Optical Engineering 44(4), 048001 (April 2005)

Distortion of Gaussian intensity profile by passing through a tilted Fabry-Perot filter

Sahnggi Park
Hyunsung Ko
Moon-Ho Park
Electronics and Telecommunications Research Institute
161, Kajong-Dong, Yusong-Gu
Taejon 305-600, Korea
E-mail: sahnggi@etri.re.kr

Abstract. As a lightwave having a Gaussian intensity profile passes through a tilted Fabry-Perot filter, the profile is distorted severely depending on the conditions of the parameters. This distortion has not been studied and reported in depth, though the propagation of a Gaussian wave itself has been discussed well in the literature. We show our results of quantitative calculation and discuss it, which we believe will help significantly design waveguides containing gratings or filters. [DOI: 10.1117/1.1886833]

Subject terms: Gaussian intensity profile; Gaussian wave propagation; Fabry-Perot filter.

Paper 040461 received Jul. 13, 2004; revised manuscript received Oct. 27, 2004; accepted for publication Oct. 30, 2004; published online Apr. 8, 2005.

1 Introduction

The Gaussian intensity profile is the most frequent beam shape in the field of lightwave technology. The lightwave passing through a planar waveguide or an optical fiber is often assumed to have a Gaussian intensity profile. A Fabry-Perot filter is also one of most frequent elements in any optics or optical communication experiments in which multiwavelengths are involved.[1–4] In the case where the Fabry-Perot filter is tilted with respect to the incident light artificially or accidentally, the transmitted wave has a significant distortion of intensity profile, depending on the conditions of related parameters.

The propagation of a Gaussian beam has been described well in any literature on lasers or articles.[5,6] A lot of research has also been done on Gaussian beams with specific purposes, for example, to get a profile of a flattop,[7] a phase conjugation,[8] or to study nonlinear light-material interactions.[9,10]

No research except by Wu et al.[11] has described a quantitative calculation of distortion of Gaussian intensity profile induced by passing through tilted filters or gratings. In 2003, Wu et al. reported a calculation of a Gaussian beam incidenting nonnormally on a Fabry-Perot etalon. Here, we present a different approach from them and new results on the transmitted beams. While the propagation of a whole Gaussian beam was traced using an (x,y,z) coordinate function in the work of Wu et al., the incident Gaussian beam was sliced by N and each slice was traced and summed using the etalon formula in our work. Although this approach needs more parameters, the results are more flexible to show various graphs with changes of various parameters. The numerical equations are presented in a form so that practical users can plot immediately using any math programs. The calculations are validated by comparing incident and outgoing beams for three different conditions, zero incident angle, sufficiently large incident angle, and double passes through the filter. Although the derivation of the required equation is simple, the results presented will help to design optical experiments or integrated circuits.

2 Calculation

In the case where a tilted Fabry-Perot filter is used to tune a specific wavelength band of the light, which has a Gaussian intensity profile, the profile is distorted by multiple reflections and walk-offs from the center in each round trip. The quantitative amount of distortion can be obtained by slicing and collecting the transmitted light through each slot after a specific number of walk-offs. Figure 1 depicts an incident and outgoing wave that is sliced by N. The wave passing through slot number n is a collection of all the waves that were walked off zero to n/s times, where s is the number of slots shifted by a single walk-off. Gaussian shapes of the input electric field may be expressed as a function of slot numbers,

$$E(x)=E_0 \exp[-x^2/\omega^2], \tag{1}$$

$$E_i=E_0 \exp\left[-\left(1-\frac{i}{N/2}\right)^2\cdot\frac{1}{c^2}\right], \tag{2}$$

with $i=0\cdots N$, where c is a constant that decides the number q of slots within the width of beam waist ω by $q=cN/2$. The width of a single slot is ω/q μm.

As the Fabry-Perot filter is tilted by an angle θ and if its length is L, then the distance shifted by a single walk-off is given by $2L\sin\theta$. To decide the number s of slots shifted by a single walk-off, we use the ratio,

$$\frac{s}{2L\sin\theta}=\frac{N}{20\omega}, \tag{3}$$

where the number 20 was chosen arbitrarily to get the total number of slots in the range of 500 to 1000, which are reasonable numbers for a personal computer and for relatively good accuracy, as shown in the following sections. This ratio equation means that, as one slot width is equal to

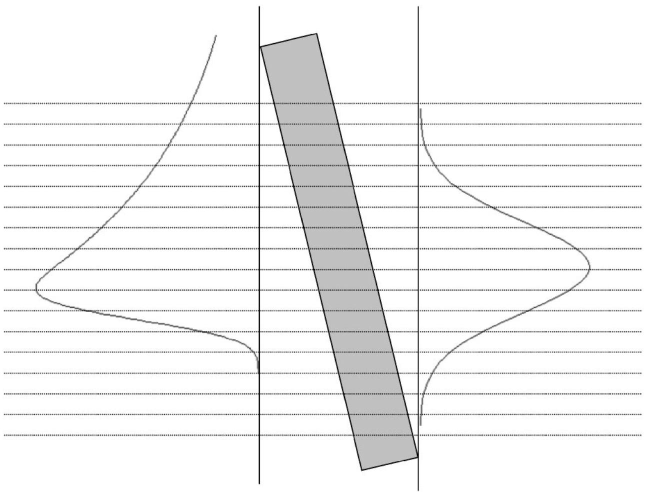


Fig. 1 Incident and outgoing intensity profiles sliced by *N*.

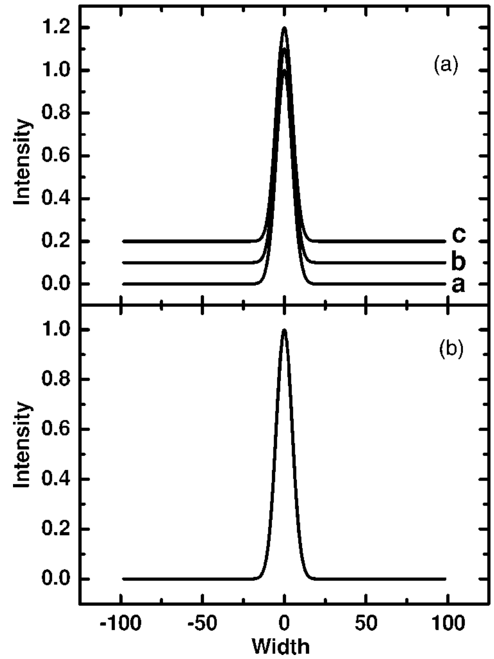


Fig. 2 (a) Small amount of shifts to distinguish each other, a input: b transmission, and c returned curve. (b) All three curves are superimposed on top of each other.

one walk-off distance, the number of slots within the width ω is one twentieth of the total number of slots N. To get a feel with Eq. (3), let us suppose $L=10\ \mu\text{m}$, $\omega=10\ \mu\text{m}$, and $N=500$, then the angle, $\theta=1.146$ deg makes one slot width equal to one walk-off distance. The input Gaussian field E_i has 25 slots within the width ω, and the constant c is 0.1.

Now the calculation is straightforward. The transmission of electric field is related to the reflectivity by,

$$t=(1-r^2-A)^{1/2}, \tag{4}$$

where A is the absorption and loss of the Gaussian field intensity through the Fabry-Perot filter. The wave passing through slot number n is given by the collection of all the waves walked off zero to n/s times in Fig. 1,

$$\begin{aligned}E_n=&E_0t^2\exp(-\alpha L/\cos\theta)+E_1t^2r^2\exp(-3\alpha L/\cos\theta)\\&+E_2t^2r^4\exp(-5\alpha L/\cos\theta)\\&+E_3t^2r^6\exp(-7\alpha L/\cos\theta)+\cdots,\end{aligned} \tag{5}$$

where if $s=1$, the first term on the right comes from the n'th slot of input beam with no walk-off, the second term from the $(n-1)$'th slot with 1 walk-off, the third term from the $(n-2)$'th slot with 2 walk-offs, etc. Equation (5) is expressed numerically by,

$$\begin{aligned}E_n=&\left[\sum_{i=0}^{n}E_it^2r^{2\cdot\mathrm{Int}(|n-i/s|)}\right.\\&\left.\times\exp\left\{-\alpha\left[2\cdot\mathrm{Int}\left(\left|\frac{n-i}{s}\right|\right)+1\right]L/\cos\theta\right\}\right]\cdot\frac{1}{s},\end{aligned} \tag{6}$$

where the whole summation is divided by s because a single walk-off jumps s slots. The factor $\mathrm{Int}(|(n-i)/s|)$ is also figured out by considering that if $i=0$, the number of walk-offs by the first slot is an intervalue of n/s. With factors of intensity normalization to the total input intensity, the transmitted Gaussian intensity profile is given by,

$$I_n=\frac{E_n^2\cdot P}{Q}, \tag{7}$$

where,

$$P=\sum_{i=0}^{N}E_i^2, \tag{8}$$

$$Q=\sum_{n=0}^{N}E_n^2. \tag{9}$$

P and Q are the total intensities of input and output Gaussian fields, respectively.

From Eq. (7), it is a small modification to calculate the profile of field that returns back through the same Fabry-Perot filter after reflection from a mirror, which we assume is located behind the filter. In this case, the walk-off will be in the opposite direction and the transmitted wave can be predicted to be symmetric.

$$\begin{aligned}E_m=&\left[\sum_{n=0}^{m}E_{N-n}t^2r^{2\cdot\mathrm{Int}(|m-n/s|)}\right.\\&\left.\times\exp\left\{-\alpha\left[2\cdot\mathrm{Int}\left(\left|\frac{m-n}{s}\right|\right)+1\right]L/\cos\theta\right\}\right]\cdot\frac{1}{s}.\end{aligned} \tag{10}$$

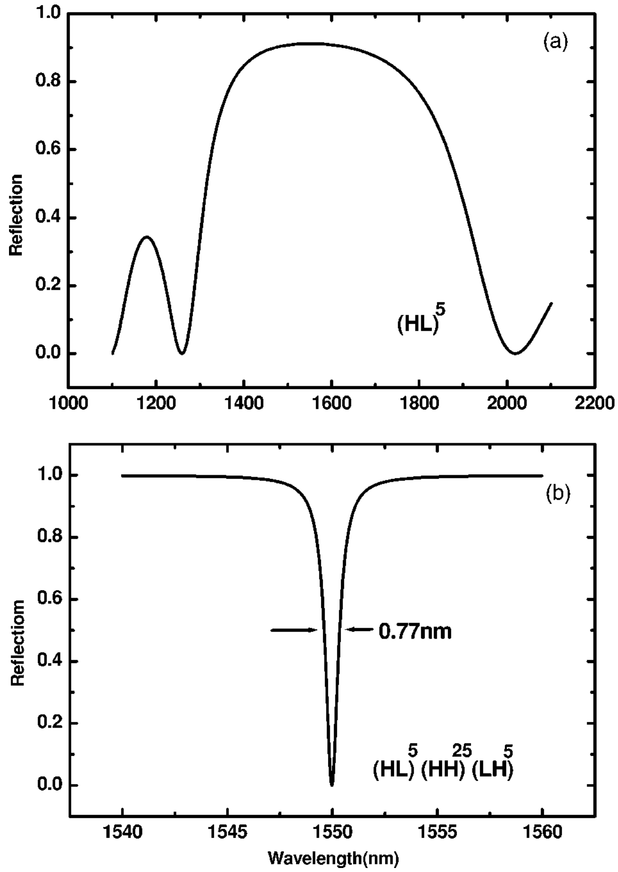


Fig. 3 Reflection curves of (a) five pairs of dielectric mirrors and (b) a single-cavity filter, which have the structures shown in the figures.

The returned intensity profile is given by,

$$I_m = \frac{E_m^2 \cdot P}{U}, \tag{11}$$

where U is a total intensity of the returned field,

$$U = \sum_{m=0}^{N} E_m^2. \tag{12}$$

The parameter s may take not only integers but also any positive noninteger values. As the tilted angle is sufficiently small, the parameter s may take a noninteger value smaller than 1, which means that multiple round trips take place within one slot width. The validness of the earlier equations can be checked by the calculation at the limit $\theta = 0$. As the angle θ approaches zero, the transmitted intensity profiles, Eqs. (7) and (11), should approach the input intensity profile. Figure 2 shows input and transmission curves with $\theta = 0.01$ deg, plotted in Fig. 2(a) with a small amount of shifts to distinguish each other and in Fig. 2(b) superimposed on top of each other. The complete accordance confirms the validity of the calculation. The horizontal axis represents the width of the curve with a unit of micrometers or millimeters, depending on the units of L and ω. The input intensity curve E^2 has full width at half maximum (FWHM) of 11.8 μm or mm. In the calculation before and hereafter, absorption is assumed to vanish $\alpha L = 0$, for simplicity. The accordance in Fig. 2(b) was confirmed by any values of parameters chosen arbitrarily except the angle, which should be sufficiently small.

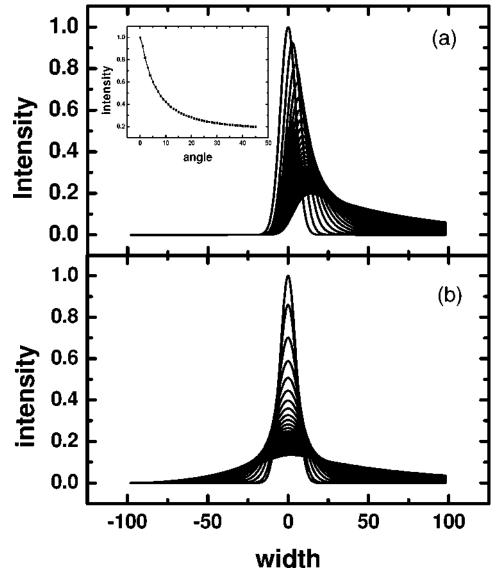


Fig. 4 Intensity profiles of (a) single transmission and (b) double transmissions with round pass, for different angles from 0.01 to 44 deg with 1-deg step.

3 Discussion

Before plotting transmitted curves as a function of tilted angle, let us describe an appropriate Fabry-Perot dielectric filter that has specific values of parameters. For simplicity, the filter is assumed to have a single-cavity symmetric structure, of which the calculation can be extended to any complicated structures as far as mirror reflections and the cavity length are given. Figure 3(a) shows a typical reflection curve of a dielectric mirror consisting of five pairs of SiO_2 ($n = 1.44$) and Ta_2O_5 ($n = 2.1$), which has 91% of intensity reflection at 1550 nm. Figure 3(b) shows a reflection

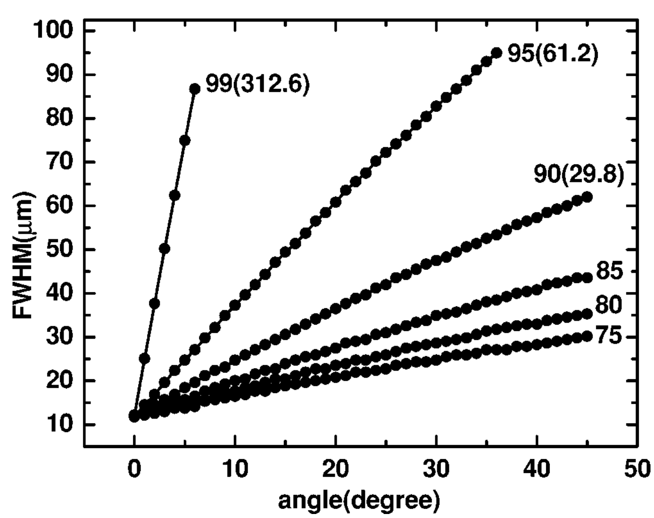


Fig. 5 Asymmetric FWHM as a function of tilted angle. The numbers show reflections with percent and finesses in parentheses.

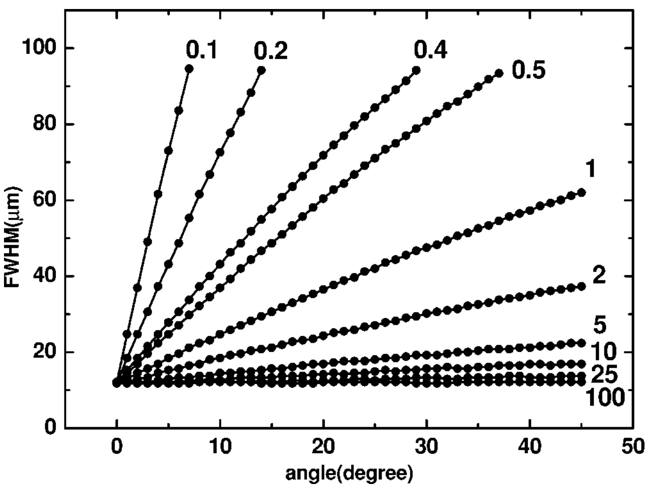


Fig. 6 Distortions for various ratios of the input beam width to the cavity length D/L, where the numbers show ratios.

curve of a single-cavity symmetric filter that has a bare cavity length of 9.23 μm, mirror thickness of 2.27 μm, spectral width of transmission peak $\Delta\lambda = 0.77$ nm, the free spectral range of 57 nm, and the effective cavity length of 10.03 μm calculated from the free spectral range.

With the filter conditions having $L = 10\,\mu$m and $R = 91\%$, as described before, and $\omega = 10$ μm, Eq. (7) was plotted in Fig. 4(a) for different angles from 0.01 to 44 deg with 1-deg step. As expected, we can see a consistent change of distortions that the peak amplitude decreases and the asymmetric beam diameter increases. At only 10 deg, for example, the asymmetric beam diameter increases more than twice, from 11.8 to 24.7 μm. As the beam diameter is comparable to the filter length, we see that the distortion is significant, even at a relatively small angle. The figure in the inset shows the variation of peak amplitude as a function of the tilted angle. It is remarkable that the slope at small angles, for example, less than 10 deg, is much steeper than at large angles. Figure 4(b) shows intensity profiles of the beam returned back to the input position plotted using Eq. (9). As mentioned before, the profiles are symmetric, except for the large angles that should also be symmetric, if the number of slots were large enough to take whole tails into account.

Figure 5 shows a quantitative calculation of distortion represented by an asymmetric beam diameter, FWHM, as a function of tilted angle for various mirror reflections. The input beam waist and the cavity length were assumed to be $\omega = 10$ μm and $L = 10\,\mu$m. The numbers in parentheses represent finesses of the cavity. At high reflections, higher than 95%, for example, we see that the distortion is extremely sensitive to angles, even at small angles.

Figure 6 also shows a quantitative calculation of distortion for various ratios of the input beam width to the cavity length ω/L, where the mirror reflectivity was assumed to be 90%. Since the quantities ω and L are the only variables that have a length dimension in the previous calculation, the plots in Figs. 5 and 6 can be applied for any units as long as they have the same unit. The distortion is very sensitive to the angle, as the ratio is smaller than one, but the distortion is negligible at the large values of ratios, larger than 25, for example. FWHM at 30 deg and ratio of 25 is 13.3 μm, a 13% increase with respect to the angle 0 deg.

In the case where the filters are located between collimated beams of which diameters are on the order of millimeters, it is common to have the ratio larger than 25, and the distortion of the Gaussian beam profile is negligible. Beam diameters, however, in a waveguide structure are comparable to the filter length. If the filter or a grating that has an appreciable angle and sufficiently high reflectivity is involved in the waveguide circuit, then the circuit should be designed with great caution to avoid an unexpected big loss. We believe that Eqs. (6) and (10) will be a valuable guidance to estimate the distortion for arbitrary values of parameters.

4 Conclusion

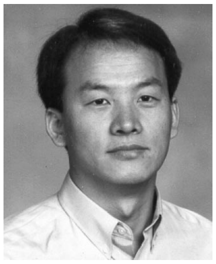
The equations to calculate distortion of a Gaussian beam by passing through a tilted Fabry-Perot filter is derived and plotted using simple math. Most significant factors are the mirror reflectivity and the ratio of the input beam diameter to the effective cavity length. At high reflectivity, for example, 95%, the angle of only 5 deg changes the beam diameter asymmetrically from 11.8 to 24.7 μm, more than twice, with 10 μm of ω and L. To get a negligibly small distortion, the ratio of input beam diameter to the effective cavity length should be high, at least higher than 25 at 90% reflectivity, for example.

Acknowledgment

This work was supported by the Ministry of Information and Communication of the Korean Government. The authors acknowledge the colleagues at the Electronics and Telecommunications Research Institute for fruitful discussions.

References

1. E. Garmire, "Theory of quarter-wave-stack dielectric mirrors used in a thin Fabry-Perot filter," *Appl. Opt.* **42**, 5442–5449 (2003).
2. P. Xindong and C. Roychoudhuri, "Design of high finesse, wideband Fabry-Perot filter based on chirped fiber Bragg grating by numerical method," *Opt. Eng.* **39**(7), 1858–1862 (2000).
3. M. Mulot, M. Swillo, M. Qiu, M. Strassner, M. Hede, and S. Anand, "Fabry-Perot cavities based on two-dimensional photonic crystals fabricated in InP membranes," *J. Appl. Phys.* **95**, 5928–5930 (2004).
4. K. Yiannopoulos, K. Vyrsokinos, E. Kehayas, N. Pleros, K. Vlachos, H. Avramopoulos, and G. Guekos, "Rate multiplication by double-passing Fabry-Perot filtering," *IEEE Photonics Technol. Lett.* **15**, 1294–1296 (2003).
5. A. E. Siegman, *Lasers*, University Science Books, California (1986).
6. M. Mansuripur, "Gaussian beam optics," *Opt. Photonics News* **12**, 44–47 (2001).
7. M. D. McNeill and T. C. Poon, "Gaussian-beam profile shaping by acousto-optic Bragg diffraction," *Appl. Opt.* **33**, 4508–4515 (1994).
8. G. Manneberg and M. Kull, "Polychromatic phase conjugation in Fabry-Perot etalons," *IEEE J. Quantum Electron.* **25**, 602–606 (1989).
9. D. V. Petrov, A. S. L. Gomes, and C. B. de Araujo, "Reflection of a Gaussian beam from saturable absorber," *Opt. Commun.* **123**, 637–641 (1996).
10. C. W. Tarn, "Gaussian-beam profile deformation via anisotropic photorefractive gratings formed by diffusive, photovoltaic, and drift mechanisms: a system transfer function approach," *Opt. Eng.* **37**(1), 229–236 (1998).
11. Z. Wu, G. Xia, H. Zhou, J. Wu, and M. Liu, "Transmission of a Gaussian beam after incidenting nonnormally on a Fabry-Perot etalon," *Opt. Laser Technol.* **35**, 1–4 (2003).



Sahnggi Park received his PhD degree from the Physics and Optical Science Center, University of Arizona, in 1999, and both MS and BS degrees in physics from Kyungpook National University, Korea, in 1988 and 1986, respectively. Since March 2001, he has been a senior research engineer at the Electronics and Telecommunication Research Institute. From 2000 to 2001, he was a senior research engineer with Samsung Electronics Company. His research projects have been in the fields of semiconductor detectors, semiconductor lasers, optical fibers and optical communication devices, DRAM and memory devices, and optics and physics of semiconductors for his PhD.

Hyunsung Ko received PhD and MS degrees from Seoul National University, Korea, in 1994 and 1999, respectively. In 2000, he joined the Electronics and Telecommunications Research Institute, where he has been engaged in the development of compound semiconductor devices for lightwave communication application.

Moon-Ho Park received a PhD degree from Teaxs A&M University in 1996. Since 1987, he has been employed as a researcher at the Electronics and Telecommunication Research Institute, Korea. He does research for semiconductor laser diodes.