# Exo-planet detection via stellar intensity correlation interferometry

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## ABSTRACT

This paper considers the Hanbury Brown-Twiss effect and its application to astrometry in the service of extra-solar planet detection, particularly terrestrial planets at a range of 15 pc or less. The system considered comprises several modest-sized telescopes (light collectors) each equipped with photodetection apparatus and the means to record the photodetector signal time-history. At some convenient location, the cross-correlations of the individual light collector photodetection histories is computed to yield, in turn, a collection of values for the magnitudes of the mutual coherence of the target scene at various measurement baselines. With this type of observation system, we show that if there are known guide stars within the picture frame, the computed coherence magnitudes may be used to infer the apparent motion of the target star. Provided sufficiently large measurement baselines, the resolution of the target star motion can be very fine.

We first compute the signal-to-noise (SNR) ratio of a single coherence magnitude measurement and then, using simple models of the telescope array and the target star gravitational perturbation due to a terrestrial planet, we compute the SNR for determination of the planet orbit parameters, up to the determinacy afforded by astrometric measurements. We have provided expressions for the region in the (planetary mass-orbital semi-major axis) plane for which SNR is above a desired value. With these results, we can determine the sensitivity and range of the overall instrument for astrometry in planet detection. Moreover, one can assess the relative advantages of this technique in comparison with amplitude interferometry.

**Key words** : exo-planets, detection, interferometry, intensity correlation, Brown-Twiss

## 1. INTRODUCTION: THE SYSTEM CONSIDERED

Here we reconsider the Hanbury Brown-Twiss effect[1-4] and its potential uses in the service of planetary astronomy. In particular, we examine the possibility that this effect could provide the basis for an inexpensive but large scale intensity correlation interferometry array with sufficient precision to detect terrestrial-sized planets within 50 light-years, and if possible to form multi-pixel images of a some of the detected planets.

Consider a system composed of several, $N>1$, light-gathering telescopes, each equipped with a photodetector and apparatus to record the time histories of the photodetector output signals. The telescopes are distributed at various locations on the surface of the earth or in space with, perhaps large distances between them, but with the stipulation that all telescopes are capable of simultaneously collecting light from the object under study. In the case of most interest here, the objective might be to determine the location of a star to see if its motion is perturbed by a nearby planet. We suppose that there is no physical connection among the telescopes, nor is there any propagation of collected light between them, as there would be if the system were a Michelson interferometer. With each telescope operating as an independent unit, photon arrivals are separately recorded and the data communicated to some convenient location where the cross-correlation statistics of the $N$ signals are computed. From these statistics, imaging or astrometry data are, in turn, computed. We attempt to answer the various questions that inevitably arise: Is the statistical data of photodetection events sufficient for imaging or astrometry? What is the sensitivity of the

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Techniques and Instrumentation for Detection of Exoplanets II, edited by Daniel R. Coulter, Proceedings of SPIE Vol. 5905
(SPIE, Bellingham, WA, 2005) · 0277-786X/05/$15 · doi: 10.1117/12.612921

entire observation system, and what sort of astronomical objects could it profitably study? What are the demands on the data-handling system and how precisely does one need to determine the relative positions of the telescopes? We specifically address all the above questions, with particular emphasis on astrometry.

To begin, we first consider the photoelectric counting statistics of a fluctuating intensity field created by a thermal source. Explaining the basic assumptions concerning our system, we derive the basic relation of the Hanbury Brown-Twiss effect[1-4] which links the cross-correlation between photodetector output currents collected at two spatially separated locations and the magnitude of the mutual coherence of the incident radiation collected at the two locations. Considering in detail the process used to compute the cross-correlations from photodection record, we determine the signal-to-noise ratio of the measured coherence magnitudes. From this we can determine the sensitivity and range of the overall instrument for astrometry in planet detection.

## 2. PHOTODETECTOR OUTPUT STATISTICS: THE HANBURY BROWN-TWISS EFFECT

Each telescope of the system receives light from an unpolarized thermal source and directs this light onto a photodetection device. Since the intensity of the optical field fluctuates, the probability $p(n,t,T_d)$ of detecting $n$ photoelectric emissions in the time interval $t$ to $t+T_d$, where $T_d$ is the detector response time, is given by (reference [5], Section 9.7)

$$p(n,t,T_d)=\left\langle \frac{1}{n!}W^n e^{-W}\right\rangle, \quad W \triangleq \eta \int_t^{t+T_d} I(\mathbf{r},t')\,dt' \tag{2.1}$$

Where $\eta$ is the detector quantum efficiency and $I(\mathbf{r},t')$ is the intensity field of the source as observed at the location of the telescope, $\mathbf{r}$.

For the detection apparatus considered here, $T_d \gg T_c$. Thus, $I(\mathbf{r},t')$ under the integral undergoes many changes in time and the time integral divided by $T_d$ can be approximated by a time average. Assuming the fluctuating intensity is an ergodic process, the time integral in $W$ is proportional to the ensemble average:

$$W \triangleq \eta \int_t^{t+T_d} I(\mathbf{r},t')\,dt' \cong \eta \langle I(\mathbf{r},t)\rangle T_d \tag{2.2}$$

Then it follows that the statistics of the number of photons collected over time $T_d$ is Poissonian and the probability of $n$ photons is;

$$p(n,t,T_d)=\frac{1}{n!}\left(\eta\langle I\rangle T_d\right)^n e^{-\eta\langle I\rangle/T_d} \tag{2.3}$$

and the moments of $n$ are given by:

$$\begin{aligned} E\left[n^k\right] &= \frac{1}{i^k}\left[\frac{\partial^k C(s,\mu)}{\partial s^k}\right]_{s=0} \\ C(s,\mu) &= \exp\left(\left(e^{is}-1\right)\mu\right) \\ \mu &= \eta\langle I\rangle T_d \end{aligned} \tag{2.4}$$

Now to consider the operation of the detector in more detail, we suppose that every photoelectron emitted at time $t'$ produces a current pulse $k(t-t')$ which is nonzero only for $t-t'\geq 0$. Then the photodetector output current , $J(t)$, is given by:

$$J(t) = \sum_j k(t - t_j) \tag{2.5}$$

Here the sum is taken over the several random emission times. We can now follow a process similar to the derivation of the generalized Campbell theorem to determine the correlations of *J(t)* for the photodetectors of our system.

In particular (see [5], eq. 9.8-11), the cross-correlation of the fluctuations of the output currents obtained from two spatially separated detectors is:

$$\left\langle \Delta J_1(t)\Delta J_2(t+\tau)\right\rangle = \eta_1\eta_2 \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt'' k(t')k(t'')\left\langle \Delta I_1(t)\Delta I_2(t+t'-t''+\tau)\right\rangle \tag{2.6}$$

The condition $T_d \gg T_c$ also implies that we can treat the intensity correlations under the above time integral as being approximately proportional to a delta function. Then, setting, $\left\langle \Delta I_1(t)\Delta I_2(t+\tau)\right\rangle = \left\langle \Delta I_1 \Delta I_2\right\rangle T_c \delta(\tau)$ we find that the cross-correlation of the photodetector current fluctuations recorded at two spatially separated detectors is related to the correlation of the light intensity fluctuations at the two locations by:

$$\left\langle \Delta J_1(t)\Delta J_2(t+\tau)\right\rangle \cong \eta_1\eta_2 \left\langle \Delta I_1 \Delta I_2\right\rangle T_c \int_{-\infty}^{\infty} k(t')k(t'+\tau)\,dt' \tag{2.7}$$

Note that we observe thermal sources. Consequently:

$$\begin{aligned} \left\langle (\Delta I)^2 \right\rangle &= \tfrac{1}{2}\left\langle I\right\rangle^2 \\ \left\langle \Delta I_1 \Delta I_2\right\rangle &= \tfrac{1}{2}\left\langle I_1\right\rangle\left\langle I_2\right\rangle \left|\gamma(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \tau)\right|^2 \end{aligned} \tag{2.8}$$

where $\gamma(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \tau)$ is the normalized mutual coherence for the two detector locations. Using these expressions, the cross correlation of the photodetector current fluctuations becomes:

$$\left\langle \Delta J_1(t)\Delta J_2(t+\tau)\right\rangle \cong \tfrac{1}{2}\eta_1\eta_2 \left\langle I_1\right\rangle\left\langle I_2\right\rangle \left|\gamma(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \tau)\right|^2 T_c \int_{-\infty}^{\infty} k(t')k(t'+\tau)\,dt' \tag{2.9}$$

This is the essence of the Hanbury Brown-Twiss effect. Also, setting the detector locations equal and the time delay to zero, gives:

$$\left\langle \left[\Delta J_k(t)\right]^2 \right\rangle \cong \tfrac{1}{2}\eta_k \left\langle I_k\right\rangle^2 T_c \int_{-\infty}^{\infty} k^2(t')dt' \tag{2.10}$$

Let us next define the single-time normalized correlation coefficient:

$$C(d) \triangleq \frac{\left\langle \Delta J_1(t)\Delta J_2(t)\right\rangle}{\sqrt{\left\langle [\Delta J_1]^2\right\rangle\left\langle [\Delta J_2]^2\right\rangle}} \tag{2.11}$$

Then making use of the above expressions to evaluate *C(d),* we obtain:

$$\begin{aligned} C(d) &= \frac{\sqrt{\delta_1\delta_2}}{\sqrt{(1+\delta_1)(1+\delta_2)}} \left|\gamma(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, 0)\right|^2 \\ \delta_k &\triangleq \tfrac{1}{2}\eta_k \left\langle I_k\right\rangle T_c \;,\; k = 1,2 \end{aligned} \tag{2.12}$$

The basic idea is to measure *C(d)* and then invert to above formula to obtain the magnitude of the mutual coherence. Here we shall suppose that the two detectors have identical characteristics. Thus:

$$\left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right| \cong \sqrt{\frac{1+\delta}{\delta}C_{meas}\left(d\right)}, \qquad \delta \triangleq \tfrac{1}{2}\eta\left\langle I\right\rangle T_c \tag{2.13}$$

Where $C_{meas}(d)$ denotes the measured value. This measurement is carried out by averaging $\Delta J_1\left(t\right)\Delta J_2\left(t\right)$, etc. over some time period of duration $T_a$ where we assume that $T_a \gg T_d$ .

It is necessary to evaluate the statistics of these empirical time averages, particularly the standard deviation of the fluctuations of $C_{meas}(d)$, in order to assess the accuracy of the estimated coherence magnitude. This entails the lengthy calculation given in the Appendix. Here we briefly review the results. Retaining only terms of the first order in the fluctuations:

$$C_{meas}\left(d\right) = C\left(d\right)\left[1+\Im\right]; \quad \sigma\left[\Im\right] = \frac{2}{\sqrt{\eta\left\langle I\right\rangle T_a}} \tag{2.14}$$

Hence we have:

$$\left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right| \cong \sqrt{\tfrac{1+\delta}{\delta}}\sqrt{C\left(d\right)}\left[1+\tfrac{1}{2}\Im\right] \triangleq \left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right|_{\text{signal}} + \left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right|_{\text{noise}} \tag{2.15}$$

or, in other words:

$$\left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right|_{\text{signal}} \triangleq \sqrt{\frac{1+\delta}{\delta}}\sqrt{C_{meas}\left(d\right)}, \qquad \sigma\left[\left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right|_{\text{noise}}\right] \cong \sqrt{\frac{1+\delta}{\delta\eta\left\langle I\right\rangle T_a}} \tag{2.16a,b}$$

Now the signal is of order unity. Consequently, the signal-to-noise ratio is approximately:

$$\text{SNR}_{|\gamma|} \cong \sqrt{\frac{\delta\eta\left\langle I\right\rangle T_a}{1+\delta}}, \quad \delta \triangleq \tfrac{1}{2}\eta\left\langle I\right\rangle T_c \tag{2.17}$$

Finally, suppose that the frequency band being collected is reasonably narrow. Then:

$$\left\langle I\right\rangle = n_p \Delta\nu \tag{2.18a,b}$$

where $n_p$ is the number of photons per second, per Hertz collected by the receiver. Moreover, the correlation time of the collected light may be estimated as $T_c \cong 1/\Delta\nu$ . Using these relations, the signal-to-noise ratio becomes:

$$\left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right|_{\text{signal}} \triangleq \sqrt{\tfrac{1+\delta}{\delta}C_{meas}\left(d\right)}, \qquad \sigma\left[\left|\gamma\left(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0\right)\right|_{\text{noise}}\right] \cong \sqrt{1+\delta}\Big/\sqrt{2\delta^2\Delta\nu T_a}, \qquad \text{SNR}_{|\gamma|} \cong \sqrt{2\delta^2\Delta\nu T_a\big/\left(1+\delta\right)}, \ \delta \triangleq \tfrac{1}{2}\eta n_p \tag{2.19a,b,c}$$

This is the signal-to-noise ratio of the coherence magnitude measurement. The above expression bears a strong resemblance to the signal-to-noise result derived for an optical heterodyne receiver[6-8], where the unity in the denominator arises from vacuum state fluctuations. This should not be surprising since both heterodyne detection and intensity correlation entail measurement of two non-commuting observables and therefore must display uncertainty principle noise.

## 3. RELATIONS BETWEEN IMAGE INTENSITY AND COHERENCE

We wish to reconstruct the intensity distribution deposited by an extended incoherent source on a nearby *image plane*, *I*, using light collected in a set of regions, $A_1$, $A_2$, *etc.*, (the entry pupils of our several telescopes) situated on a very distant observation surface, *O*. Let $\mathbf{r}$ and $\mathbf{r}'$ denote the position vectors of points on *O* and *I*, respectively. Consider one polarization state and let *U* denote some component of the electric field of quasi-monochromatic light with mean frequency $\bar{\nu}$. Provided that for all points at which the field is evaluated, the mutual path differences between points within *O* or within *I* are small compared with the coherence length, an approximate expression of the Huygens-Fresnel principle is (see [5], section 4.4.3):

$$U(\mathbf{r},t) = \int_I d^2\mathbf{r}' \frac{e^{ikR}}{R} \Lambda U(\mathbf{r}',t) \tag{3.1}$$

apart from a phase factor, where $\Lambda$ is the inclination factor and $k$ denotes $2\pi\bar{\nu}/c$. To obtain the energy preserving estimate of the inverse of this given that light is collected over a limited entry pupil, one can reverse the time direction in the imaging situation of Fig.1, treating light collectors as projectors, reverse the roles of surfaces *I* and *O* and again apply the Huygens-Fresnel principle to obtain[9]:

$$U^e(\mathbf{r}',t) = \sum_m \int_{A_m} d^2\mathbf{r} \frac{e^{-ikR}}{R} \Lambda^* \, U(\mathbf{r},t) \tag{3.2}$$

$U^e(\mathbf{r}',t)$ is the field on the focal plane of an optical instrument having the specified entry pupil. The above classical result carries over into the quantum theory by substitution of the electric field operator, $\hat{U}$ for *U* (where the notation $(\hat{\bullet})$ distinguishes the operator of an observable from the observed numerical value).

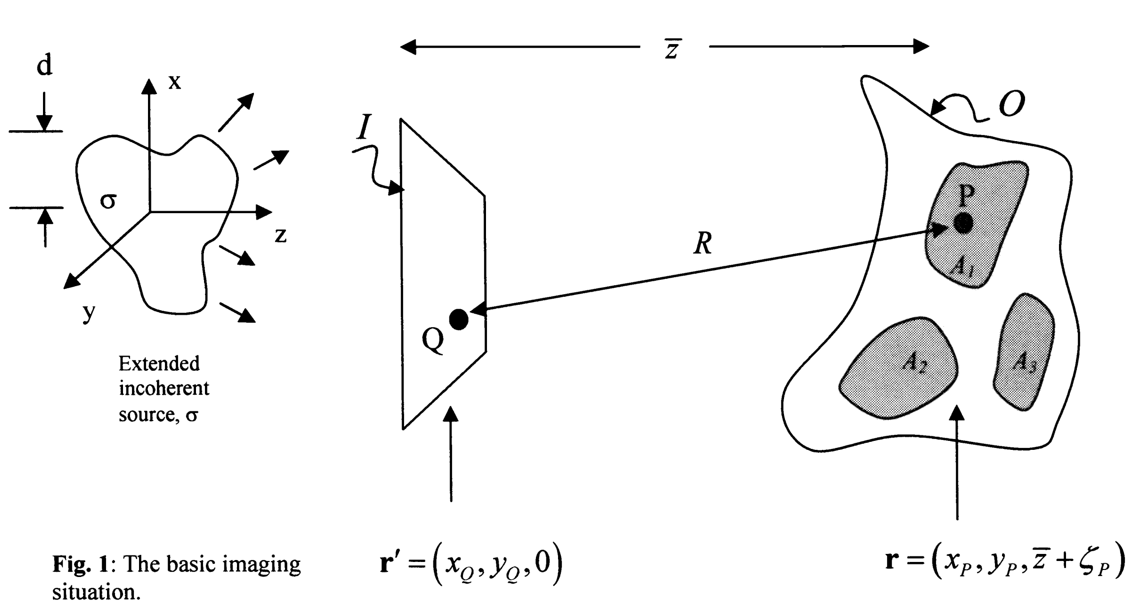


**Fig. 1**: The basic imaging situation.

Denoting the field state by $|\Psi\rangle$, the energy density that would be accumulated by a photodetector at image plane point $\mathbf{r}'$ over detection time $\Delta t_a$ is proportional to $I^{(e)}(\mathbf{r}',t)\,\Delta t_a$, where:

$$I^{(e)}(\mathbf{r}',t) = \langle\Psi|\hat{U}^{e\dagger}(\mathbf{r}',t)\hat{U}^{e}(\mathbf{r}',t)|\Psi\rangle$$
$$\hat{U}^{e}(\mathbf{r}',t) = \sum_{m}\int_{A_m} d^2\mathbf{r}\,\frac{e^{-ikR}}{R}\Lambda^{*}\,\hat{U}(\mathbf{r},t) \tag{3.3.a,b}$$

A slight manipulation gives:

$$I^{(e)}(\mathbf{r}',t) = \sum_{m,n}\int_{A_m} d^2\mathbf{r}_1\int_{A_n} d^2\mathbf{r}_2\,\frac{e^{-ik(R_1-R_2)}}{R_1R_2}\Lambda_1^{*}\Lambda_2\,\mathrm{J}(\mathbf{r}_1,\mathbf{r}_2,t)$$
$$\mathrm{J}(\mathbf{r}_1,\mathbf{r}_2,t) \triangleq \langle\Psi|\hat{U}^{\dagger}(\mathbf{r}_2,t)\hat{U}(\mathbf{r}_1,t)|\Psi\rangle = \sqrt{I(\mathbf{r}_1,t)I(\mathbf{r}_2,t)}\;\gamma(\mathbf{r}_1,\mathbf{r}_2,0) \tag{3.4.a,b}$$

Thus we have the direct relation between measured values of mutual coherence and the energy-preserving estimate of the image intensity given coherence values only over a restricted set of apertures. In comparison, if we were to acquire coherence data over the entire observation surface, *O*, the actual image intensity could be found from:

$$I(\mathbf{r}',t) = \int_{O} d^2\mathbf{r}_1\int_{O} d^2\mathbf{r}_2\,\frac{e^{-ik(R_1-R_2)}}{R_1R_2}\Lambda_1^{*}\Lambda_2\,\mathrm{J}(\mathbf{r}_1,\mathbf{r}_2,t)$$
$$\mathrm{J}(\mathbf{r}_1,\mathbf{r}_2,t) \triangleq \sqrt{I(\mathbf{r}_1,t)I(\mathbf{r}_2,t)}\;\gamma(\mathbf{r}_1,\mathbf{r}_2,0) \tag{3.5a,b}$$

The above formulae are valid for any configuration of light collectors, including significant in-range displacements among the component telescopes. Although these results can be retained in their full complexity it is convenient for further developments to specialize these relations to the case in which *I* and *O* are planes normal to the line-of-sight. Then we recover the well-known formulae:

$$I^{(e)}(\mathbf{r}',t) = \sum_{m,n}\int_{A_m-A_n} d^2\mathbf{u}\,\exp(-2\pi i\mathbf{u}\bullet\boldsymbol{\theta})\,\mathrm{J}(\mathbf{u})$$
$$I(\mathbf{r}',t) = \int_{O} d^2\mathbf{u}\,\exp(-2\pi i\mathbf{u}\bullet\boldsymbol{\theta})\,\mathrm{J}(\mathbf{u}) \tag{3.6a,b,c}$$

where:

$$\mathbf{u} = \Delta\mathbf{r}/\lambda$$

and the integral in the first relation is taken over all the relative position vectors among all the subapertures. $\boldsymbol{\theta}$ is the lateral component of a unit vector pointing toward a point on the image plane, i.e., a look-angle vector. Obviously one can invert the second equation to express the coherence in terms of the image intensity:

$$J(\mathbf{u},t) = \int_{I} d^2\boldsymbol{\theta}\,\exp(2\pi i\mathbf{u}\bullet\boldsymbol{\theta})\,I(\boldsymbol{\theta}) \tag{3.7}$$

## 4. PHASE RETRIEVAL FROM *A PRIORI* INFORMATION – IMAGING EXAMPLE

As is clear from (2.19), intensity correlation measurements only furnish the magnitude, but not the phase, of the mutual coherence. However, in many situations, one has sufficient *a priori* constraints on the image that, with sufficient coherence magnitude estimates, one can retrieve the phase and, hence, reconstruct the image or desired astrometric information. In this section, we show a relatively sophisticated instance of this, involving multi-pixel imaging of an object with bounded support. In the next section, we pick up the planet detection theme and show, in a much more elementary fashion, how guide star knowledge can be used to obtain astrometric measurements.

Suppose we are trying to obtain a multi-pixel image of an object with bounded positive support, i.e., an illuminated planet against a black background, using only coherence magnitude estimates on a

$N_1 \times N_2$ grid. Suppose also, that we know that a subset of pixels in the image plane must have a particular value of intensity, e.g. zero intensity (representing the black background of space surrounding a planetary disk). Our task is to use the image constraints and the coherence magnitude measurements to estimate coherence phase and then reconstruct the full image.

Using the formulation of (3.6,7), it is evident that the image intensity may be related to the coherence via a discrete Fourier transform. Employing Kronecker algebra notation, $vec(J)$ denotes the $N_t \triangleq N_1 N_2$ -dimensional vector obtained by stacking the columns of the matrix of coherence values, $J \in \mathbb{C}^{N_1 \times N_2}$. Likewise, let $vec(I) \in \mathbb{R}^{N_t}$ be the vector formed from the $N_1 \times N_2$ array of image intensity. It is evident from (3.6) that we can readily define a unitary transformation, $F \in \mathbb{C}^{N_t \times N_t}$ such that:

$$vec(I) = F\, vec(J) \tag{4.1}$$

Also, we can define a projection (idempotent matrix), $\tau \in \mathbb{R}^{N_t \times N_t}$, which projects elements of $\mathbb{C}^{N_t}$ onto the subspace containing the pixels whose intensity is known to be zero. Using this, we can express the constraints on pixels that are known to be black by the condition:

$$\tau\, vec(I) = 0 \tag{4.2}$$

Now, $vec(J)$ may be expressed as $|vec(J)| \odot \exp(i\varphi)$, where $(\ldots)\odot(\ldots)$ denotes the Hadamard product and $\varphi \in \mathbb{R}^{N_t}$ is the vector of unknown coherence phases. Considering the hermitian symmetry of $J$, it is clear that the measured values of coherence magnitude together with image constraints may suffice to determine the phase vector provided that the number of constrained pixels is more than half the total number of pixels in the picture frame. Given this condition, we are still faced with a difficult nonlinear estimation problem. However, we may have recourse to various recursive schemes. We display the simplest gradient-descent scheme here.

Let $\varphi^{(e)}$, and $vec\left(I^{(e)}\right) \triangleq F\left|vec(J)\right| \odot \exp\left(i\varphi^{(e)}\right)$ denote the currently estimated phase vector and the correspondingly reconstructed image intensity. A suitable measure of the extent to which the image constraints are violated is given by:

$$P \triangleq \left\| \tau\, vec\left(I^{(e)}\right) \right\|^2 \tag{4.3}$$

The gradient of $P$ with respect to the k$^{th}$ element of the phase vector is readily found to be:

$$\frac{\partial P}{\partial \varphi_k^{(e)}} = 2i\left[\left(\tau\, vec\left(I^{(e)}\right)\right)^T \tau F\right]_k \left(F^H vec\left(I^{(e)}\right)\right)_k \tag{4.4}$$

With this result the most straightforward approach is to recursively update the phase estimates via a repeated substitution of the form:

$$\varphi_k^{(e)} \leftarrow \varphi_k^{(e)} - \mu \frac{\partial P}{\partial \varphi_k^{(e)}} \tag{4.5}$$

Figure 2 shows results obtained from a simple algorithm of this kind, in a plot of $P$ versus iteration number.. It is supposed that the pixels at least one pixel width distant from the planetary disk are known to be black. We start the algorithm assuming $I$ is a uniform intensity disk having the measured value of total flux. It is seen that after some tens of iterations, the algorithm converges rapidly to the actual image. Of course, much can be done to improve the convergence behavior. The point here is that prior image information combined with the Hanbury Brown-Twiss technique can successfully reconstruct image data even in the most complex of situations. In the next section, we consider the much simpler problem of determining astrometry data given guide star information.

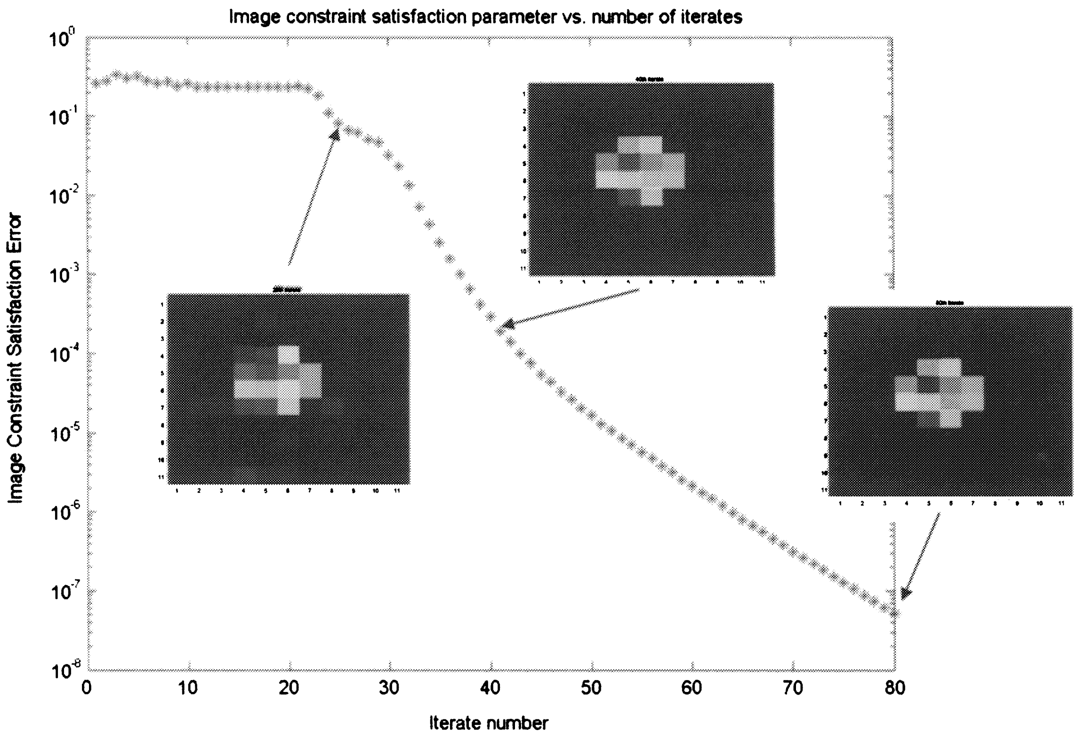


**Figure 2:** Illustration of the convergence of a simple phase retrieval algorithm.

## 5. INTENSITY CORRELATION INTERFEROMETRY FOR ASTROMETRY

Now we return to the principal theme of this paper and show how intensity correlation measurements, combined with *a priori* guide star information can be used to determine complete astrometric measurement of a target star. Such a measurement is accordingly used to infer the existence of unseen planetary companions.

Suppose the sky presents the picture sketched in Fig. 3 to the observer. We have a target star whose averaged position defines the origin of the picture frame coordinates. In addition, there are several point sources (more distant stars) with positions shown within the picture frame which we here assume are known to be fixed over the observation time. Then it is evident that the image plane intensity distribution takes the form:

$$I(\boldsymbol{\theta},t) = B_T\delta(\boldsymbol{\theta}-\boldsymbol{\theta}_T) + \sum_k B_k\delta(\boldsymbol{\theta}-\boldsymbol{\theta}_k) \tag{5.1}$$

Where $B_T, B_k,\ k = 1,2,...$ denote the brightnesses of the various objects. Using Eq.(3.6), we find that the mutual coherence produced by this collection of point sources is:

$$J(\mathbf{u},t) = B_T \exp(2\pi i\mathbf{u}\bullet\boldsymbol{\theta}_T) + \sum_k B_k \exp(2\pi i\mathbf{u}\bullet\boldsymbol{\theta}_k) \tag{5.2}$$

Consequently, the magnitude of the coherence that we could hope to measure via intensity correlations is:

$$|\gamma(\mathbf{u},t)|^2 = \left|\frac{J(\mathbf{u},t)}{J(\mathbf{0},t)}\right|^2 = \frac{1}{\overline{B}^2}\left\{\begin{array}{l} B_T^2 + 2\sum_k B_T B_k \cos(2\pi\mathbf{u}\bullet(\boldsymbol{\theta}_T-\boldsymbol{\theta}_k)) \\ +2\sum_k\sum_j B_k B_j \cos(2\pi\mathbf{u}\bullet(\boldsymbol{\theta}_j-\boldsymbol{\theta}_k)) \end{array}\right\} \tag{5.3a,b}$$

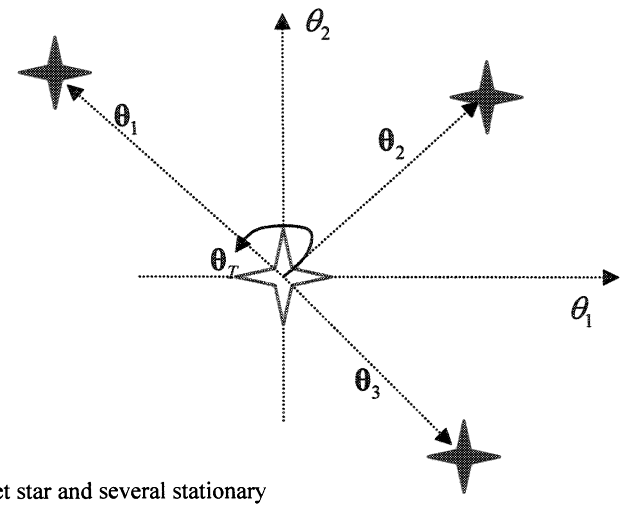


**Fig. 3 :** Target star and several stationary guide stars in the picture frame

where $\overline{B} \triangleq B_T + \sum B_k$ . Now suppose we make observations of the square of the magnitude of coherence at two successive times, $t - T_0$ and $t$ and then compute the change in results:

$$\begin{aligned}\Delta_T\left(\left|\gamma(\mathbf{u},t)\right|^2\right) &\triangleq \left|\gamma(\mathbf{u},t)\right|^2 - \left|\gamma(\mathbf{u},t-T_0)\right|^2 \\ &= \frac{2}{\overline{B}^2}\sum_k B_T B_k \left[\cos\left(2\pi\mathbf{u}\bullet\left(\boldsymbol{\theta}_T(t)-\boldsymbol{\theta}_k\right)\right) - \cos\left(2\pi\mathbf{u}\bullet\left(\boldsymbol{\theta}_T(t-T_0)-\boldsymbol{\theta}_k\right)\right)\right] \\ &\cong \left\{\frac{4\pi}{\overline{B}^2}\sum_k B_T B_k \sin\left(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_k\right)\right\}\mathbf{u}\bullet\Delta_T\left(\boldsymbol{\theta}_T(t)\right)\end{aligned} \tag{5.4}$$

where we assume $\Delta_{T_0}\left(\boldsymbol{\theta}_T\right) \ll 1$, and $\boldsymbol{\theta}_T \ll \boldsymbol{\theta}_k$. Thus by tracking the changes in the coherence magnitude, we uncover the time-varying component of the target star position. Suppose we make measurements of $\Delta_{T_0}\left(\left|\gamma(\mathbf{u},t)\right|^2\right)$ at several different relative positions of the light collecting telescopes, i.e. for $\mathbf{u} = \mathbf{u}^{(1)}, \mathbf{u}^{(2)}, \ldots, \mathbf{u}^{(M)}$, etc. and denoting the components of the vectors $\mathbf{u}^{(n)}$, and $\boldsymbol{\theta}_T$ by $\left(u_1^{(n)}, u_2^{(n)}\right)$ and $\left(\theta_{T1}, \theta_{T2}\right)$, respectively, we obtain the relations:

$$\begin{bmatrix} u_1^{(1)} & u_2^{(1)} \\ u_1^{(2)} & u_2^{(2)} \\ \vdots & \vdots \\ u_1^{(M)} & u_2^{(M)} \end{bmatrix} \begin{Bmatrix} \Delta_{T_0}\left(\theta_{T1}(t)\right) \\ \Delta_{T_0}\left(\theta_{T2}(t)\right) \end{Bmatrix} = \frac{\overline{B}^2}{4\pi B_T} \begin{Bmatrix} \Delta_T\left(\left|\gamma\left(\mathbf{u}^{(1)},t\right)\right|^2\right) \Big/ \sum_k B_k \sin\left(2\pi\mathbf{u}^{(1)}\bullet\boldsymbol{\theta}_k\right) \\ \Delta_{T_0}\left(\left|\gamma\left(\mathbf{u}^{(2)},t\right)\right|^2\right) \Big/ \sum_k B_k \sin\left(2\pi\mathbf{u}^{(2)}\bullet\boldsymbol{\theta}_k\right) \\ \vdots \\ \Delta_T\left(\left|\gamma\left(\mathbf{u}^{(M)},t\right)\right|^2\right) \Big/ \sum_k B_k \sin\left(2\pi\mathbf{u}^{(M)}\bullet\boldsymbol{\theta}_k\right) \end{Bmatrix} \tag{5.5}$$

It is evident that under broad conditions, this relation can be inverted to yield a unique determination of $\Delta_{T_0}\left(\boldsymbol{\theta}_T\right)$. Indeed, some algebraic manipulation yields:

$$\begin{bmatrix} \wp_{11} & \wp_{12} \\ \wp_{12} & \wp_{22} \end{bmatrix} \begin{Bmatrix} \Delta_{T}\left(\theta_{T1}(t)\right) \\ \Delta_{T}\left(\theta_{T2}(t)\right) \end{Bmatrix} = \frac{\overline{B}^2}{4\pi B_T} \begin{Bmatrix} \sum_{m=1}^{M} u_1^{(m)} \Delta_{T_0}\left(\left|\gamma\left(\mathbf{u}^{(m)},t\right)\right|^2\right) \Big/ \sum_k B_k \sin\left(2\pi \mathbf{u}^{(m)} \bullet \boldsymbol{\theta}_k\right) \\ \sum_{m=1}^{M} u_2^{(m)} \Delta_{T_0}\left(\left|\gamma\left(\mathbf{u}^{(m)},t\right)\right|^2\right) \Big/ \sum_k B_k \sin\left(2\pi \mathbf{u}^{(m)} \bullet \boldsymbol{\theta}_k\right) \end{Bmatrix}$$

where: $$\wp_{kj} = \sum_{m=1}^{M} u_k^{(m)} u_j^{(m)}$$

(5.6a,b)

Clearly as long as the two $M$-dimensional vectors $\left[u_1^{(1)}, u_1^{(2)}, \ldots, u_1^{(M)}\right]^T$ and $\left[u_2^{(1)}, u_2^{(2)}, \ldots, u_2^{(M)}\right]^T$ are distinct, $\wp_{12} < \sqrt{\wp_{11}\wp_{22}}$ and the matrix $\wp$ is nonsingular.

With regard to minimum requirements, no more than one guide star is needed and a minimum of two coherence measurements are necessary. The latter requirement implies that a minimum of three telescopes must be used. For example, under the above minimal constraints, if the relative positions of the telescopes are chosen so that $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$ are orthogonal and we take the coordinate axes parallel to $\mathbf{u}^{(1)}$ and $\mathbf{u}^{(2)}$, then the above expression becomes:

$$\begin{Bmatrix} \Delta_{T_0}\left(\theta_{T1}(t)\right) \\ \Delta_{T_0}\left(\theta_{T2}(t)\right) \end{Bmatrix} = \frac{\overline{B}^2}{4\pi B_T B_1} \begin{Bmatrix} \Delta_{T_0}\left(\left|\gamma\left(\mathbf{u}^{(1)},t\right)\right|^2\right) \Big/ u_1^{(1)} \sin\left(2\pi \mathbf{u}^{(1)} \bullet \boldsymbol{\theta}_1\right) \\ \Delta_{T_0}\left(\left|\gamma\left(\mathbf{u}^{(2)},t\right)\right|^2\right) \Big/ u_2^{(2)} \sin\left(2\pi \mathbf{u}^{(2)} \bullet \boldsymbol{\theta}_1\right) \end{Bmatrix} \quad (5.7)$$

Thus provided that neither $\sin\left(2\pi\mathbf{u}^{(1)}\bullet\boldsymbol{\theta}_1\right)$, nor $\sin\left(2\pi\mathbf{u}^{(2)}\bullet\boldsymbol{\theta}_1\right)$ vanish, $\Delta_{T}\left(\boldsymbol{\theta}_T\right)$ is uniquely determined.

Having derived the foregoing expressions, let us determine the capabilities of this approach to astrometry with regard to planet detection, in particular, the anticipated signal-to-noise ration of the final astrometric measurement. Let us assume the conditions of the above example, with one reference star and two measurements with distinct baselines. Return to expression (5.4) and consider measurement of the coherence magnitude along only one baseline vector. Let:

$$\mathbf{u} = \frac{D_B}{\lambda}\hat{\mathbf{u}}, \quad \left|\hat{\mathbf{u}}\right| = 1 \quad (5.8)$$

And denote $\hat{\mathbf{u}}\bullet\boldsymbol{\theta}_T$ by $\hat{\theta}_T$ and recognize that in employing the above formulae, it is the measured values of the coherence magnitude that we must use. Hence, $\hat{\theta}_T$ must also have a fluctuating component, call it $\Delta\left(\Delta_T\left(\hat{\theta}_T(t)\right)\right)$. Using relation (5.4):

$$\begin{aligned} &\Delta_{T}\left(\left[\left|\gamma(\mathbf{u},t)\right|_{signal} + \left|\gamma(\mathbf{u},t)\right|_{noise}\right]^2\right) \\ &\cong \left\{\frac{2}{\overline{B}^2} B_T B_1 \sin\left(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1\right)\right\} \sin\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T}\left(\hat{\theta}_T(t)\right) + \Delta\left(\Delta_{T}\left(\hat{\theta}_T(t)\right)\right)\right)\right) \end{aligned} \quad (5.9)$$

Or expanding both sides and retaining only first order terms in the fluctuations:

$$\begin{aligned} &\Delta_{T_0}\left(\left|\gamma(\mathbf{u},t)\right|^2_{signal}\right) + 2\left|\gamma(\mathbf{u},t)\right|_{signal}\left[\left|\gamma(\mathbf{u},t)\right|_{noise} - \left|\gamma(\mathbf{u},t-T_0)\right|_{noise}\right] \\ &\cong \left\{\frac{2}{\overline{B}^2} B_T B_1 \sin\left(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1\right)\right\}\left[\sin\left(\frac{2\pi D_B}{\lambda}\left(\left(\Delta_{T_0}\left(\hat{\theta}_T(t)\right)\right)_{signal}\right)\right) + \sin\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}\left(\hat{\theta}_T(t)\right)\right)_{noise}\right)\right] \end{aligned}$$

(5.10)

Then separating the signal and noise components, we obtain:

$$\Delta_{T_0}\left(|\gamma(\mathbf{u},t)|^2_{noise}\right) \cong \left\{\frac{2}{\overline{B}^2} B_T B_1 \sin(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1)\right\} \sin\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{signal}\right)$$

$$2|\gamma(\mathbf{u},t)|_{signal}\left[|\gamma(\mathbf{u},t)|_{noise} - |\gamma(\mathbf{u},t-T_0)|_{noise}\right]$$
$$\cong \left\{\frac{2}{\overline{B}^2} B_T B_1 \sin(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1)\right\} \cos\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{signal}\right) \sin\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{noise}\right) \quad (5.11a,b)$$

Evaluating the standard deviation of both sides of the second equation above gives:

$$4|\gamma(\mathbf{u},t)|_{signal}\,\sigma\left[|\gamma(\mathbf{u},t)|_{noise}\right]$$
$$\cong \left|\frac{2}{\overline{B}^2} B_T B_1 \sin(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1)\right| \cos\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{signal}\right) \frac{2\pi D_B}{\lambda}\sigma\left[\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{noise}\right] \quad (5.12)$$

Then we obtain:

$$SNR_{\Delta_{T_0}(\theta_T)} \triangleq \left|\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{signal}\right| \Big/ \sigma\left[\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{noise}\right]$$
$$\cong \left\{\frac{1}{2\overline{B}^2} B_T B_1 \left|\sin(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1)\right|\right\} \left|\sin\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)_{signal}\right)\right| \Big/ \sigma\left[|\gamma(\mathbf{u},t)|_{noise}\right]$$
$$\cong \sqrt{\frac{2\delta^2\Delta\nu T_a}{1+\delta}} \left\{\frac{B_T/B_1}{2(B_T/B_1+1)^2}\left|\sin(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1)\right|\right\} \left|\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)\right| \times \begin{cases} \cos\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)\right), & \text{if } \frac{2\pi D_B}{\lambda}\left|\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)\right| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \quad (5.13)$$

Where we have used expression (2.19b) for $\sigma\left[|\gamma(\mathbf{u},t)|_{noise}\right]$.

Now, with the above expression, we can ascertain appropriate design values for various parameters and determine the observational limits of this technique.

## 6. CAPABILITIES FOR PLANET DETECTION

Considering the one-baseline measurement situation with one guide star, it is evident that we can readily improve the signal-to-noise ratio, $SNR_{\Delta_{T_0}(\theta_T)}$ by choosing the guide star and baseline combination so that $\left|\sin(2\pi\mathbf{u}\bullet\boldsymbol{\theta}_1)\right| \cong 1$. Moreover, $SNR_{\Delta_{T_0}(\theta_T)}$ is also maximized if we can acquire a guide star with luminosity approximately equal to that of the target. Assuming these choices are possible:

$$SNR_{\Delta_{T_0}(\theta_T)} \cong \frac{1}{8}\sqrt{\frac{2\delta^2\Delta\nu T_a}{1+\delta}} \left|\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)\right| \times \begin{cases} \cos\left(\frac{2\pi D_B}{\lambda}\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)\right), & \text{if } \frac{2\pi D_B}{\lambda}\left|\left(\Delta_{T_0}(\hat{\theta}_T(t))\right)\right| \le \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases} \quad (6.1)$$

Then the $SNR_{\Delta_{T_0}(\theta_T)}$ is limited only by the photon count and the baseline distance, $D_B$. To relate the detector signal-to-noise ratio to instrument variables, suppose that each telescope is of circular aperture and employ a simple black-body model of the target star:

$$\delta \triangleq \tfrac{1}{2}\eta n_p \cong \frac{\eta\pi^2 D_T^2}{4\lambda^2}\left(\frac{R_{star}}{H}\right)^2 \frac{1}{\left(e^{ch/KT_{star}\lambda}-1\right)}$$

$$\Delta\nu = c/R_{spectr}\lambda \qquad (6.2a,b)$$

Given the above choices, we next consider the relationship between $SNR_{\Delta_{T_0}(\theta_T)}$ and the accuracy with which the orbital parameters of the unseen planet may be estimated from the measurements of $\Delta_{T_0}\left(\hat{\theta}_T(t)\right)$. This latter task is the principal objective of the observational system. The general estimation problem is too complex to be described in its entirety here, but to obtain rough estimates of the capabilities of the intensity correlation approach, we confine attention to the case in which the stellar perturbation due to a single planet may be isolated, and the stellar orbit is apparently circular with angular radius $\hat{a}_S$ and period $P_S$. Let the time between correlation observations, $T_0$, be some fraction, $f_0$, of $P_S$. Then the standard deviation of the estimated angular radius is roughly:

$$\sigma\left[\hat{a}_S^{(e)}\right] \approx \frac{1}{\sqrt{f_0}}\sigma\left[\Delta_{T_0}\left(\hat{\theta}_T(t)\right)\right];\ \left(\hat{a}_S^{(e)}\right)_{signal} \approx \frac{\left|\Delta_{T_0}\left(\hat{\theta}_T(t)\right)\right|_{signal}}{2\pi f_0} \qquad (6.3)$$

Hence:

$$SNR_{\hat{a}_S} \cong \frac{1}{2\pi\sqrt{f_0}} SNR_{\Delta_{T_0}(\theta_T)} \cong \frac{1}{16\pi\sqrt{f_0}}\sqrt{\frac{2\delta^2\Delta\nu T_a}{1+\delta}}\left|\frac{(2\pi)^2 D_B}{\lambda} f_0\hat{a}_S\right| \begin{cases}\cos\left(\frac{(2\pi)^2 D_B}{\lambda} f_0\hat{a}_S\right), & \text{if } \frac{(2\pi)^2 D_B}{\lambda} f_0\hat{a}_S \le \frac{\pi}{2}\\ 0, & \text{otherwise}\end{cases} \qquad (6.4)$$

Further, the averaging time for each correlation measurement, $T_a$, must be some fraction, $f_a$, of the time period between observations. Thus $T_a = f_a T_0 = f_a f_0 P_S$ and making these substitutions, we get:

$$SNR_{\hat{a}_S} \cong \frac{1}{16\pi}\sqrt{\frac{2\delta^2\Delta\nu f_a P_S}{1+\delta}}\left|\frac{(2\pi)^2 D_B}{\lambda} f_0\hat{a}_S\right| \times \begin{cases}\cos\left(\frac{(2\pi)^2 D_B}{\lambda} f_0\hat{a}_S\right), & \text{if } \frac{(2\pi)^2 D_B}{\lambda} f_0\hat{a}_S \le \frac{\pi}{2}\\ 0, & \text{otherwise}\end{cases} \qquad (6.5)$$

Necessarily, $f_0 \ge 1/3$ because at least three measurements are needed to estimate the angular radius. Next, $\hat{a}_S$ is related to the radius of the planet orbit by:

$$\hat{a}_S = \frac{a_s}{H} = \frac{M_P}{M_S}\frac{a_P}{H} \qquad (6.6)$$

where the ratio of masses and the actual planet orbit radius, $a_P$, can be determined from the measured apparent stellar orbit radius, the measured orbit period and the estimated stellar mass. Making substitutions:

$$SNR_{\hat{a}_s} \cong \frac{1}{16\pi}\sqrt{\frac{2\delta^2 \Delta\nu f_a P_S}{1+\delta}}\left|\Gamma\right| \times \begin{cases}\cos(\Gamma), \text{ if } \Gamma \le \frac{\pi}{2} \\ 0, \text{ otherwise}\end{cases}, \quad \Gamma \triangleq \frac{(2\pi)^2 D_B}{\lambda} f_0 \frac{M_P}{M_S}\frac{a_P}{H},$$

$$\delta \cong \frac{\eta\pi^2 D_T^2}{4\lambda^2}\left(\frac{R_{star}}{H}\right)^2 \frac{1}{\left(e^{ch/KT_{star}\lambda}-1\right)}, \quad \Delta\nu = c/R_{spectr}\lambda \tag{6.7a-c}$$

And note:

$$P_S = T_{yr}\sqrt{\frac{M_\oplus + M_S}{M_P + M_S}}\left(\frac{a_P}{AU}\right)^{3/2} \tag{6.7c}$$

Where $T_{yr}$ is the duration of a year in seconds, $M_\oplus$ is the mass of the Earth and $AU$ denotes the astronomical unit.

To illustrate the search space over which one might achieve reasonable levels of $SNR_{\hat{a}_S}$ we consider a star with solar parameters;

$$M_S = 1.989\times10^{30}kg,\ R_{star} = 6.955\times10^8 m,\ T_{star} = 6000^\circ K \tag{6.8 a,b,c,d}$$

At a distance of $H = 15pc$ we observe over a broad band centered at near-IR:

$$R_{spectr} \cong 1,\ \lambda = 1\,\mu m \tag{6.9 a,b}$$

And assume telescope , detector and observation period parameters as :

$$D_T = 0.5m,\ f_a = 0.3,\ f_0 = 0.1,\ \eta = 0.8 \tag{6.10 a,b,c,d}$$

Then, using the above relations, we can map the regions in the $\frac{M_P}{M_\oplus}$ - $\frac{a_P}{AU}$ plane for which differing values of the measurement baseline, $D_B$, gives $SNR_{\hat{a}_S}$ greater than a set value, say, in this case, $SNR_{\hat{a}_S} \ge 10$.

Figure 4 shows the regions for which $SNR_{\hat{a}_S} \ge 10$ for six values of $D_B$. The positions of most of the solar planets are shown for comparison. It is seen that for a given measurement baseline, the search region of acceptable SNR is an ellipse-like shape with long axis parallel to $M_P/M_\oplus \cong 10\,a_P/AU$. The ellipses extend only to some lower bound value of $a_P/AU$ and the value at which this occurs is approximately independent of $M_P/M_\oplus$. The reason for this behavior is that at the tips of the ellipses, the quantity $\Gamma \triangleq \frac{(2\pi)^2 D_B}{\lambda} f_0 \frac{M_P}{M_S}\frac{a_P}{H}$ assumes always the fixed value that maximizes $|\Gamma|\cos(\Gamma)$, namely the positive root of $\tan\alpha = 1/\alpha$, or $\alpha = 0.8603$. Thus the location of the tip depends only upon $\sqrt{2\delta^2\Delta\nu f_a P_S/1+\delta}$ and even for large planetary masses, $P_S$ has a very weak dependence upon $M_P/M_\oplus$. Hence the location of the tips of the acceptable SNR regions is characterized by a nearly fixed value of $a_P/AU$.

It is seen that different ranges of planetary sizes and orbit radii are covered by different values of the measurement baseline. In particular, planets of the size of Earth, Venus and Mars from a distance of 15 pc could be characterized with $SNR_{\hat{a}_s} \ge 10$ by the single baseline of $D_B \cong 200\,km$, as Figure 4 illustrates.

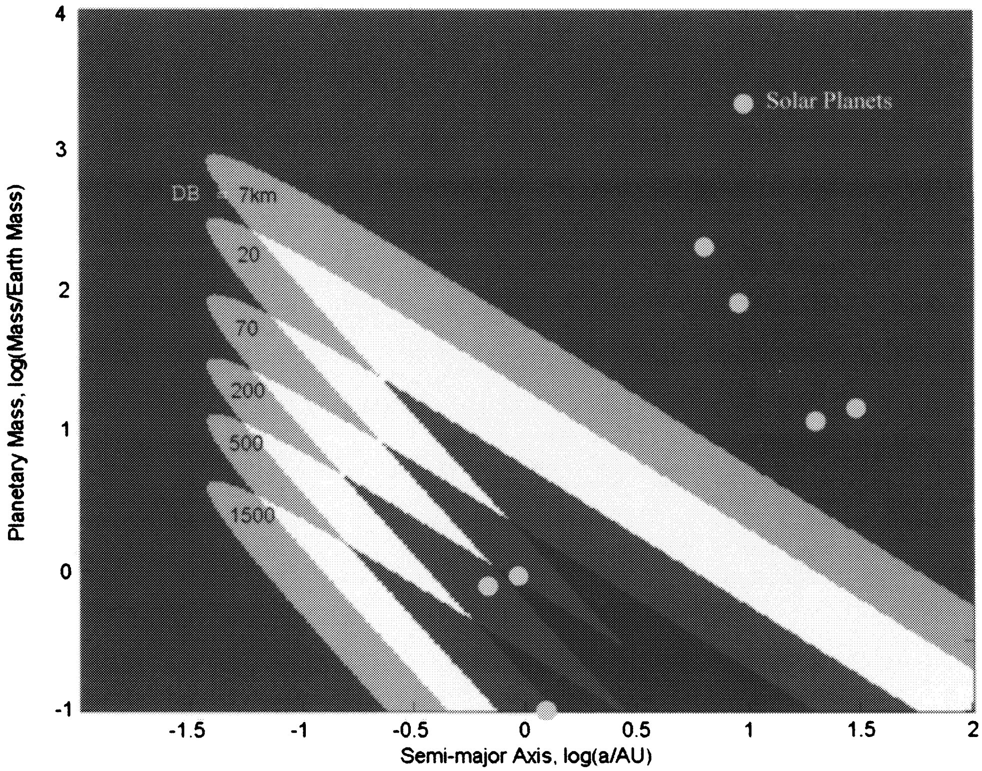


**Fig. 4:** Regions in the $M_P / M_\oplus$ - $a_P / AU$ plane for which differing values of the measurement baseline, $D_B$, gives $SNR_{\hat{a}_s} \geq 10$.

## 7. CONCLUSION AND FURTHER WORK

This paper reconsidered the Hanbury Brown-Twiss effect and its application to astrometry in the service of extrasolar planet detection, particularly terrestrial planets at a range of 15 pc or less. The system considered comprises several modest-sized telescopes (light collectors) each equipped with photodetection apparatus and the means to record the photodetector signal time-history. At some convenient location, the cross-correlations of the individual light collector photodetection histories is computed to yield, in turn, a collection of values for the magnitudes of the mutual coherence of the target scene at various measurement baselines. In many instances, this data can be combined with *a priori* image information to retrieve coherence phase and thus achieve full image reconstruction. This was illustrated with a relatively complex imaging example. In a much simpler context, we have shown that with this type of observation system, if there are known guide stars within the picture frame, the computed coherence magnitudes may be used to infer the apparent motion of the target star. Provided sufficiently large measurement baselines, the resolution of the target star motion can be very fine.

We first computed the signal-to-noise (SNR) ratio of a single coherence magnitude measurement and then, using simple models of the telescope array and the target star gravitational perturbation due to a terrestrial planet, we computed the SNR for determination of the planet orbit parameters, up to the determinacy afforded by astrometric measurements. We have provided expressions for the region in the (planetary mass-orbital semi-major axis) plane for which SNR is above a desired value. In particular for a solar-mass star at 15 pc, the parameters of an Earth, Venus and Mars may be ascertained with SNR > 10 with measurement baselines 200 km or less.

Of course, direct, homodyne, amplitude interferometry provides both magnitude and phase of the mutual coherence and thus astrometric data in a more direct way. However, the slightest change in the relative positions of optical elements within an amplitude interferometer can produce path-difference changes of many wavelengths. Atmospheric turbulence can produce even larger fringe variations, which must be corrected via an extensive adaptive optics system. The demands on path-length control challenge even space-based systems where turbulence is removed as an obstacle but the nanometer-level control of path lengths over baselines of the order of hundreds to thousands of kilometers remains a difficult and expensive undertaking.

The photoelectric correlation technique that this paper discusses provides an alternative method with a number of advantages. The light collecting telescopes are completely independent, not even the propagation of collected beams to some central combiner is needed. Only the *data* on the several photoelectric signals are brought together. As a consequence, the optical path differences do not have to be maintained strictly constant and slight optical element motions and atmospheric turbulence have a very small effect. Thus the observation system considered here could be deployed in space or established at the Earth's surface. The requirements on the *a priori* knowledge of the relative positions of the telescopes are extremely benign, being confined to a precision equal to some small fraction of the maximum baseline divided by the square root of the number of pixels that are desired in the final image result. Furthermore, the light-collecting telescopes need not be of very high optical quality, since their chief function is merely to direct the light to a photodetector at the focus.

## APPENDIX

In calculating the desired statistical characteristics of $C_{meas}(d)$, one could follow the procedure in reference [5] adopted to develop the foregoing formulae in the text. However, since $T_a \gg T_d$ we resort to simplifying approximations. Assume that $k(t)$ rises rapidly to a plateau and then falls rapidly to zero at $t=T_d$, and therefore approximates a rectangular pulse:

$$k(t) = \begin{cases} \kappa, & 0 \le t \le T_a \\ 0, & \text{otherwise} \end{cases} \tag{A.1}$$

Where $\kappa$ is a positive constant denoting the detector peak response. With this form for the detector response function, it is evident from that $J(t)$ is proportional to the number of photon arrivals in $[t\text{-}T_d,\ t]$:

$$J_k(t) = \kappa n_k(t - T_d, t) \tag{A.2}$$

This relation considerably simplifies calculation of the statistics of $C_{meas}(d)$.

Then, to begin with, define the time average measurement of $\langle J_k \rangle_{meas}$ as:

$$\langle J_k \rangle_{meas} = \frac{1}{T_a}\int_{t-T_a}^{t} dt' J_k(t') = \frac{\kappa}{T_a}\int_{t-T_a}^{t} dt' n_k(t' - T_d, t') \tag{A.3}$$

However, since $T_a \gg T_d$ the above expression can be approximated by:

$$\langle J_k \rangle_{meas} = \frac{\kappa}{T_a}\int_{t-T_a}^{t} dt' n_k(t' - T_d, t') \cong \frac{\kappa}{M}\sum_{m=1}^{M} n_k\left(t - T_a + (m-1)T_d, t - T_a + mT_d\right) \tag{A.4}$$

Where $M = T_a / T_d$ and there are approximately $M$ terms whose fluctuations about their mean values are uncorrelated.

Using the above expressions and approximations, we obtain, for example:

$$E\left[\langle J_k\rangle_{meas}\right] = \frac{\kappa}{T_a}\int_{t-T_a}^{t} dt' E\left[n_k\left(t'-T_d,t'\right)\right] = \frac{\kappa\mu}{T_a}\int_{t-T_a}^{t} dt' = \kappa\mu$$

$$\sigma\left[\Delta\left(\langle J_k\rangle_{meas}\right)\right] = \sqrt{E\left[\left(\frac{\kappa}{T_a}\int_{t-T_a}^{t} dt' n_k\left(t'-T_d,t'\right) - \kappa\mu\right)^2\right]}$$

$$\cong \sqrt{E\left[\left(\frac{\kappa}{M}\sum_{m=1}^{M} n_k\left(t-T_a+(m-1)T_d, t-T_a+mT_d\right)-\kappa\mu\right)^2\right]} = \kappa\sqrt{\tfrac{1}{M}E\left[\left(n_k\left(t-T_d,t\right)-\mu\right)^2\right]} = \kappa\sqrt{\mu/M}$$

(A.5.a,b)

The last line follows from $\left\langle\left(n_k-\mu\right)^2\right\rangle = \mu$. Thus, in summary:

$$\langle J_k\rangle_{meas} = \kappa\mu + \Delta\left(\langle J_k\rangle_{meas}\right)$$
$$\sigma\left[\Delta\left(\langle J_k\rangle_{meas}\right)\right] \cong \kappa T_d\sqrt{\eta\langle I\rangle/T_a}$$

(A.6.a,b)

Next, considering measured cross-correlations, and employing an approximation consistent with the above:

$$\begin{aligned}
&\langle\Delta(J_1)\Delta(J_2)\rangle_{meas}\\
&\cong \frac{1}{M}\sum_{m=1}^{M}\left(\kappa n_1\left(t-T_a+(m-1)T_d, t-T_a+mT_d\right)-\langle J_1\rangle-\Delta\left(\langle J_1\rangle_{meas}\right)\right)\\
&\qquad\times\left(\kappa n_2\left(t-T_a+(m-1)T_d, t-T_a+mT_d\right)-\langle J_2\rangle-\Delta\left(\langle J_2\rangle_{meas}\right)\right)\\
&\cong \frac{1}{M}\sum_{m=1}^{M}\left(\kappa n_1\left(t-T_a+(m-1)T_d, t-T_a+mT_d\right)-\langle J_1\rangle\right)\\
&\qquad\times\left(\kappa n_2\left(t-T_a+(m-1)T_d, t-T_a+mT_d\right)-\langle J_2\rangle\right) + \text{H.O.T.}
\end{aligned}$$

(A.7.a,b)

From this we see that:

$$E\left[\langle\Delta(J_1)\Delta(J_2)\rangle_{meas}\right] \cong \kappa^2 E\left[n_1 n_2 - \mu^2\right] \cong \kappa^2 E\left[n^2-\mu^2\right] = \kappa^2\mu$$
$$\sigma\left[\Delta\left(\langle\Delta(J_1)\Delta(J_2)\rangle_{meas}\right)\right] \cong \kappa^2\sqrt{\frac{1}{M}E\left[\left((n-\mu)^2-\mu\right)^2\right]} = \kappa^2\sqrt{\frac{1}{M}\left[\mu+2\mu^2\right]}$$

(A.8.a,b)

However, the average number of photon arrivals during the response time of the detector is quite small, i.e., $\mu \ll 1$. Hence we retain only the lowest power of $\mu$ to obtain:

$$\langle\Delta(J_1)\Delta(J_2)\rangle_{meas} = \kappa^2\mu + \Delta\left(\langle\Delta(J_1)\Delta(J_2)\rangle_{meas}\right)$$
$$\sigma\left[\Delta\left(\langle\Delta(J_1)\Delta(J_2)\rangle_{meas}\right)\right] \cong \kappa^2\sqrt{\mu/M} = \kappa^2 T_d\sqrt{\eta\langle I\rangle/T_a}$$

(A.9.a,b)

Now we can give the relative magnitudes of the fluctuations of the various measured quantities considered above:

$$\sigma\left[\Delta\left(\langle J_k\rangle_{meas}\right)\right]\Big/\langle J_k\rangle \cong \tfrac{1}{\sqrt{\eta\langle I\rangle T_a}}\,;\ \sigma\left[\Delta\left(\left\langle\Delta(J_k)\Delta(J_j)\right\rangle_{meas}\right)\right]\Big/\left\langle\Delta(J_k)\Delta(J_j)\right\rangle \cong \tfrac{1}{\sqrt{\eta\langle I\rangle T_a}}$$

(A.10.a,b)

With these results, we may now evaluate the statistics of the fluctuations in the measured values of the correlation coefficient, $C_{meas}(d)$. In the following, we retain only terms of the first order in the fluctuations.

$$C_{meas}(d)=\frac{\langle\Delta J_1\Delta J_2\rangle_{meas}}{\sqrt{\langle\Delta J_1^2\rangle_{meas}\langle\Delta J_2^2\rangle_{meas}}}\cong C(d)\left[1+\frac{\Delta\left(\langle\Delta J_1\Delta J_2\rangle_{meas}\right)}{\langle\Delta J_1\Delta J_2\rangle}-\frac{1}{2}\frac{\Delta\left(\langle\Delta J_1^2\rangle\right)\langle\Delta J_2^2\rangle+\Delta\left(\langle\Delta J_2^2\rangle\right)\langle\Delta J_1^2\rangle}{\langle\Delta J_1^2\rangle\langle\Delta J_2^2\rangle}\right]$$
$$=C(d)[1+\Im];\ \sigma[\Im]=2\Big/\sqrt{\eta\langle I\rangle T_a}$$

(A.11. a-c)

Hence we have:

$$\left|\gamma(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0)\right|\cong\sqrt{\tfrac{1+\delta}{\delta}C(d)}\left[1+\tfrac{1}{2}\Im\right]\ \triangleq\ \left|\gamma(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0)\right|_{\text{signal}}+\left|\gamma(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0)\right|_{\text{noise}} \qquad \text{(A,12)}$$

Now the signal is of order unity and:

$$\sigma\left[\left|\gamma(\vec{\mathbf{r}}_1,\vec{\mathbf{r}}_2,0)\right|_{\text{noise}}\right]\cong\sqrt{\frac{1+\delta}{\delta\eta\langle I\rangle T_a}} \qquad \text{(A.13)}$$

Consequently, the signal-to-noise ratio is approximately:

$$\text{SNR}_{|\gamma|}\cong\sqrt{\delta\eta\langle I\rangle T_a\big/1+\delta}\ ,\quad \delta\triangleq\tfrac{1}{2}\eta\langle I\rangle T_c \qquad \text{(A.14.a,b)}$$

Finally, suppose that the frequency band being collected is reasonably narrow. Then:

$$\langle I\rangle=n_p\Delta\nu,$$
$$n_p\triangleq\text{The number of photons per second per hertz collected by the receiver} \qquad \text{(A.15.a,b)}$$

Moreover, the correlation time of the collected light may be estimated as $T_c\cong 1/\Delta\nu$. Using these relations, the signal-to-noise ratio becomes:

$$\text{SNR}_{|\gamma|}\cong\sqrt{\delta\eta n_p\Delta\nu T_a\big/(1+\delta)},\quad \delta=\tfrac{1}{2}\eta n_p \qquad \text{(A.16.a,b)}$$

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