

Michelson Interferometer

Purpose

Interferometers are basic optical tools used to precisely measure wavelength, distance, index of refraction, and temporal coherence of optical beams. We will construct a Michelson interferometer, study the fringe patterns resulting from both a point source and a parallel beam, and make a precise measurement of the wavelength of the He-Ne laser.

Outline of the Experiment

1. Preliminary
 - Identify the new optical and mechanical components you will use.
 - Be sure that you understand the precautions to be taken with mirrors and beam splitters.
2. Set up the Michelson interferometer
 - Mount the laser on the optical table with the beam parallel to the table surface at a height of 6-1/4".
 - The moveable mirror will be the one in the direct path of the laser beam. Mount this mirror on the translation stage.
 - Adjust the optical paths to be equal to a few millimeters, with about 6 cm from the beam splitter to each mirror.
 - Align the mirrors using the laser beam.
3. Measure the wavelength of the He-Ne laser.
 - Try to achieve an accuracy of one part in 10^4 .
 - Set up the photodiode to measure the intensity of a fringe of equal thickness. Use a lens to magnify the fringe.
 - Translate the moveable mirror at a slow and steady rate. Count the number of fringes that pass the photodiode using a Schmitt trigger and an electronic counter. Count about 10,000 fringes.
 - Calculate the wavelength from $\lambda = 2\Delta d/n$, where Δd is the change in position that causes n fringes to pass the photodiode.

References

1. Welford Sections 6.1 and 6.2
2. Heavens and Ditchburn, Section 4.15 and Chapter 17.

Problems

- Design of the interferometer for circular fringes. Refer to Figure 5.6. Let $d_1=d_2=6$ cm, $d_3=10$ cm, and $d_E=30$ cm. Assume that the diameter of the reflected beams are restricted by a 1 cm diameter hole at the beam splitter.

 - What is the maximum illuminated diameter on the screen, E? Draw to scale the actual system you will use showing the beam envelope at each point in the system.
 - For a 1 mm diameter laser beam, calculate the focal length of the lens L1 that illuminates an area on screen, E, of the size you obtained in part a).
 - For circular fringes obtained with a point source, S, at the focus of L1, calculate the diameter of the inner three fringes for distances $\Delta d = (d_1 - d_2)$ of 10, 100, and 1000 microns. Assume in each case that the center of the screen has the maximum.
 - Suppose that the mirror, M2, is tilted as shown in Figure 5.1. Where on the screen would the center of the displaced circular fringes lie? Assume that $\Delta d = 2\text{mm}$.
- Design the interferometer for fringes of equal thickness. You will need to expand the laser beam using L1 and another lens, L2 (see Figure 5.4).

 - What focal length should you use for L2 to produce a 1 cm diameter beam?
 - Suppose that the mirror, M2 is tilted as shown in Figure 5.1. Calculate the fringe spacing that will be observed on the screen.
- Measurement of the wavelength. Suppose that you determine the wavelength of an argon ion laser using the interferometer set to display parallel fringes as in Figure 5.8. A photodiode is placed at the center of the screen and its output is connected to an electronic counter. When the moving mirror, M1, is driven a distance of 5.120 mm, a total of 19,368 fringes is counted. What is the wavelength of the light from this laser?

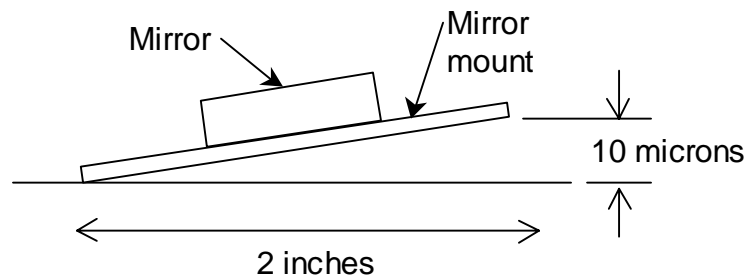


Figure 5.1

The Michelson Interferometer

With an optical interferometer you can measure physical distances directly in terms of wavelengths of light by counting interference fringes that move when one or the other of two objects are displaced. The beams must be mutually coherent for fringes to be seen. There must be a definite phase relationship between them. Mutual coherence is obtained in the Michelson interferometer by splitting light that originates from a single source with a partially reflecting mirror known as a beam splitter (BS). The reflected (R) and transmitted (T) waves are redirected by ordinary mirrors to the output where they are superposed to form fringes. This process is known as interference by division of amplitude.

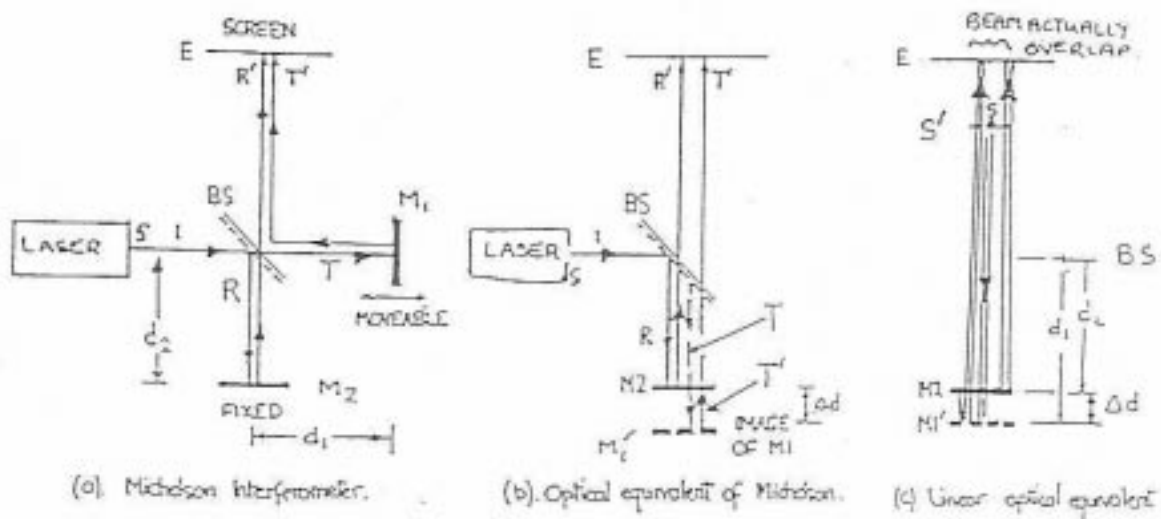


Figure 5.2. Michelson Interferometer.

In the basic Michelson interferometer (Figure 5.2 (a)) the light from a source, S , is divided by a 50% beam splitter oriented at 45 degrees to the beam. The transmitted beam travels to mirror $M1$ where it is back reflected to BS. 50% of the returning beam is deflected by 90 degrees at BS and it then strikes the screen, E (the other 50% is transmitted back towards the laser and is of no further interest here). The reflected beam travels to mirror $M2$ where it is reflected. Again, 50% of it then passes straight through BS and reaches the screen (the remaining 50% is reflected towards the laser and is again of no further interest here). The two beams that are directed towards the screen, E , interfere to produce fringes on the screen.

A linear optical equivalent of the Michelson helps to understand the optical path differences. First, replace mirror $M1$ with its virtual image, $M1'$ seen when looking into the beam splitter from the laser. The laser aperture, S , is then replaced by its virtual image, S' , as seen looking into

BS from the position of M2. The fringes are the same as would be generated by a single source, S' , that is reflected from two distinct mirrors, M2 and M1', separated by a distance Δd .

Suppose that mirror M1 is moveable. The optical path difference is $2\Delta d$ between the rays reflected from M1' and M2. The center of the screen is bright when the optical path difference is an integral number of wavelengths. The condition for a maximum of intensity at the center of the screen is:

$$2\Delta d = n\lambda$$

FRINGE FORMATION:

To understand the fringe pattern away from the center of the screen we must distinguish between two cases. When the light source, S , is a point source, we will see circular fringes on the screen. If, on the other hand, we feed the interferometer with a parallel beam (either the original laser beam or an expanded beam) then we will observe fringes of equal thickness. Before discussing these two cases, we review the behavior of mirrors and beam splitters.

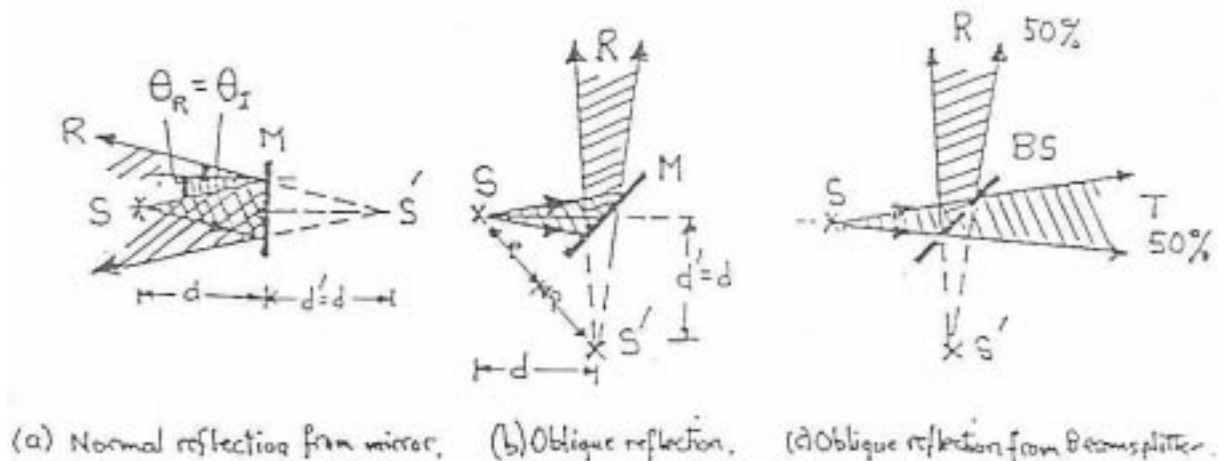


Figure 5.3. Mirrors and Beam Splitters.

In Fig. 5.3 (a), rays from a point source, S , that are reflected from the plane mirror, M , appear to diverge from the virtual image, S' , that is as far behind the mirror as the real source is in front. The mirror surface is the perpendicular bisector of the line, SS' .

The same rule applies to oblique reflection (b) where the mirror is extended to intersect SS' . In the beam splitter BS the transmitted beam, T , diverges from the original source, S , while the reflected beam, R , appears to diverge from the virtual source, S' . There is a fixed and constant phase relation between the waves R and T since they both are derived from the single real source, S . Waves R and T are mutually coherent.

The geometry for fringes of equal thickness is shown in Figs. 5.4 and 5.5. A parallel beam from the laser or beam expander is fed to the interferometer. In Fig. 5.4 (b) a plane wave is incident on the mirrors M2 and M1'. There are two reflected waves with an optical path difference of $2\Delta d$. A point on the screen E will be bright if $2\Delta d = n\lambda$, and dark if $2\Delta d = (n + 1/2)\lambda$. If the two mirrors are precisely parallel, then the whole illuminated area of the screen will be uniformly lit. If M2 is tilted at an angle $\theta/2$, then straight fringes will appear parallel to the axis of tilt.

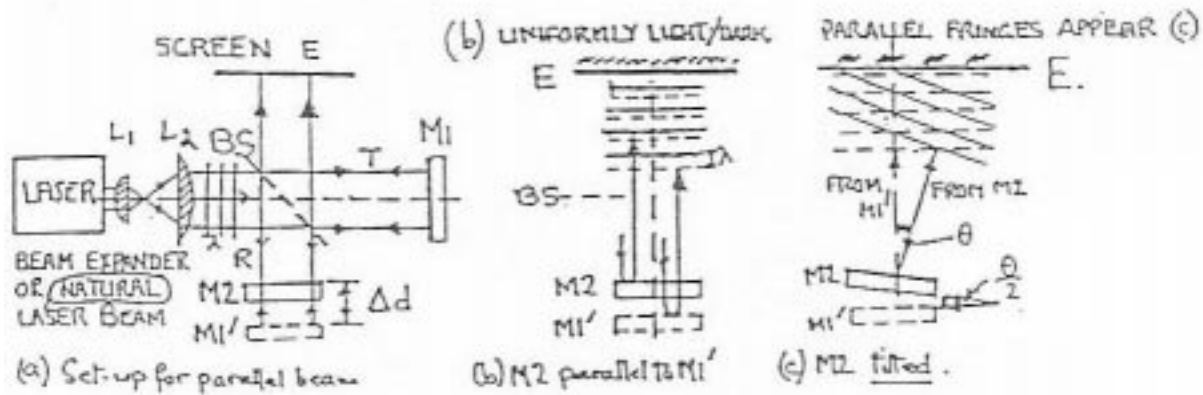


Figure 5.4. Generation of fringes of equal thickness.

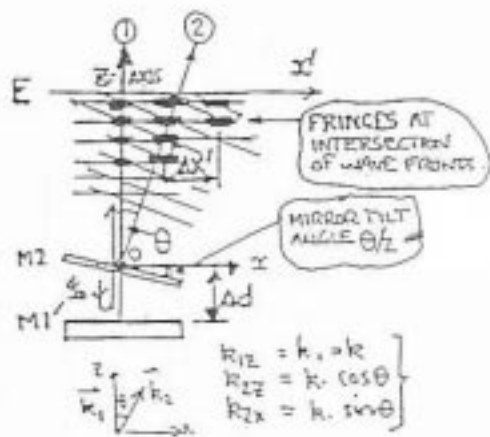


Figure 5.5.

To calculate the fringe spacing, we refer to Fig. 5.5. Let the origin of coordinates be at the center of M2. Waves are reflected normally by M1' (wave 1) and at an angle θ to the z-axis by M2 (wave 2). The resulting plane waves are:

$$E_1 = E_0 \exp i [k(z + 2\Delta d) - \omega t]$$

$$E_2 = E_0 \exp i [kz \cos \theta + kx' \sin \theta - \omega t]$$

In the region where the two beams overlap, the resultant electric field is:

$$\begin{aligned} E_{tot} &= E_1 + E_2 \\ &= E_0 e^{-i\omega t} \left\{ \exp [ik(z + 2\Delta d)] + \exp [ikz \cos \theta + kx' \sin \theta] \right\} \end{aligned}$$

The intensity can now be calculated using:

$$I_{tot} \propto E_{tot} \cdot E_{tot}^*$$

which yields the result:

$$I_{tot} = 4I_0 \cos^2 k \left(\Delta d - \frac{x'}{2} \sin \theta + z \sin^2 \frac{\theta}{2} \right)$$

Therefore, for fixed Δd , we have fringes at intervals $\Delta x'$ given by:

$$\Delta x' = 2\lambda / \sin \theta$$

When mirror M1' is moved in a direction normal to its face, the fringe pattern moves as Δd changes. The number, n , of the fringes that cross the center of the screen when M2 is moved a distance Δd is given by:

$$n = 2\Delta d / \lambda$$

The geometry for circular fringes produced by a point source, S, is shown in Fig. 5.6. The source S may be replaced by its virtual image, S', formed by reflection in the beam splitter. Reflection M1' gives S1'', while reflection in M2 gives S2''. Since S1'' and S2'' are derived from the same source, S, they are sources for a pair of mutually coherent spherical waves. Interference between spherical waves of different radii (R1 and R2) give rise to circular fringes in the plane of E (Fig. 5.6 (c)).

We can calculate the fringe radii as follows: Consider a point P lying on a circular fringe. The optical path difference at P is $\delta = (R_1 - R_2) = 2\Delta d \cos \theta$. The point P lies on a bright fringe if $\delta = n\lambda$ (n is called the order of the fringe). The polar angle, θ_n , where the n^{th} order fringe occurs is given by:

$$2\Delta d \sin \theta_n = n\lambda \tag{1}$$

where we have made the simplifying assumption that R1 and R2 are much greater than Δd . Suppose now that we label the fringes by the subscript p , with $p=0$ referring to the center of the screen. Suppose further that we adjust Δd until the center of the screen is at maximum brightness. Then, the fringe orders are:

$p=0$	at center of screen	order $n_0 = 2\Delta d / \lambda$
$p=1$	first fringe out from center	order $n_0 - 1$
$p=2$	next fringe out	order $n_0 - 2$
p^{th}	fringe out	order $n_0 - p$

The value of θ_p and $\cos \theta_p$ for the p^{th} fringe is:

$$\theta_p \approx \frac{r_p}{R_0} \quad \cos \theta_p \approx 1 - \frac{\theta_p^2}{2} = 1 - \frac{r_p^2}{2R_0^2}$$

where

$$R_0 = d_E + d_s + 2d_2$$

and r_p is the fringe radius on the screen. Substituting into Eqn. (1) above then gives:

$$n_p \lambda = 2\Delta d \left(1 - \frac{r_p^2}{2R_0^2} \right)$$

For the central fringe, this result yields:

$$1 - \frac{n_0 \lambda}{2\Delta d} = 0$$

For the general fringe, we find:

$$1 - \frac{n_p \lambda}{2\Delta d} = \frac{r_p^2}{2R_0^2}$$

Subtracting the last two expressions finally yields:

$$r_p = R_0 \sqrt{\frac{p\lambda}{\Delta d}}$$

This expression can be compared with measured fringe radii, for example by plotting the fringe radii against the index, p .

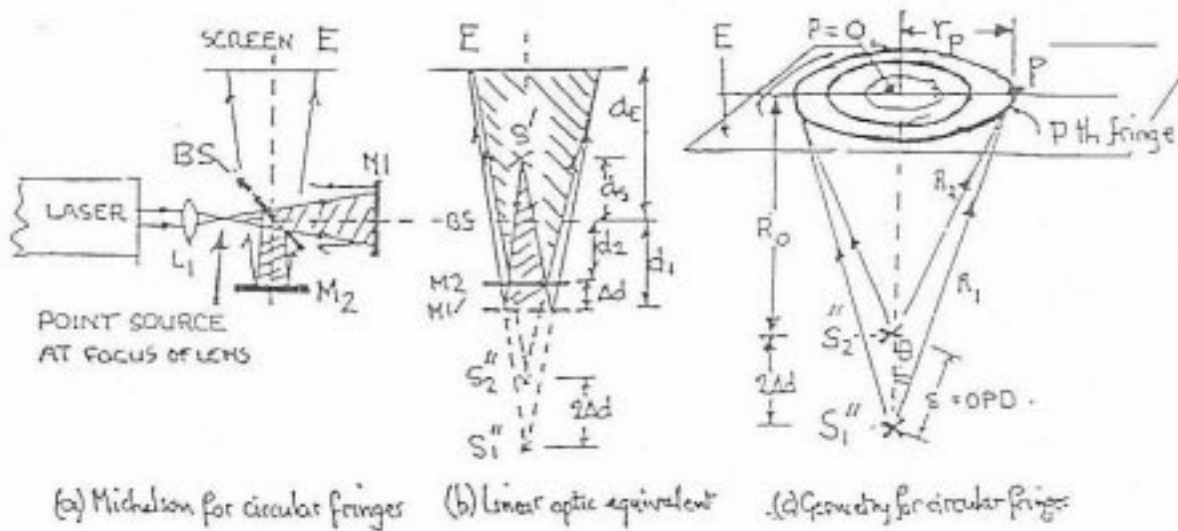


Figure 5.6. Generation of circular fringes.

Technical Details of the Experiment

1. New components

- Optical Table

The experiment will be set up on a 3' by 4' optical table. The table is rigid so that the relative positions of optical components attached to it will remain constant to less than one wavelength. Anti-vibration pads should be inserted between the optical table and the bench, to isolate the table from floor vibrations.

- Magnetic clamps

These clamps hold the components in fixed positions on the optical table. Depress the locking button to fix the clamp in place. Push the button the opposite way to free it.

Components are attached to the magnetic clamp by means of a post/post-holder combination. The post holder is attached by a 1/4-20 screw set in the tapped hole in the top of the clamp.

- Front surface mirrors and beam splitters

The faces of the special mirrors and beam splitters are flat within $\lambda/10$. The reflecting surface is a multi-layer dielectric coating that gives higher reflectivity than is possible with metallic coatings. The mirrors are >99% reflecting while the plitters are 50% reflecting at 45 degree angle of incidence.

TRY NOT TO TOUCH THE REFLECTING SURFACES!!

If the surfaces are already dirty, leave them in that condition and talk with your instructor to see whether they are usable. The mirror and beam splitters surfaces should be handled carefully to prevent damaging the surfaces. If you DO touch the surface, a quick cleaning with lens tissue and solvent will usually avoid permanent damage. Leaving finger grease on the surface will eventually cause surface etching and permanent damage.

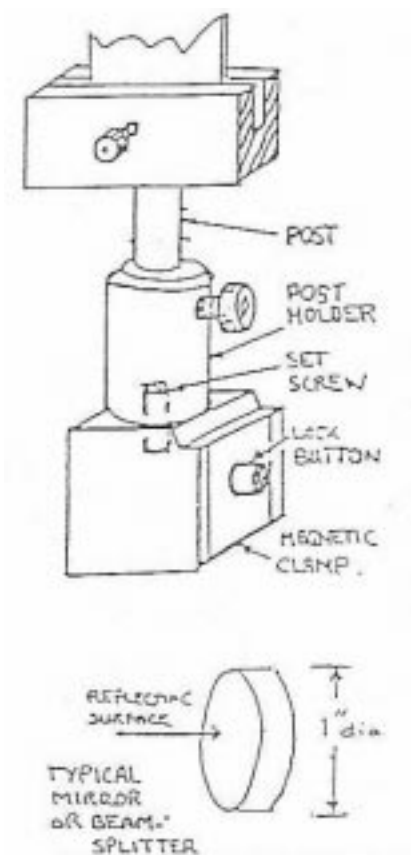


Figure 5.7. Magnetic clamp and mirror.

- Combining the various components

The beam is best set at a height of around 6-1/4" above the surface of the optical table. The different components can be matched to this height by the following combinations:

- For mirrors and beam splitters mounted in the adjustable knematic mounts, use a 3" post and a 3" post holder.
- For lenses and other components mounted on 2x2" plates, use 2" posts and 2" post holders together with the slide holder.

2. Apparatus - Laboratory Version

The lab arrangement for using the laser beam to align the mirrors of the interferometer is shown in Fig. 5.8 (a). The direction of the cross arm is at 90 degrees to the laser beam.

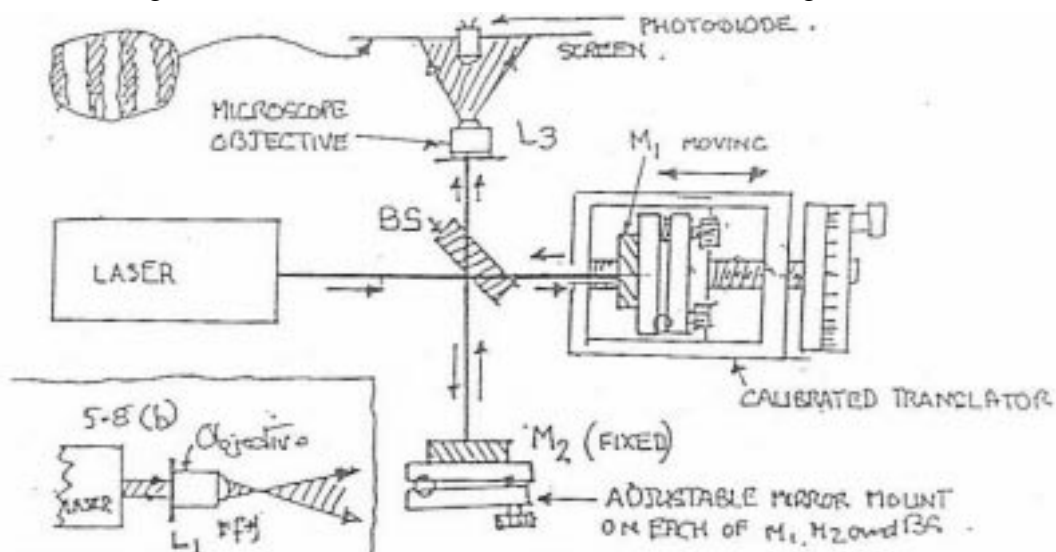


Figure 5.8 Laboratory version of the Michelson interferometer.

The microscope objective, lens L3, is removed for the initial alignment, which is best done with the 1 mm laser beam. The objective is inserted to make fringes larger. The lens that forms the point source, S, for circular fringe generation is shown in Fig. 5.8 (b).

Alignment Procedure

- Set up the laser with the beam 6-1/4" above the optical table and parallel to its surface.
- Adjust the heights of M1, M2, and BS so that the beam hits the center of each when you slide each into the beam. Mark the center of the screen E at the same

- height. Set the reflecting face of the BS away from the laser.
- c. Position M1, M2, BS, and E by eye using a ruler to set BS to M2 distance at about 6 cm. Set the optical path BS to M1 equal to BS to M2 within 1 mm or so if possible. Remember to add to d_2 an amount equal to the additional optical path of d_1 due to the glass in BS.
 - d. Adjust M1 to be normal to the beam. First block the light reflected from M2 with a card. Orient M1 so that the beam reflected back to the laser is about 2 mm directly above the laser exit prot. (Light returned into the laser can cause instability by positive feedback). Ignore the weaker reflected spots that arise from reflections off the uncoated side of the BS.
 - e. Adjust orientation of BS so that it reflects the beam returning from M1 into the center of the screen.
 - f. Uncover M2. Note especially the appearance of an additional bright spot on the screen. Again, ignore the weak spots.
 - g. Adjust the orientation of M2 so that the brightest spot on the screen due to the beam from M2 precisely coincides with the brightest spot from M1. Interference often causes the brightness of the combined spot to flicker. Insert a 40x microscope objective lens (L3) between BS and the screen to make fringes visible.

The interferometer should now be ready to observe interference fringes of various kinds.

3. Production of the basic types of interference fringe with the interferometer

- a. Production of circular fringes.

Refer to Fig. 5.8(b) for the experimental arrangement. Use a lens, L1, of the focal length you calculated in the homework to illuminate as large a part as possible on screen E. Do not use L3 for this part. Circular fringes should now appear on the screen. They could appear as arcs of circles if the center of the fringe system is displaced very far by excessive tilt of either mirror. Re-orient M2 to center the fringes on the screen. Choose for yourself a set of measurements that will provide a satisfactory test of the theory of circular fringes.
- b. Production of fringes of equal thickness.

Install the beam expander to generate a highly parallel beam about 1 cm in diameter. If the mirror N2 is tilted relative to the image, M1', then parallel linear fringes will appear on the screen. Each fringe represents a contour of constant thickness in the air wedge between M1 and M2'. When the surfaces of M1' and

M2 are precisely parallel, a single fringes should completely fill the illuminated area on the screen. This condition provides a technique for achieving a parallelism. Any deviation of the mirrors from perfect flatness will show up as distortion of these fringes. The mirrors should be flat to within $\pm \lambda/10$. Devise and carry out a procedure to test the theory of fringes of equal thickness.

4. Precise measurement of the wavelength of the He-Ne laser beam.

a. Set up the interferometer to produce fringes of equal thickness with the natural laser beam.

b. Insert a photodiode through a hole in the screen to measure the intensity of the light in the center fringe. Connect the photodiode via a Schmitt Trigger to a DC 504A counter-timer.

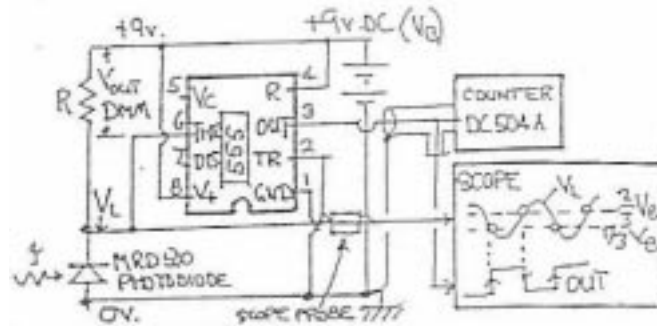


Figure 5.9 Optical detector system.

The Schmitt, based on a 555 timer chip should reduce the spurious counts due to noise that appears in the intensity (what causes this noise?). Set the controls of the counter to totalize the number of counts. In addition, display the photodiode and trigger circuit outputs on an oscilloscope to check the behavior of these signals. The scope will be particularly useful in studying the signals while translating mirror M1.

c. Install a slow speed motor to drive the translator. Connect the translator with a flexible torque cable to a low speed variable speed motor. Do not touch the table

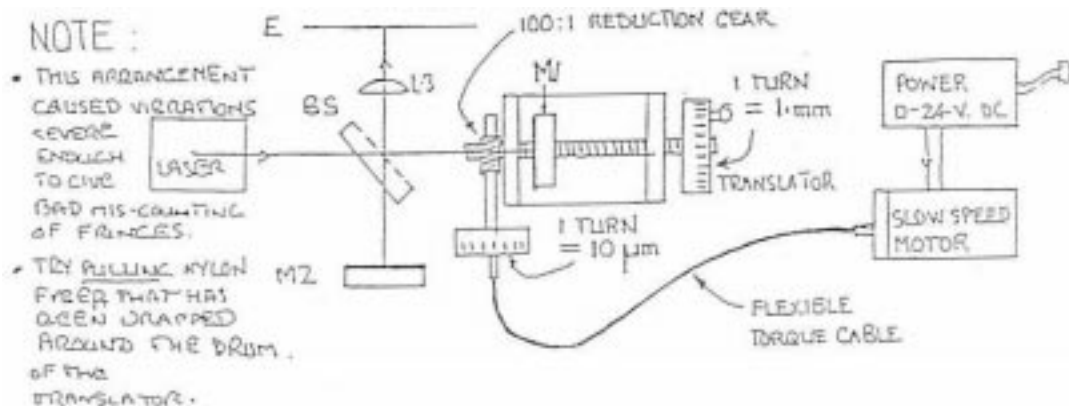


Figure 5.10. Michelson interferometer with slow speed drive.

while making measurements as touching can cause spurious shifts of several fringes.

- d. First count fringes by eye to get a 1% result for wavelength and to check that things are working properly.
- e. Count the number of fringes detected by the photodiode when you drive the mirror M1 through a measured distance of at least a few mm. The wavelength is given by $\lambda = n/(2\Delta d)$. Repeat your measurement several times and compare your values of wavelength while at the bench. Vibration on the table can cause spurious counts and can lead to major discrepancies between different measurements. Vibration and spurious counting is the major problem to be overcome in this part of the experiment.
- f. Measure the distance traveled. The screw-driven stages travel 1 mm per turn of the screw. The reduction gear gives 10 microns per turn of the small wheel. It is advisable to check the calibration of the translator against the traveling microscope.

Again, the primary difficulty in this experiment is reducing vibration and the associated false counts as the fringes wiggle and jump across the photodiode. Your job is to experiment with the motorized driving of M1 to produce a smooth and reproducible motion. In solving the problem of vibrations, it is extremely helpful to have an oscilloscope to view the photodiode circuit and the 555 timer output. Use these signals as your major source of feedback on the mirror motion and vibration. Remember, mirror motion of only 300 nm is enough to produce a complete transition from bright to dark on the photodiode. Therefore, your Michelson is an exquisitely sensitive tool for studying vibrations and stage motion. When dealing with vibrational problems, there are two general concepts to keep in mind: First, vibration is the result of FORCES applied to your system, but the motion itself arises from the associated accelerations. Recall that $F=ma$! Even if you cannot reduce the vibrational forces (which you usually can if you figure out where they come from), the acceleration can be controlled through the use of large masses. Second, vibration implies oscillatory motion. If damping is high enough, then the amplitude of oscillation can be dramatically reduced without any change in the drive forces. If the motion is lightly damped, the vibrational motion can grow enormously.

Talk to your instructors and other colleagues in the lab. Experiment with the addition of mass and damping. With a bit of care, you can produce excellent results for the He-Ne wavelength.