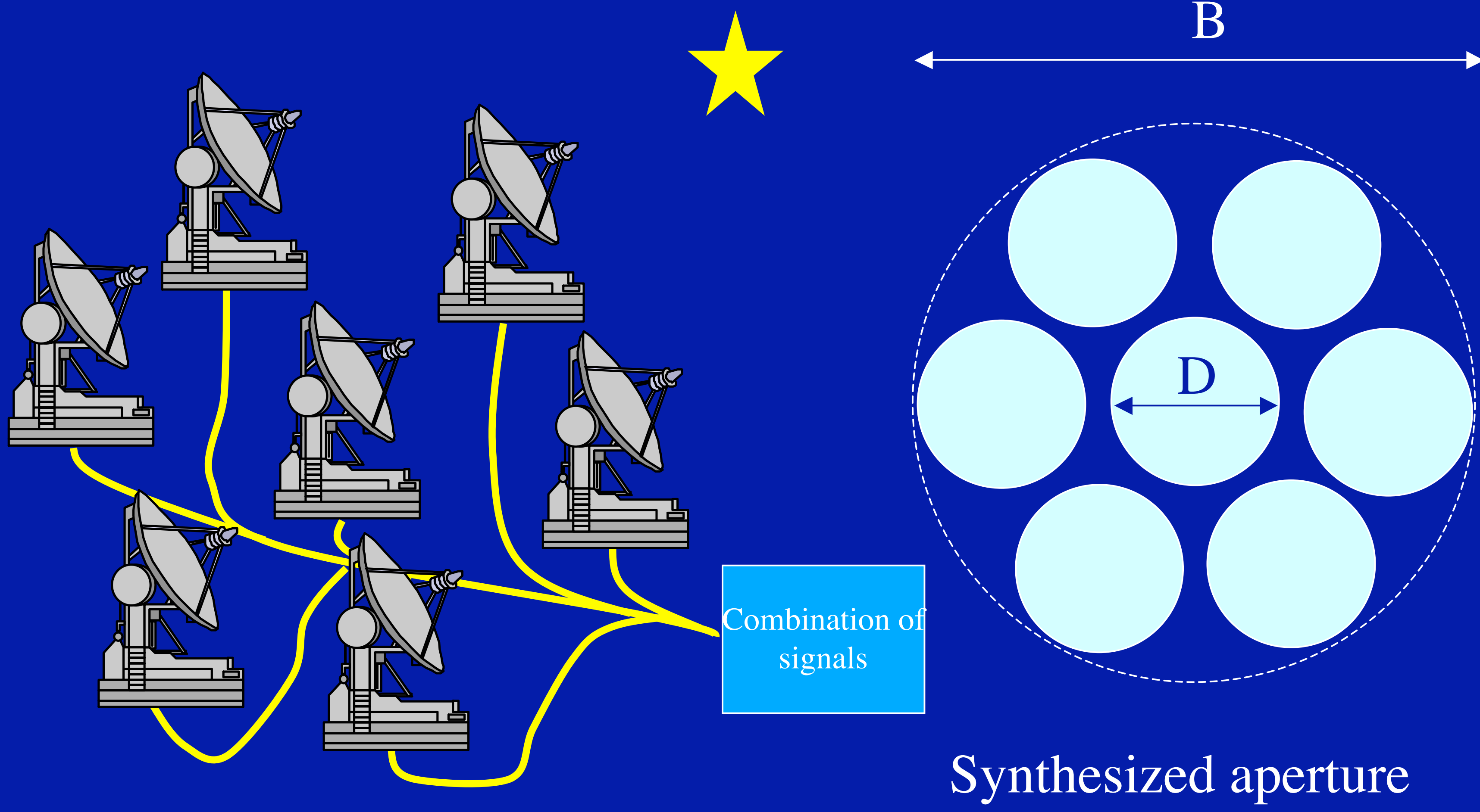


resolution : λ/B

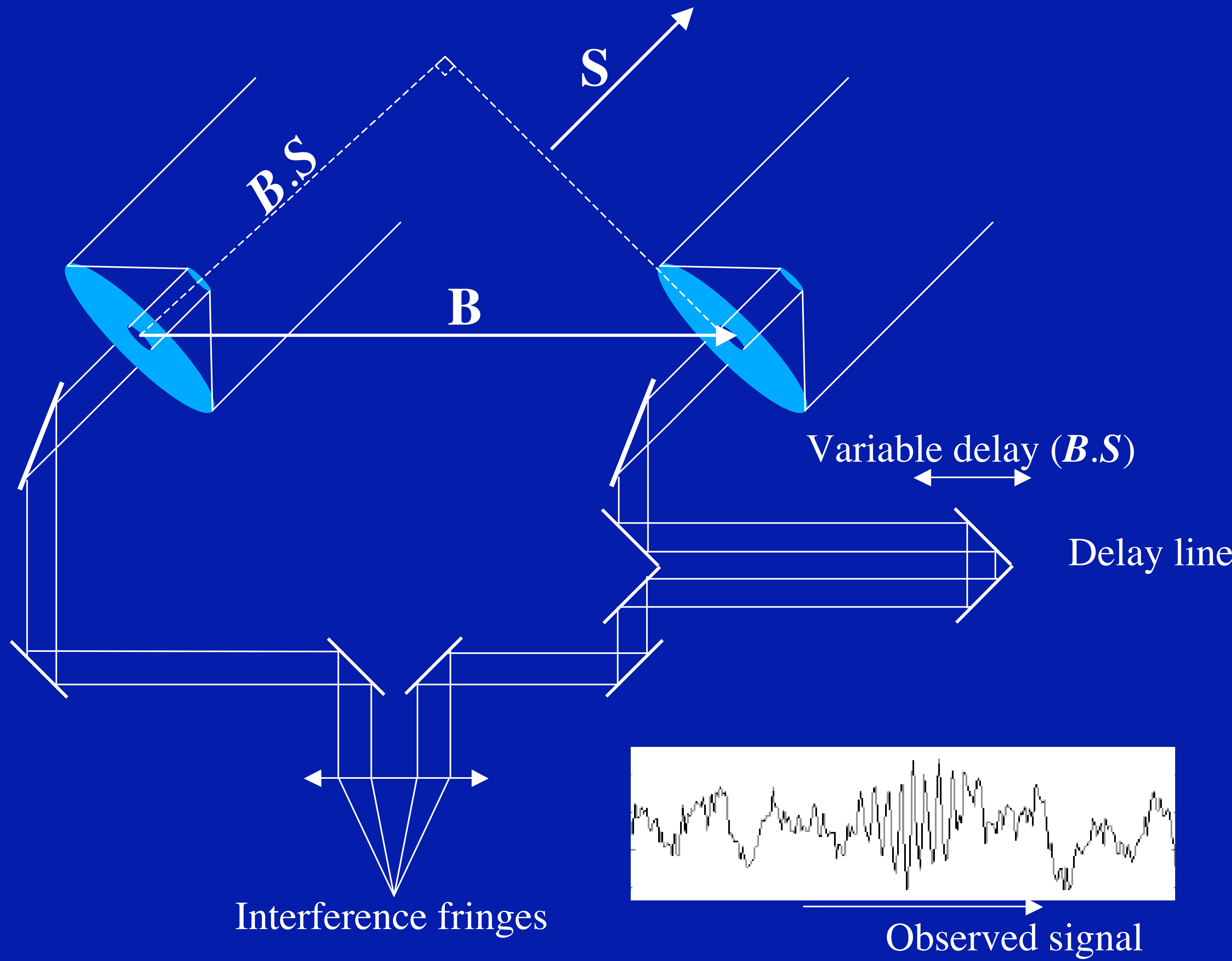
sensitivity : $\sim D^2$

Information about the structure of objects on the sky is contained in the interference fringes (contrast and phase)

Aperture synthesis and image formation



A practical interferometer



What does the interferometer do?

An interferometer measures interference fringes which contain information on the structure and position of the object. Its functions are:

- ✓ Recording of the signal (observation of an object located at the direction **S** with telescopes separated by baseline **B**)
- ✓ Optical path matching (compensation of the geometrical delay with the delay lines $\mathbf{d_1, d_2}$)
- ✓ Addition of signals (electric fields) from interferometer elements (telescopes)
- ✓ Fringe detection, measurement of contrast and phase

Signal processing in a 2-element interferometer

Electric fields received by the two apertures:

$$E_1 = Ae^{ik(s \bullet B + d_1)} e^{-i\omega t}$$

$$E_2 = Ae^{ikd_2} e^{-i\omega t}$$

Time-averaged intensity after summation:

$$\langle EE^* \rangle = \langle (E_1 + E_2)(E_1 + E_2)^* \rangle \propto 2 + 2\cos(kD)$$

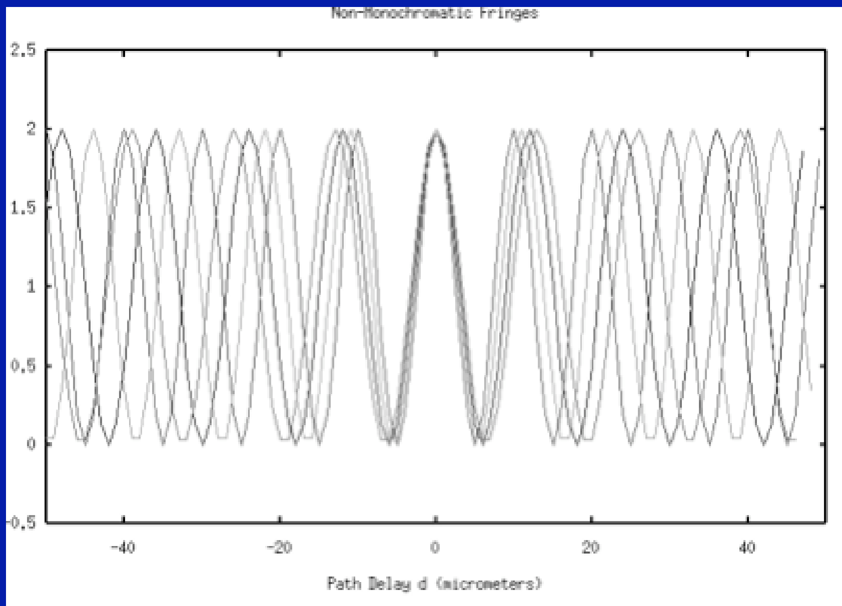
where:

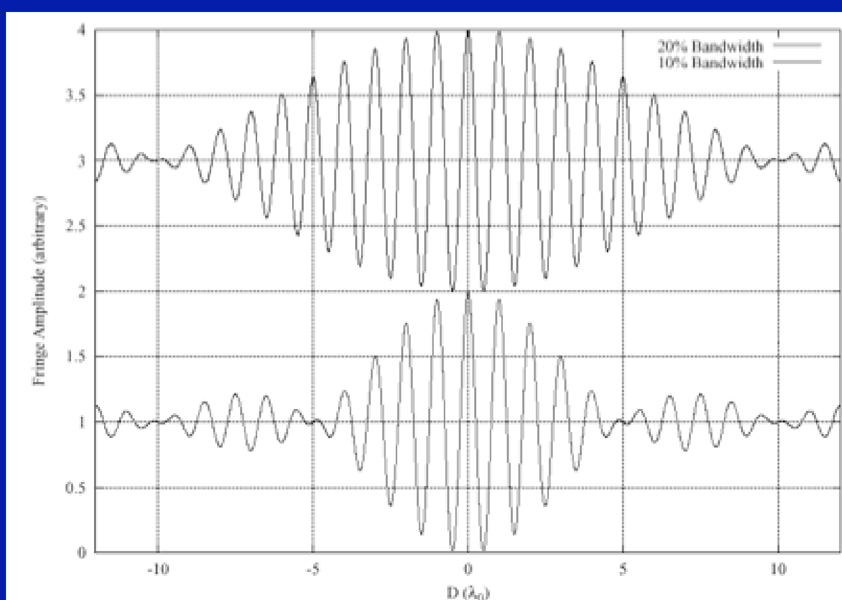
$$D = s \bullet B + d_1 - d_2$$

$$k = \frac{2\pi}{\lambda}$$

Cosinusoidal interference fringes are a function of direction of the signal, baseline, and the net geometrical delay in the signal path

Interference of a broadband signal





If the instrument accepts a range of wavelengths $\Delta\lambda$, fringes get washed out

$$P \propto \int_{\lambda_0-\Delta\lambda/2}^{\lambda_0+\Delta\lambda/2} [2 + 2\cos(kD)]\, d\lambda$$

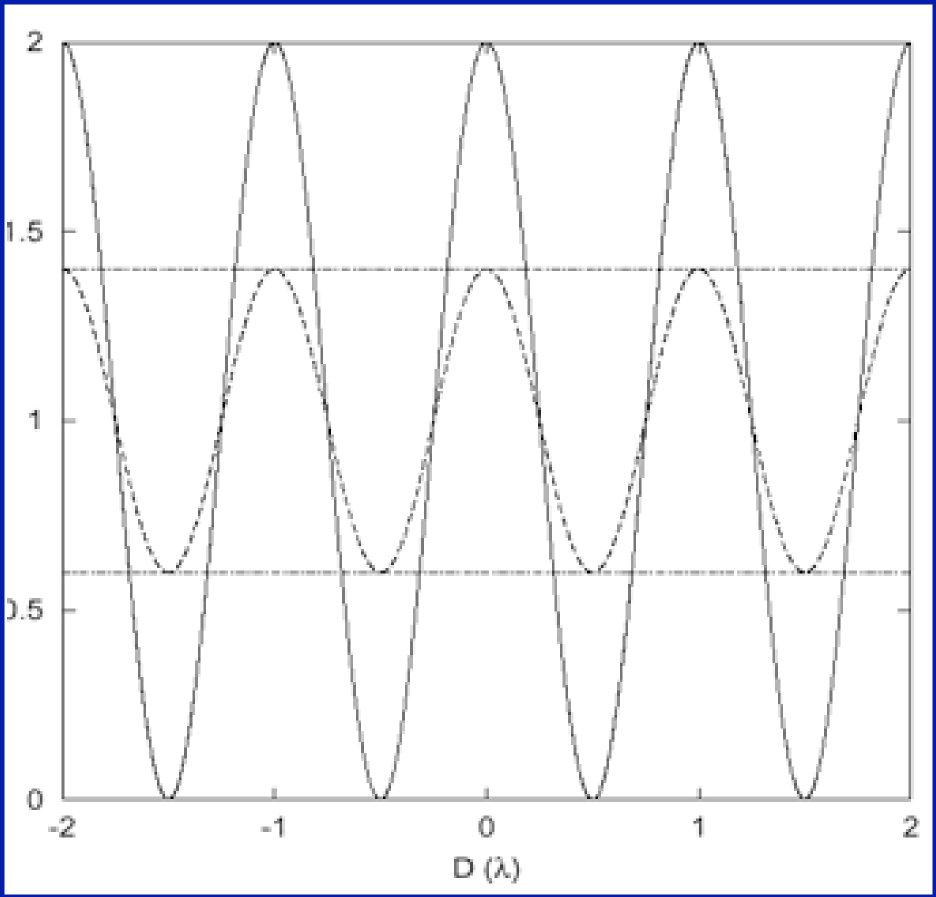
Evaluation of this integral gives:

$$= \Delta\lambda \left[1 + \frac{\sin \pi D \Delta\lambda / \lambda^2_0}{\pi D \Delta\lambda / \lambda^2_0} \cos k_0 D \right]$$

$$= \Delta\lambda \left[1 + \frac{\sin \pi D / \Lambda_{coh}}{\pi D / \Lambda_{coh}} \cos k_0 D \right]$$

Meaning that fringes are modulated by an envelope of the form sin*x/x* with a characteristic width
$\Lambda_{coh}=\lambda_o^2/\Delta\lambda$ (signal coherence)

Parameters of interference fringes



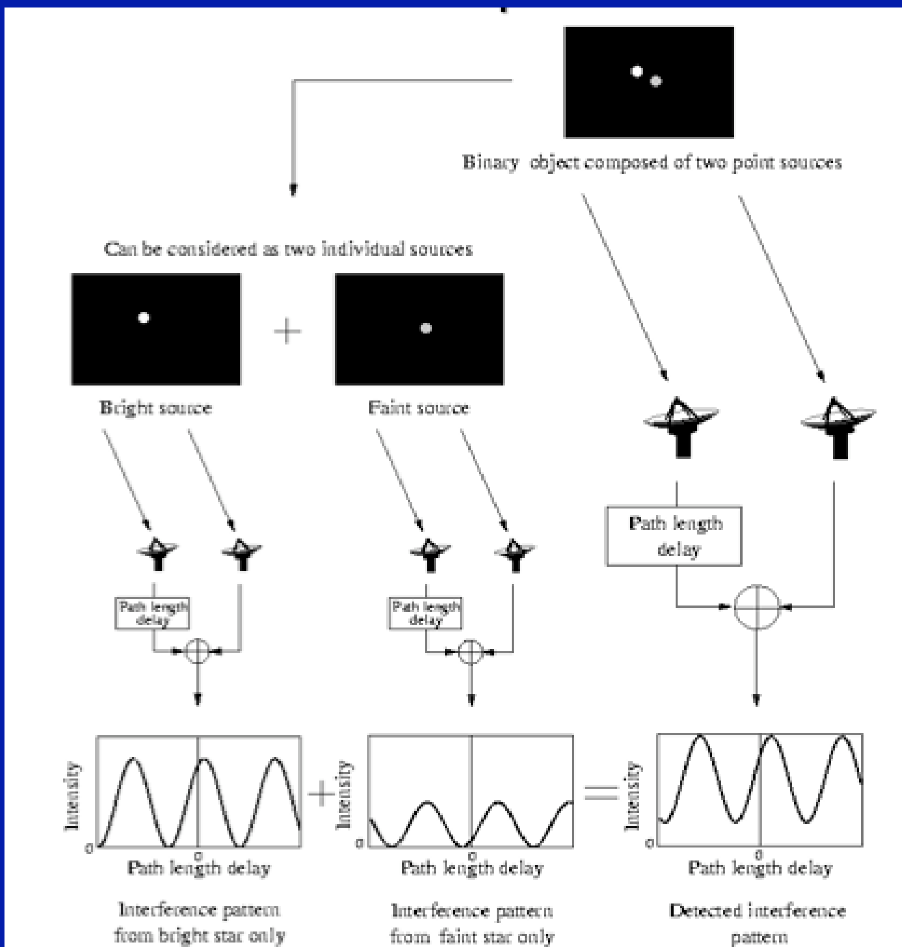
- Fringe measurement requires their stabilization: $k(s^*B+d_1-d_2)=0$
- Measured parameters: fringe contrast and phase

Fringe visibility function:

$$V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

Fringe phase: fringe location with respect to a reference phase

Double stars and extended sources



- Two separately observed stars are point sources with a deep modulation of fringe amplitude
- The same stars observed together give a lower amplitude modulation because fringes do not add together at the same phase and amplitude
- Locations of the two stars are encoded in fringe phases
- Generally, an extended source is treated as a sum of interference patterns, each with a different amplitude and phase
- Structure of the source is encoded in the amplitude and phase of the fringes

Visibility function of interference fringes

Consider an extended source, whose intensity is described by I(s). Let's write it as $I(s_o+\Delta s)$, where Δs is a vector perpendicular to line of sight s_o. Then, the power of the received signal is:

$$P(s_0,B) \propto \int I(s)(1+\cos kD)\,d\Omega \propto \int I(\Delta s)[1+\cos k(\Delta s \bullet B)]\,d\Omega'$$

Define the complex visibility function as:

$$V(k,B) = \int I(\Delta s)e^{-ik\Delta s \bullet B}\,d\Omega'$$

It can be definead in terms of angular coordinates on the sky, α,β and coordinates $u=B_x/\lambda$ i $v=B_y/\lambda$, which are baseline projections on the plane of the sky called spatial frequencies:

$$V(k,B) = \int I(\alpha,\beta)e^{[-i2\pi(\alpha u+\beta v)]}\,d\alpha d\beta$$

Van Cittert-Zernike theorem

- The above equation says that the complex visibility function is a Fourier transform of the brightness distribution of the source in the sky
- An interferometer measures: $P(s_o,B,\delta)=I_{tot}+Re[V\exp(-ik\delta)]$, where δ is a phase shift
- Real and imaginary components of V are obtained by meaasuring its V with $\delta=0$ i $\delta=\lambda/4$

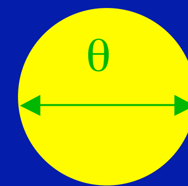
Van Cittert-Zernike theorem:

The output signal of an interferometer is a Fourier transform of the observed brightness distribution of a source on the sky

An example of visibility function

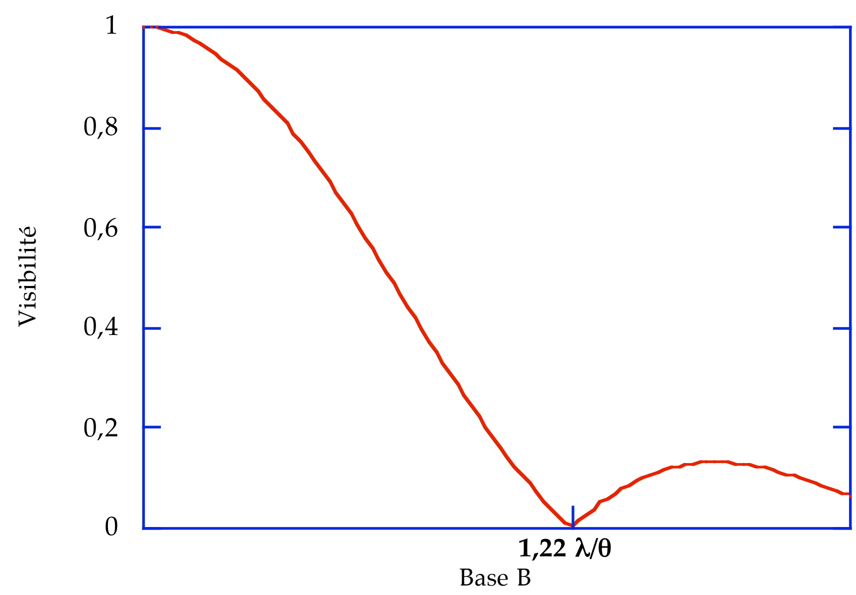
Observed source:
an uniformly illuminated disc

$$I(\vec{S}) = \Pi(\frac{S}{\theta})$$



A corresponding visibility function:

$$V(\vec{B}) = \frac{2J_1\left(\frac{\pi\theta B}{\lambda}\right)}{\frac{\pi\theta B}{\lambda}}$$



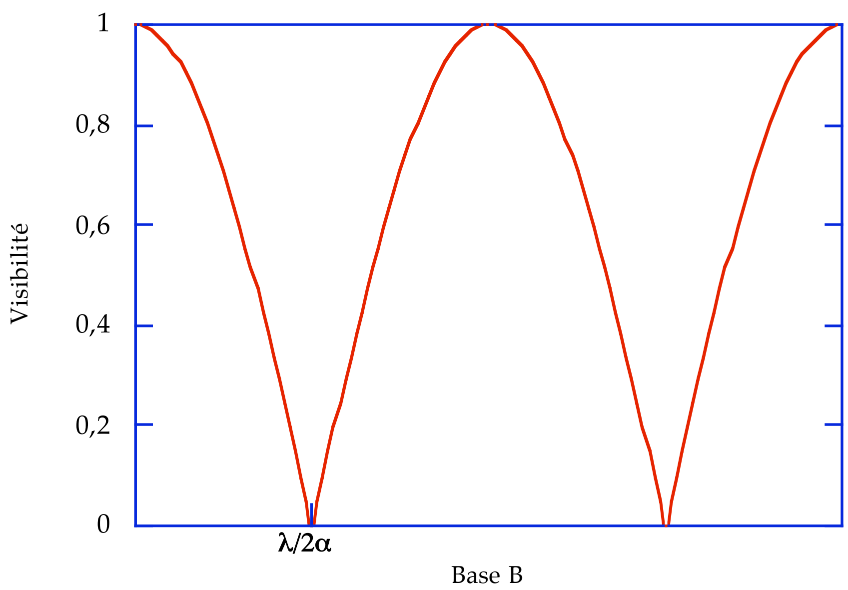
One more example…

Binary system
separated by α

$$I_t(\vec{S}) = \delta\left(\vec{S} - \frac{1}{2}\vec{\alpha}(t)\right) + \delta\left(\vec{S} + \frac{1}{2}\vec{\alpha}(t)\right)$$

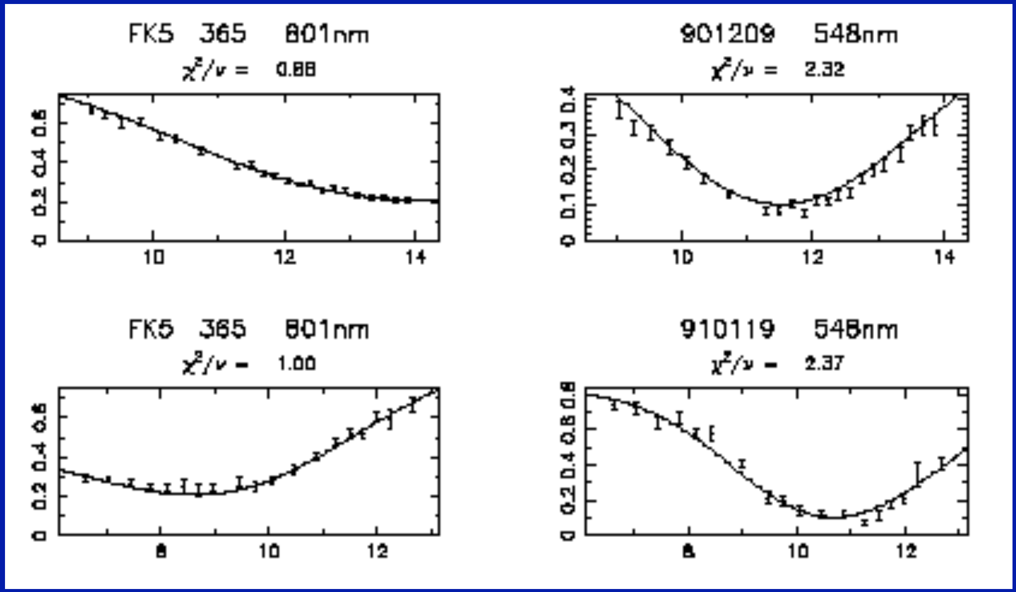
…and its visibility function

$$V_t(\vec{B}) = \cos\left(\frac{\pi\vec{\alpha}(t).\vec{B}}{\lambda}\right)$$

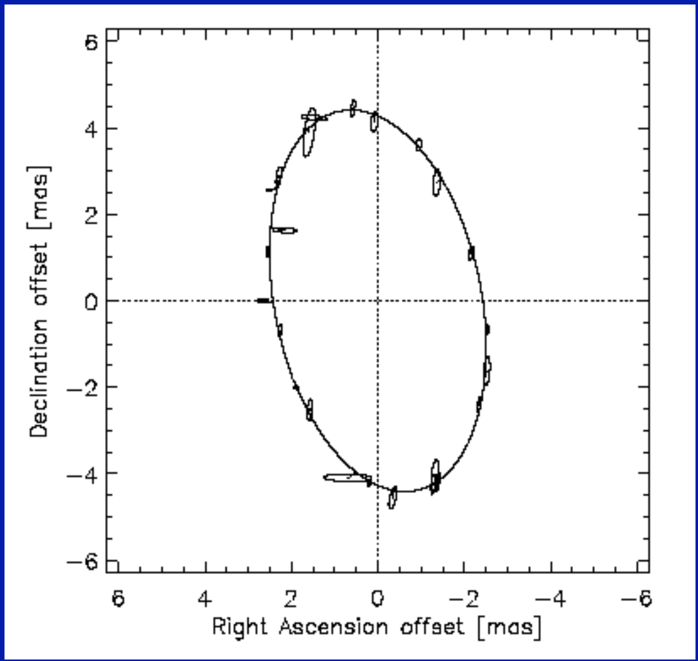


Orbit of a binary star
(Hummel et al. 2001: ο Leo)

Fringe visibility

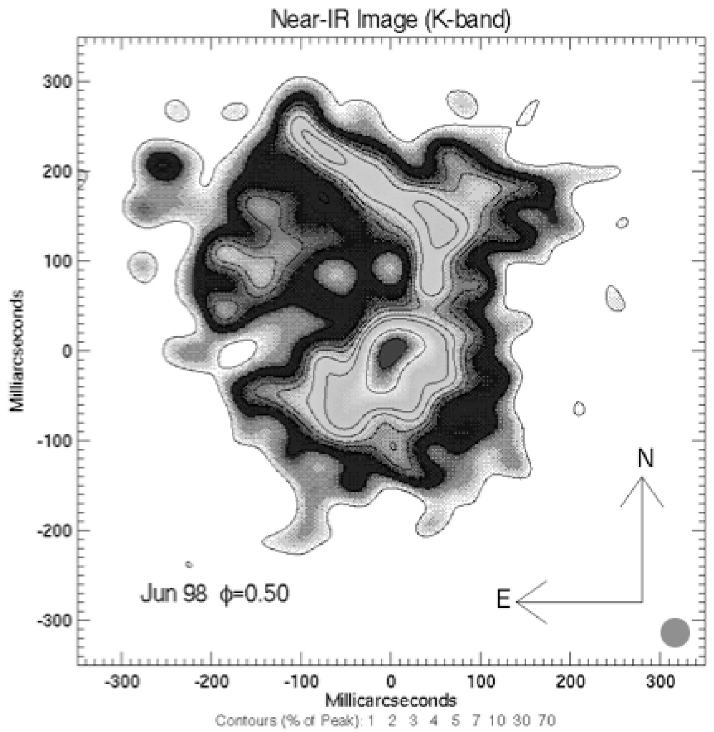


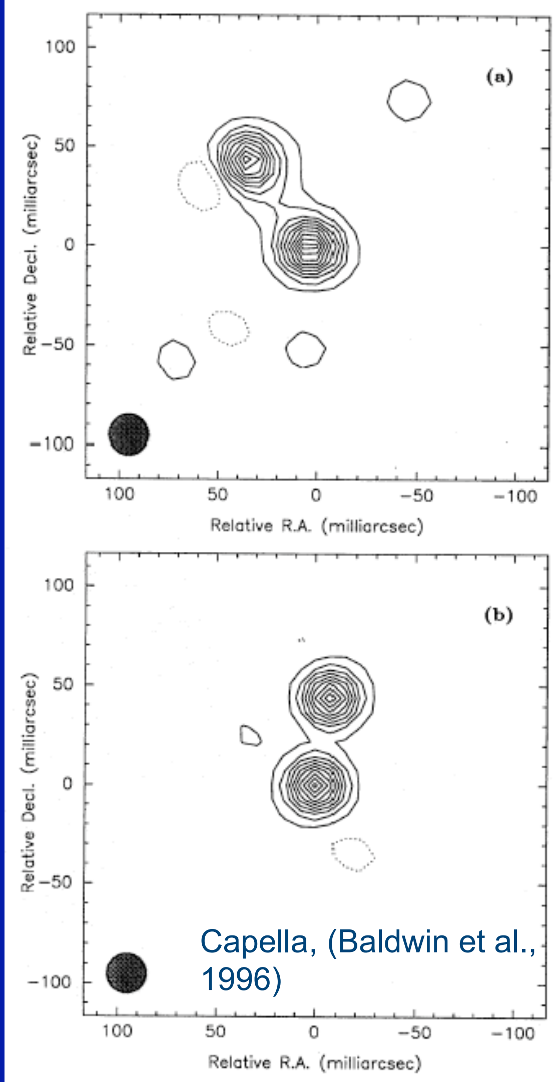
Orbit



Examples of extended sources

K-band image of IRC+10216. Image courtesy of Peter Tuthill and John Monnier.





Capella, (Baldwin et al., 1996)

Homework #1,2

Homework #1

- Go to the course webpage, download the data set stored under the “Data” link. The data are a series of radial velocity measurements of a star (col. 2), taken over ~200 days (col. 1)
- Using your favorite method (eyeballing, folding at trial periods, periodogram analysis, etc.) determine a period of radial velocity variation in this data set. Write up a one-page report on your work including: (i) a description of the method you have used to measure the period, (ii) the value you have come up with and, (iii) a graph showing the data folded modulo the period.

Homework #2

- Search the internet for reports on the recently discovered Neptune-mass planets using the radial velocity method. Summarize these discoveries in a short paper (max. 4 pages incl. figures, single, 1.5, or double spaced, 11pt font or larger, include references)
- Both homeworks are due on Feb 14. You can form teams up to 3 students do it! All team members will get the same grade for their work.