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Coherence and Fluctuations in Light Beams*

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A quasithermal, quasmonochromatic lamp is described which serves as a highly degenerate light source with adjustable coherence time between 10^{-5} sec and 1 sec. This lamp is used for several demonstration experiments concerning the relations between coherence and fluctuations: The intensity interferometer of Hanbury Brown and Twiss is applied to measure the correlations between intensity fluctuations. The double slit experiment of Young serves to stress the role of fluctuations for classical interferometry. Interference patterns from two independent quasithermal lamps are presented.

I. INTRODUCTION

The concept of "coherence" for light beams was originally based only upon interference phenomena: Two light beams were considered coherent if their superposition results in a spatially fixed interference field. Hanbury Brown and Twiss[1] have shown that this concept of coherence is closely linked to the correlation of intensity fluctuations in both beams. The authors have proved this relationship and have demonstrated it experimentally with their intensity interferometer. This leads to the modern definition of coherence by means of fluctuations: *Two beams are coherent if their fluctuations are correlated* (see Born and Wolf[2]). Mandel[3] has reviewed this subject and has given a detailed list of the references prior to February 1963.

This concept of coherence has been developed in order to describe the radiation field of thermal light sources in which the probability distribution for the field strength is Gaussian.[4–6]

Glauber[4] has given a more general treatment. His coherence functions of higher order which are given in full quantum terms contain the older concept of coherence as a first order. Wolf[5] has discussed radiation fields of higher-order coherence in classical terms.

In this publication we describe demonstration experiments elucidating the concept of coherence. For this purpose we use a novel light source, which behaves like a monochromatic thermal lamp with adjustable coherence time. This greatly reduces experimental difficulties such that the following experiments can easily be performed with ordinary laboratory equipment.

II. CONDITIONS FOR COHERENCE

The conditions for getting spatially-fixed interference patterns (coherence conditions of classical optics[7]) can be summarized as follows: "*Each oscillation mode interferes only with itself.*" The volume of an oscillation mode in phase space is h^3. For a photon in this mode the ranges in coordinate (x,y,z) and momentum (p_x,p_y,p_z) are given by

$$\Delta x \Delta p_x = h, \quad (1a)$$

$$\Delta y \Delta p_y = h, \quad (1b)$$

$$\Delta z \Delta p_z = h. \quad (1c)$$

For a mode propagating in the z direction the relations (1a) and (1b) can easily be rewritten in terms of the dimensions of the light source Δx, Δy, and the angular apertures $2\Delta u_x$, $2\Delta y_y$ of the beam in the x–z and the y–z plane. Using $\sin\Delta u_x = \Delta p_x/|p|$, $\sin\Delta u_y = \Delta p_y/|p|$ and $h/|p|$

* The experiments have been demonstrated at the Frühjahrstagung 1964 des Regionalverbandes Hessen–Mitterlrhein–Saar der Deutschen Physikalischen Gesellschaft, Bad Nauheim, April, 1964.

[1] R. Hanbury Brown, and R. G. Twiss, Phil. Mag. [Ser. 7] **45**, 663 (1954); Nature **177**, 27 (1956); Proc. Roy. Soc. (London) [A]**242**, 306 (1957); Proc. Roy. Soc. (London) [A]**243**, 291 (1957).

[2] M. Born, and E. Wolf, *Principles of Optics* (Pergamon Press, London, 1959), p. 255.

[3] L. Mandel, in *Progress in Optics II*, edited by E. Wolf (North-Holland Publishing Company, Amsterdam, 1963), pp. 181–248.

[4] R. J. Glauber, Phys. Rev. Lett. **10**, 84 (1963); Phys. Rev. **130**, 2529 (1963); Phys. Rev. **131**, 2766 (1963).

[5] E. Wolf, in *Proceedings of the Symposium on Optical Masers* (Polytechnic Institute of Brooklyn 1963), pp. 29–42.

[6] L. Mandel, in *Proceedings 3rd International Conference on Quantum Electronics* (Dunod Cie, Paris, 1964), p. 101.

[7] R. W. Pohl, Optik und Atomphysik, (Springer-Verlag, Berlin, 1963), 11. Aufl. p. 65.

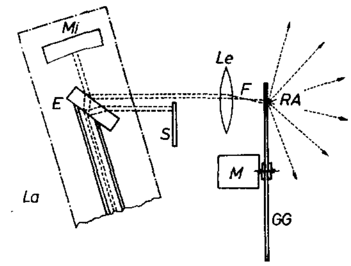

FIG. 1. Arrangement of quasithermal lamp. La Laser with end plate E and mirror Mi; GG movable ground glass screen with motor M; RA radiant area of quasithermal lamp. The diameter Δx of the area can be adjusted by means of lens Le. Shade S screens off the beam which is reflected at the lower surface of E.

$=\lambda$ we get

$$\Delta x \sin\Delta u_x=\lambda, \tag{2a}$$

$$\Delta y \sin\Delta u_y=\lambda. \tag{2b}$$

Further, if we put $\Delta p_z=h\Delta\nu/c$ and $\Delta z=c\Delta t$, where $\Delta\nu$ is the frequency spread of the light and Δt is the time during which the beam is observed, we obtain from (1*c*)

$$\Delta\nu\Delta t=1. \tag{2c}$$

Those values of angular apertures $2\Delta u_x$, $2\Delta u_y$ which fulfill (2a) and (2b) are called coherence angles, that value of observation times Δt, which fulfills (2c) is called coherence time. A light beam with aperture 2α and duration T smaller than the coherence angle $2\Delta u$ and coherence time Δt, respectively, can not be generated. But it is possible to consider a section out of a beam for

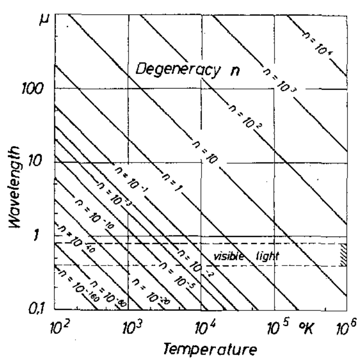

FIG. 2. Degeneracy of blackbody radiation for varying temperatures and wavelengths.

which the spatial and temporal conditions

$$\alpha_x\ll\Delta u_x, \tag{3a}$$

$$\alpha_y\ll\Delta u_y, \tag{3b}$$

and

$$T\ll\Delta t \tag{3c}$$

are fulfilled. (3a) and (3b) are the conditions for *spatial* coherence, (3c) for *temporal* coherence.

The intensity of thermal light sources fluctuates in space and time because of the statistical nature of the emission processes. However, there

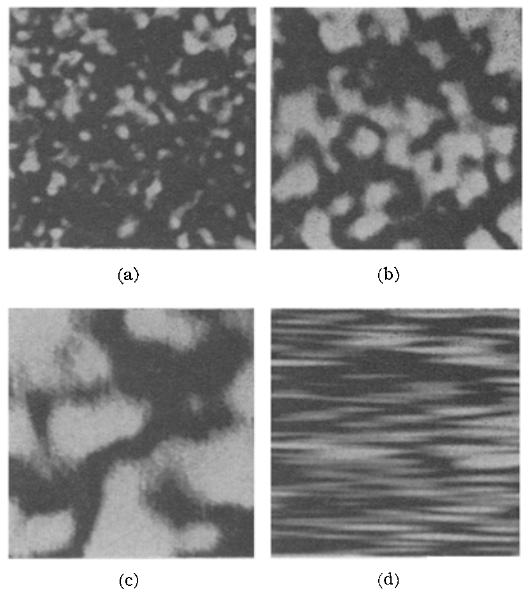

FIG. 3. Granulation for various sizes of the quasithermal lamp. The diameter Δx of the radiant area was (a) 0.5 mm (b) 0.2 mm (c) 0.1 mm. Fig. 3(d) was made with a radiant rectangle $0.1\cdot 1$ mm². Distance radiant area-photographic plate 45 cm. Linear amplification 1.7.

are no fluctuations within one oscillation mode. Therefore, a spatially coherent section of a light beam never shows fluctuations in space, but if it is observed during times $T\gg\Delta t$, there will be fluctuations in time. On the other hand, if we observe a temporally coherent beam ($T\ll\Delta t$), no fluctuations in time are possible, but if the aperture is sufficiently large, the intensity will be randomly distributed over the various directions in space.

III. METHODS FOR DETERMINING COHERENCE ANGLES AND COHERENCE TIMES

Coherence angle and coherence time can be measured—at least in principle—directly from the spatial and temporal fluctuations. However, there are two basic experimental difficulties. The first one is given by the low time resolution of the detectors. The coherence times of all thermal sources are shorter than 10^{-8} sec even after an optimal spectral resolution. Secondly, there is a limit because of the low intensity of ordinary thermal light sources. As a consequence of this small intensity, the fluctuations in question are not detectable because they are below the shot noise level.

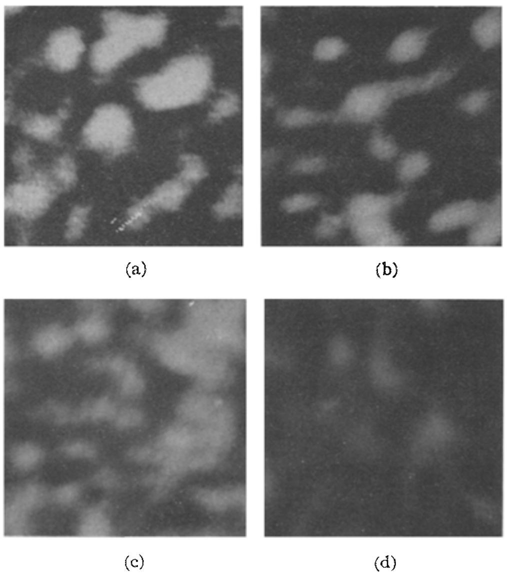

FIG. 4. Granulation for various ratios exposure time/coherence time. (a) $T/\Delta t=0.4$ (b) 1.9 (c) 4.5 (d) 25. Exposure time always 30 msec, coherence time was varied and determined from the time dependence of the photo current (see Fig. 5).

Besides these direct methods which are demonstrated in Sec. 5, there are indirect methods for determining the fluctuations and coherence properties by using interferometers. In all interferometers the beam to be investigated is split in two or more parts. Hanbury Brown and Twiss measure the correlation of the intensity fluctuations in the divided beams with their intensity interferometer. This interferometer is demonstrated in Sec. 6. The classical interferometers (Young, Fresnel, Michelson, etc.) superimpose the divided beams and measure the stationary interference pattern. An example is discussed in Sec. 7.

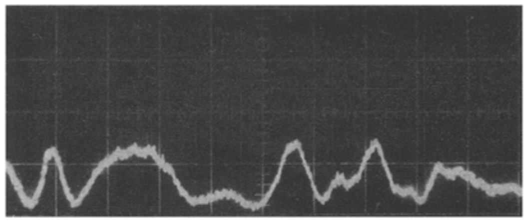

FIG. 5. Time dependence of photo current for detector with small aperture. Time axis 5 msec/large unit. The mean half-width determines a coherence time of 4 msec. Diameter Δx of lamp 0.2 mm.

IV. THE QUASITHERMAL, QUASIMONOCHROMATIC LAMP

In this section we describe the setup of a lamp which behaves like a thermal source with a long coherence time. This light source[8] overcomes the two difficulties mentioned above; direct observation of fluctuations now is feasible.

A narrow beam is obtained from a standard thermal light source, such as an arc or a mercury lamp. The spatial intensity fluctuations within

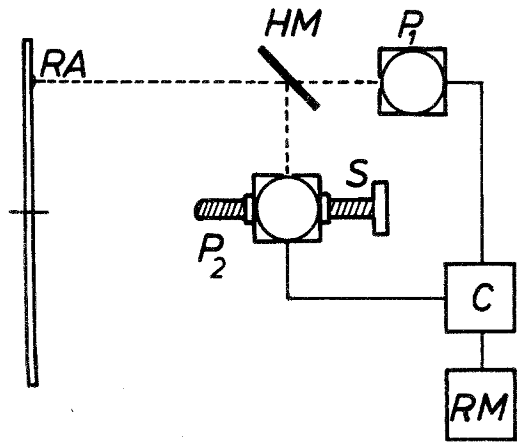

FIG. 6. Experimental arrangement after Hanbury Brown and Twiss[1] for detection of correlations in the intensity fluctuations. HM half-silvered mirror; P_1, P_2 photomultipliers 1P28; S turn screw for shifting P_2; C coincidence stage; RM rate meter.

[8] We have dedicated this lamp to Prof. R. W. Pohl for his 80th birthday, 10 August 1964.

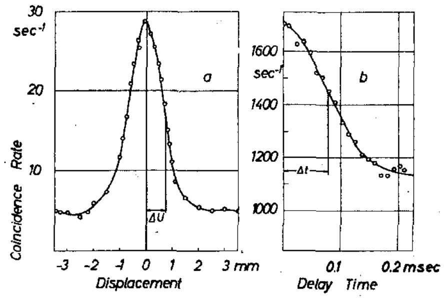

FIG. 7. (a) Coincidence rate as function of position for multiplier P_2. Δu determined from mean size of granulation spot. Coherence time $\Delta t=4$ msec; distance lamp-multiplier 70 cm, diameter of the lamp 0.2 mm. (b) Coincidence rate between direct and delayed multiplier signal. The coherence time $\Delta t=0.075$ msec is the mean half-width of pulses as determined according to scheme of Fig. 5.

this beam disappear, if the aperture is chosen smaller than the coherence angle. The fluctuations in time cannot be made to disappear, because it is experimentally impossible to filter out an adequately narrow spectral range. However, we can smooth out these fluctuations by using a sufficiently slow detector.

This beam is made to illuminate a ground glass screen, which contains a random distribution of scattering centers. The phase differences of the scattered waves are randomly distributed because of the thickness variations of the grains in the screen. The scattered light—observed with the sufficiently slow detector—shows no intensity fluctuations in time; however those fluctuations can now be generated deliberately by moving the screen within its plane. The phase differences of the beams originating from the various elements of the screen area now fluctuate randomly with a coherence time determined by the screen velocity. Our source is thus equivalent to a thermal light source[9], with an extremely long coherence time. This coherence time can easily be varied between 10^{-5} sec and 1 sec by adjusting the speed of the lateral motion of the screen.

For the actual experiments we illuminate our ground glass screen with a He–Ne laser of wavelength 0.633 μ (see Fig. 1). Gas lasers can easily be made to fulfill the coherence relation (2a, b.) All experiments to be described could also be performed by illuminating the ground glass with a standard arc lamp and by using an interference filter. However, all phenomena then become less intensive and cannot as conveniently be demonstrated.

The light from our model source appears to the slow detector as being highly degenerate. The degeneracy parameter n defined as the average number of photons in a given mode appears to be between 10^8 to 10^{13} according to the selected coherence time. Standard thermal lamps on the other hand have[10,11] an actual degeneracy parameter of less than 10^{-2}. Figure 2 depicts the degeneracy n of a blackbody as a function of temperature and wavelength.

V. FLUCTUATIONS IN SPACE AND TIME

Figure 3 shows a typical image, which is obtained when the light from our lamp with fixed ground glass screen (coherence time Δt infinite) is observed on a screen or photographic plate. The figure clearly shows a granular structure.

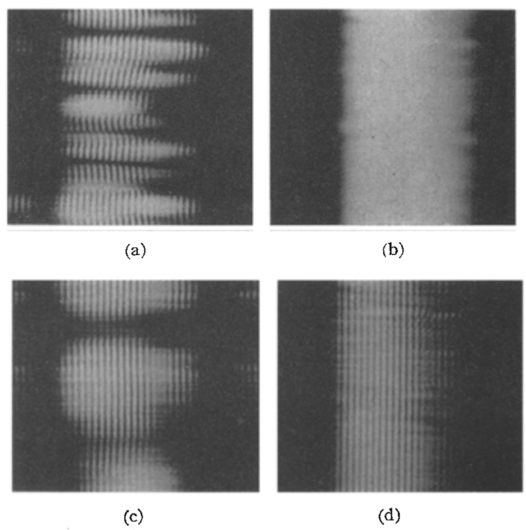

FIG. 8. Double slit experiment after Young (a) $\Delta x\cdot\sin\Delta u_x=2.15\lambda$; $\Delta t=\infty$, (b) $\Delta x\cdot\sin\Delta u_x=2.15\lambda$; $\Delta t=50$ msec, (c) $\Delta x\cdot\sin\Delta u_x=0.43\lambda$; $\Delta t=\infty$, and (d) $\Delta x\cdot\sin\Delta u_x=0.43\lambda$; $\Delta t=50$ msec. Exposure time always 150 sec; slit width 0.07 mm, distance between slits 0.7 mm.

[9] Ratcliffe in Rep. Progr. Phys. 19, 188 (1956) has already mentioned "It is often useful to think of an incoherent source of radiation as though it were a completely rough diffracting screen." We thank Dr. H. E. J. Neugebauer for drawing our attention to Ratcliffe's paper.

[10] L. Mandel, J. Opt. Soc. Am. 51, 797 (1961).

[11] D. Gabor, in *Progress in Optics I*, edited by E. Wolf (North-Holland Publishing Company, Amsterdam, 1961), pp. 146–148.

This granulation is well known[12,13]; it gained renewed interest after the invention of the laser.[14] The average area of the bright spots is essentially determined by the dimensions of the illuminated area, i.e., by the size of the lamp. Fig. 3. shows the increase of the spot size with decreasing lamp size according to Eq. (2a, b).

The transition from an infinite coherence time to a finite value is achieved by moving the ground-glass screen. The bright spots now fluctuate in their position and intensity.[12] Extremely short observation times still yield images like those of Fig. 3. With longer observation times we obtain a blurred image. Fig. 4 presents a series of photographs with varying ratios of $T/\Delta t$, where T is the exposure time. Standard thermal sources always enforce $T \gg \Delta t$; a granulation can not be observed this way.

Similar to the observations of pure spatial fluctuations with $T \ll \Delta t$ according to (2a,b), we can also investigate the pure temporal fluctuations, if a photodetector with small aperture α is used. We postulate $\alpha \ll u$, this means that we must make the detector area small compared to the area of a bright spot of the granulation. Figure 5 shows the time dependence of the photo current, indicating random fluctuations. A coherence time can be determined from Fig. 5 as the average half-width of a maximum.

The condition $T \ll \Delta t$ implicitly also contains a postulate regarding the intensity. The granulation can only be observed if sufficient quanta hit the photographic plate during the exposure time T. This means, that each oscillation mode must be occupied with several quanta, i.e., the used light must be degenerate:

$$n \gg 1. \quad (4)$$

Postulate (4) holds for all experiments to be described below except for the classical interference experiments.

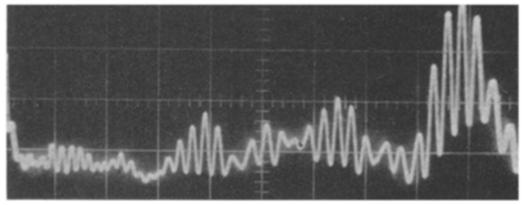

FIG. 10. Variation of photocurrent for detector with small aperture. 5 msec/large unit, distance between lamps 0.8 mm, diameter of lamps 0.2 mm each.

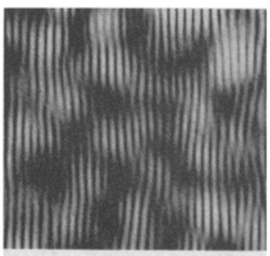

FIG. 9. Granulation with two quasithermal lamps, having infinite coherence time. Distance between lamps 0.5 mm, diameter of each lamp 0.1 mm.

VI. CORRELATIONS BETWEEN INTENSITY FLUCTUATIONS

Let us now examine the correlations between the temporal fluctuations at different points in space. The apparatus is the intensity interferometer of Hanbury Brown and Twiss[1,15,16,17] sketched in Fig. 6. The light beam is split in two by a half-silvered mirror HM. The coincidence stage[18] in connection with the rate meter RM measures the coincidence rates of the photocurrents from the two photomultipliers P_1 and P_2. These coincidences are a measure for the correlation between the intensity fluctuations of both beams. A maximum rate is found, when both multipliers are in the optical axis, where they give essentially identical responses. The coincidence rate drops [Fig. 7(a)] if one multiplier is shifted off the optical axis by turning the screw. The width of the resulting curve obeys Eq. (2a, b) and changes with apparent lamp size according to this rela-

[12] K. Exner, Wiedemanns Ann. Physik **9**, 239 (1880).
[13] M. v. Laue, Preuss. Akad. 1914, p. 1144.
[14] J. D. Rigden and E. J. Gordon, Proc. IRE. 50, 2367 (1962). J. Braunbeck, Naturwiss. **49**, 389 (1962). J. Braunbeck and M. W. Muller, Naturwiss. **50**, 325 (1963). R. V. Langmuir, Appl. Phys. Letters **2**, 29 (1963). B. M. Oliver, Proc. J. E. E. E. **51**, 220 (1963). L. Allen and D. G. C. Jones, Phys. Lett **7**, 321 (1963).
[15] G. A. Rebka and R. W. Pound, Nature **180**, 1035 (1957).
[16] E. Brannen, J. S. Ferguson, and W. Wehlau, Can. J. Phys. **36**, 871 (1958).
[17] R. Q. Twiss and A. G. Little, Australian J. Phys. **12**, 77 (1959).
[18] We are grateful to Professor Schopper, Institut für Kernphysik der Universität Frankfurt/Main for loaning us the coincidence stage.

tion. We thus conclude that the intensity fluctuations are correlated within the coherence angle.

Accordingly, we can investigate the correlations of the fluctuations at one point in space at various times. This is achieved by feeding the signal into the coincidence counter twice, once directly and secondly through a delay line.[19] Figure 7(b) shows the coincidence rate thus measured as a function of the delay time. The width of this curve agrees with the coherence time Δt determined from the time fluctuations similarly as in Fig. 5. We conclude that the temporal fluctuations at one space point are correlated within the coherence time.

VII. THE CLASSICAL INTERFERENCE EXPERIMENT

We consider Young's double slit-experiment as an example for classical interference experiments. We can repeat it by illuminating the double slit with the quasithermal lamp and by observing the pattern on a screen. Both slits are first illuminated incoherently by different continually changing granulation spots. This is provided by choosing the lamp size Δx greater than is required by Eq. (2a), where Δu_x now is the angular slit separation as seen from the lamp. The phase differences of the two beams leaving the double slit fluctuate at random, since there is no correlation between individual grains. Therefore the resulting interference pattern also fluctuates; the position of the maxima is defined only within the coherence time, as is the case for Fig. 8(a). Longer observation intervals ($T \gg \Delta t$) yield blurred images, such as shown in Fig. 8(b).

Secondly both slits may be illuminated along their x direction by one and the same granulation spot by choosing the source smaller than indicated by Eq. (2a).[20] The phase differences of the two beams leaving the double slit are now solely determined by the optical paths from double slit to screen. This defines the position of the maxima and minima in the interference pattern.

We obtain a pattern consisting of a periodic structure superimposed upon statistical intensity fluctuation. The regular structure (interference fringes) is fixed in space and time; whereas the statistical structure (granulation spots) fluctuates in space and time. Figure 8(c) presents a photograph with exposure time shorter than coherence time. Figure 8(d) on the other hand, shows a photograph where during a long exposure time nearly all points of the periodic pattern became covered. The pattern of Fig. 8(d) is typical for classical interference observations: despite temporal intensity fluctuations we see a spatially fixed regular structure represented by the interference fringes. We have an indirect method to check the coherence properties of the illuminating light by variation of the slit separation and observing the transition from case 8(d) to 8(b). The patterns can be easily detected even with very weak sources because the detector can be made to integrate over arbitrarily many coherence times; the intensity postulate $n \gg 1$ must no longer be met.

VIII. EXPERIMENTS WITH TWO INDEPENDENT SOURCES

In this last section we study the possibility to get interference effects with two independent light sources separated by arbitrary distances D. The experimental arrangement of Fig. 1 can easily be modified to yield two independent quasithermal light sources. This is done by removing the shade S. All experiments described in Secs. 5 and 6 can now be repeated with two sources instead of one. We first observe the interference pattern directly on a screen. We obtain granulations in which each bright spot displays interference fringes. Their angular spacing is given by the relation

$$\sin\gamma = \lambda / D. \qquad (5)$$

Figure 9 shows a typical pattern, it corresponds to the one described by Magyar and Mandel obtained by superposition of two ruby lasers.[21,22]

[19] We are grateful to G. Lehnert and D. Wolf, Institut für Angewandte Physik der Universität Frankfurt/Main for letting us use their delay element, which utilizes a magnetic tape recorder.

[20] Δu_y is of course much bigger than the value given by Eq. (2b); therefore the slits are still illuminated by several granulation spots along their y direction.

[21] G. Magyar and L. Mandel, Nature **198**, 256 (1963).

[22] L. Mandel, Phys. Rev. **134**, 110 (1964).

Reduction of the coherence time decreases the contrast, resulting in phenomena analogous to those described by Fig. 4.

The variations of intensity with time at a fixed point are shown in Fig. 10. This trace should be compared to the one of Fig. 5. In addition to the fluctuations of Fig. 5 a periodic structure within the maxima is now observed. Again we see the close relationship between spatial and temporal fluctuations. The spatial structure seen in Fig. 9 has its counterpart in the temporal structure of the trace in Fig. 10, both in its periodic and statistical features.

We now apply the intensity interferometer of Fig. 6 to measure coincidence rates in the field of the two light sources. Shifting of the movable multiplier P_2 reveals pronounced maxima and minima in the coincidence rate (Fig. 11). Again we get a maximum coincidence rate when both detectors are positioned on the optical axis. A rate minimum is found when the distance between the detectors is equal to the distance between a maximum and a minimum of the interference pattern in Fig. 9. In this case the first detector is just then fully illuminated when the second detector obtains no light and vice versa: both photocurrents are anticorrelated.

The number of maxima depends on the ratio of the lamp distance to the diameter of the individual lamp. The region in which extrema are observable is defined as in Sec. 6 by the coherence angle of the individual light source. Point lamps ($\Delta x \leq \lambda$) at arbitrary distance generate extrema over the entire space.

The interference phenomena of this chapter can only be obtained with short observation times $T \ll \Delta t$. Therefore, it is necessary to fulfill the intensity condition (4): $n \gg 1$. On the other hand there is no restriction by the geometrical conditions (2a, b).

The important feature of the interference experiment with two independent sources is the following. If the one light source is considered as a time standard, the interference experiment is a method for determining the *absolute phase* of the other beam, whereas the stationary pattern of a classical interferometer determines only *phase differences* which are produced by path differences. The absolute phase of the illuminating beam in classical interferometer experiments remains unknown.

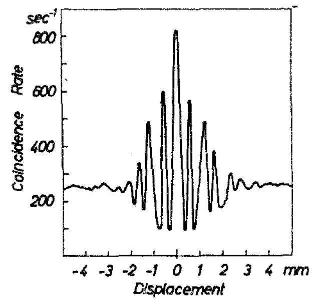

FIG. 11. Coincidence rate as function of position for multiplier P_2 with illumination by two quasithermal lamps; distance between lamps 0.8 mm, diameter of lamps 0.2 mm each; distance lamps-multiplier 70 cm.

There is some difficulty if we want to describe the difference between the experiments in this section and those of Secs. 5 and 6 by means of the coherence angle and the coherence time. These quantities have the same values in both cases. However, the periodic structure within one mode of the oscillation field—a granulation spot of Fig. 9 or the width of a maximum in Fig. 10— contains further information about the radiation field.

In Secs. 5 and 6 we have used a light source with a Gaussian distribution of intensity over the lamp area. Our two lamps can be considered as one light source with a special non-Gaussian intensity distribution; the regular structure in Fig. 9–11 indicates the deviations from the Gaussian distribution.

Analogous phenomena occur in the case that the probability distribution for the field strength is not Gaussian (nonthermal lamps). This property again will be remarked in fluctuation measurements in an analogous manner as the non-Gaussian intensity distribution over the lamp area can be observed by the additional regular structure in the granulation. For the case of nonthermal lamps the corresponding generalization of the coherence concept has been given by higher-order correlation functions.[4,5]

IX. CONCLUSIONS

(1) The classical coherence conditions (2a,b) define coherence angles and coherence times.

(2) We can observe intereference patterns which are spatially fixed for an arbitrary long

time, if the interferometer is illuminated within the coherence angle. On the other hand we can observe interference patterns within arbitrary large angles, if the observation time is within the coherence time, provided the intensity is sufficient ($n \gg 1$).

(3) The intensity of the light remains constant within the coherence angle and the coherence time. The fluctuations are spatial for angles greater than the coherence angles; they are temporal for times greater than the coherence time.

(4) The intensity fluctuates randomly in the radiation field of thermal sources, if the intensity distribution over the area of the source is Gaussian. Other distributions in addition produce more regular fluctuations.

ACKNOWLEDGMENTS

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