Calculation of Signal-to-Noise Ratio for Image Formation Using Multispectral Intensity Correlation

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ABSTRACT

In previous work, we explored the possibility of using intensity correlation techniques, based upon the Hanbury Brown-Twiss effect to perform fine resolution imaging in the service of exoplanet astronomy. Here we consider a multi-spectral variant of the Hanbury Brown-Twiss technique. At each of a number of independent, light-gathering telescopes photodetection data encompassing each of a set of frequency channels are obtained and then are communicated to some convenient computational station. At the computational station, the correlations among the photodetections in each of the frequency bands are time averaged and then further averaged over the various frequency channels to arrive at measurements of the mutual coherence magnitude for each pair of telescopes. From these statistics, imaging data are, in turn, computed via phase retrieval techniques. Here, within a modern quantum optics framework, we examine the signal-to-noise characteristics of the coherence estimates obtained in this way under a variety of non-ideal conditions. We provide step-by-step derivations of the statistical quantities needed in a largely self-contained treatment. In particular, we examine the effects of partial coherence on a scene typical of exoplanet imaging and show how partial coherence can be used to greatly attenuate the parent star. We find that the multispectral version of intensity interferometry greatly improves the signal-to-noise ratio in general and dramatically so for exoplanet detection. The results also extend the analysis of signal-to-noise to a wider variety of practical conditions and provide the basis for multispectral intensity correlation imaging system design.

Key words : exo-planets, detection, interferometry, intensity correlation, Brown-Twiss

1. INTRODUCTION: THE SYSTEM CONSIDERED

Over the past several years, much progress has been made in the development of the *entry pupil processing* (EPP) approach to ultra-fine resolution imaging [1-4]. Within the context of a synthetic aperture system employing a multiplicity of modest-sized telescopes, this approach involves the conversion of light collected at each sub-aperture into data and the data is transferred to some suitable location where mutual coherence information and, finally, the desired image are computed. In contrast to conventional Michelson interferometry, entry pupil processing eliminates the need for extreme-precision relative positioning control for path length control and the necessity of transporting collected beams to a central combiner. Each spacecraft-hosted telescope can be operated as an independent unit with metrology-derived relative position knowledge replacing the need for very precise formation control. Two principal detector technologies have been investigated for the implementation of entry pupil processing: Multi-channel optical heterodyne detectors [1], and Intensity Correlation Imaging (ICI) imaging arrays [5-9]. Recent studies [10] of sparse aperture techniques have compared the two EPP technologies with Michelson interferometry. In particular, use of ICI offers several decisive advantages. The light collecting telescopes are completely independent, not even the propagation of collected beams to some central combiner is needed. Only the *data* on the several photoelectric signals are brought together. As a consequence, the optical path differences do not have to be maintained strictly constant and slight optical element motions have a very small effect. The

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requirements on the *a priori* knowledge of the relative positions of the telescopes are extremely benign, being confined to a precision equal to some small fraction of the maximum baseline divided by the square root of the number of pixels that are desired in the final image result. Furthermore, the light-collecting telescopes need not be of very high optical quality, since their chief function is merely to direct the light to a photo detector at the focus. Finally, this approach requires only conventional photometric and processing technology.

Since ICI is based upon the Hanbury Brown-Twiss effect, we consider a system composed of several light-gathering telescopes, each equipped with a photodetector and apparatus to record the time histories of the photodetector output signals. With each telescope operating as an independent unit, the photodetector output data are communicated to some convenient location where the cross-correlation statistics of the signals are computed to yield estimates of the mutual coherence in accordance with the Hanbury Brown-Twiss relations. From these statistics, imaging data are, in turn, computed. Here we examine the signal-to-noise characteristics of the coherence estimates obtained in this way, providing fundamental, step-by-step derivations of the statistical quantities needed in a largely self-contained treatment carried out in the framework of modern quantum optics. The results extend the analysis of signal-to-noise to a wider variety of practical conditions and provide the basis for multispectral intensity correlation imaging system design.

2. PHOTODETECTOR OUTPUT STATISTICS: BASIC THEORY

We consider the following situation. Light from an extended, incoherent, thermal source is received by two afocal telescopes. The light collected by each telescope is directed to a photodetector and the output signal of the photodetector is acted upon by some signal shaping electronics. We denote the impulse response of the photodetector and the signal conditioning electronics by $k\left(t\right)$. The photodetector and conditioning electronics has response time T_d. Hence, the output of the signal conditioning electronics, $J_k\left(t\right), k=1,2$, can be expressed as:

$$J_n\left(t\right)=\sum_{j_n}k\left(t-t_{j_n}\right) \tag{2.1}$$

where the sum is to be taken over the various random photon arrival times, t_{j_n}, occurring at telescope *n*.

We determine the time average of each output. The symbol $\left\langle\cdots\right\rangle_{T_a}$ denotes the time average, defined as;

$$\left\langle\cdots\right\rangle_{T_a}=\frac{1}{T_a}\int_{t-T_a}^{t}\left(\cdots\right)d\xi \tag{2.2}$$

where T_a denotes the averaging time, where $T_a >> T_d$ is assumed here. We reserve the symbol $\left\langle\cdots\right\rangle$ for the ensemble average. We next subtract $\left\langle J_n\left(t\right)\right\rangle_{T_a}$ from $J_n\left(t\right)$ for $n=1,2$ to obtain the fluctuating component of the output of each photodetector:

$$\Delta_{T_a}J_n\left(t\right)=J_n\left(t\right)-\left\langle J_n\left(t\right)\right\rangle_{T_a} \tag{2.3}$$

More generally, the prefixes "Δ" and "Δ_{T_a}" indicate fluctuations about the ensemble average and about the time average, respectively. In other words, for some quantity, $A\left(t\right)$:

$$\Delta A\left(t\right)=A\left(t\right)-\left\langle A\left(t\right)\right\rangle,\ \Delta_{T_a}A\left(t\right)=A\left(t\right)-\left\langle A\left(t\right)\right\rangle_{T_a} \tag{2.4.a,b}$$

As the final step in this process, we multiply the two fluctuating outputs and average the result over T_a. Assuming that all processes are stationary and ergodic, then as T_a increases without bound, the time averages coincide with ensemble averages. In this limit, as discovered by Brown and Twiss, the correlation

$\left\langle \Delta_{T_a} J_1(t) \Delta_{T_a} J_2(t) \right\rangle_{T_a}$ is proportional to the square of the magnitude of the normalized mutual coherence of the incoming radiation at the two telescope locations.

Regarding the incident field, we assume that the field is quasi-monochromatic, i.e. that it is confined to a narrow spectral band, $\Delta \nu_c$, centered at frequency $\bar{\nu}$ such that $\Delta \nu_c << \bar{\nu}$. For the moment, we consider a single spectral band in evaluating the signal statistics. In a later section, we extend these results to the simultaneous correlation of multiple spectral bands. Following [11] we define the spectral range, $\Delta \nu$, from considerations of the extent of a unit cell of photon phase space. With definitions (4.3-78 to 4.3-81) in [11], pp.179-180, we have:

$$T_c \Delta \nu_c = 1 \tag{2.5}$$

where T_c is the correlation time of the light defined in (2.25) below. We make the "slow detector assumption" that $T_c << T_d$.

In calculating the desired statistical characteristics of, $\left\langle \Delta_{T_a} J_1(t) \Delta_{T_a} J_2(t) \right\rangle_{T_a}$ one could follow the procedure in reference [11], to produce results pertaining to any impulse response function, $k(t)$. However, our object is to evaluate higher order statistics than are considered in [11] and calculations for a general impulse response would prove very laborious while yielding little insight. Hence, we resort to simplifying approximations for $k(t)$.

Assume that $k(t)$ rises rapidly to a plateau and then falls rapidly to zero at $t=T_d$, and therefore approximates a rectangular pulse:

$$k(t) = \begin{cases} \kappa, \ 0 \le t \le T_d \\ 0, \ \text{otherwise} \end{cases} \tag{2.6}$$

Where κ is a positive constant denoting the output peak response. With this form for the impulse response function, it is evident from that $J_n(t), n = 1,2$ is proportional to the number of detection events in [t-T_d, t]:

$$J_k(t) = \kappa n_k(t, T_d) \tag{2.7}$$

where $n_k(t, T_d)$ is the number of detection events occurring during the time interval $\left[t - T_d, t\right)$. This relation considerably simplifies calculation of the required statistics.

The primary statistics to be calculated are the mean and standard deviation of $\left\langle \Delta_{T_a} J_1(t) \Delta_{T_a} J_2(t) \right\rangle_{T_a}$. To evaluate these quantities, we need to assemble the basic probabilistic relations for $n_k(t, T_d)$. Within the quantum theory, the joint probability density of $n_1(t, T_d)$ and $n_2(t, T_d)$ is:

$$p(n_1, n_2; t, T_d) = \left\langle \mathcal{T} : \frac{1}{n_1! n_2!} \hat{W}_1^{n_1} \hat{W}_2^{n_2} e^{-(\hat{W}_1 + \hat{W}_2)} : \right\rangle \tag{2.8}$$

where $\mathcal{T}$ and $(:\cdots:)$ are the time ordering and the normal ordering symbols and where:

$$\hat{W}_k = \alpha c \int_{S_k} d^2 x_k \int_{t-T_d}^{t} dt' \, \hat{I}(\mathbf{x}_k, t') \quad (k = 1,2) \tag{2.9}$$

Here, α is the dimensionless quantum efficiency of the detectors (assumed the same for all detectors). The two-dimensional vector, $\mathbf{x}_k$, is the position vector of points on the entrance pupil, S_k, of the k$^{\text{th}}$ telescope (where we assume here that the entrance pupil maps one-to-one onto the detector aperture). We suppose that both telescopes have total aperture area *S*. $\hat{I}(\mathbf{x}_k, t)$ is the number density per unit volume in the field incident upon the entrance pupil of either telescope:

$$\hat{I}\left(\mathbf{x}_k,t\right)=\hat{\mathbf{V}}^{\dagger}\left(\mathbf{x}_k,t\right)*\hat{\mathbf{V}}\left(\mathbf{x}_k,t\right) \tag{2.10}$$

where $\hat{V}\left(\mathbf{x}_k,t\right)$ is the configuration space photon localization operator. In writing (2.9) we assume that there is nearly normal incidence on both detectors. Note that in view of the optical equivalence theorem for normally ordered operators (2.8) can be expressed as:

$$\begin{gathered}
p\left(n_1,n_2;t,T_d\right)=\int\phi\left(\{v\}\right)\frac{1}{n_1!n_2!}W_1^{n_1}W_2^{n_2}e^{-(W_1+W_2)}d\{v\} \\
W_k=\alpha c\int_{S_k}d^2x_k\int_{t-T_d}^{t}dt'\,I\left(\mathbf{x}_k,t'\right)\quad\left(k=1,2\right) \\
I\left(\mathbf{x}_k,t\right)=\mathbf{V}^{\dagger}\left(\mathbf{x}_k,t\right)*\mathbf{V}\left(\mathbf{x}_k,t\right)
\end{gathered} \tag{2.11}$$

Where $\phi\left(\{v\}\right)$ is the phase space functional or the diagonal representation of the density operator for the field. $\mathbf{V}\left(\mathbf{x}_k,t\right)$ is the right eigenvalue of $\hat{\mathbf{V}}\left(\mathbf{x}_k,t\right)$ belonging to the coherent state $\{v\}$. For a thermal source, $\phi\left(\{v\}\right)$ is actually a probability density that is jointly normal in all the attendant random variables.

Using (2..11), it is evident that by exchanging the phase space integrations with the summations over n_1 and n_2, one can express any moment, such as $\left\langle n_1^r n_2^s\right\rangle$, in the form:

$$\begin{gathered}
\left\langle n_1^r n_2^s\right\rangle=\left\langle\left\langle n_1^r n_2^s\right\rangle_I\right\rangle_\phi \\
\left\langle n_1^r n_2^s\right\rangle_I=\left\{\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\frac{n_1^r n_2^s}{n_1!n_2!}W_1^{n_1}W_2^{n_2}e^{-(W_1+W_2)}\right\}_{I(\mathbf{x}_1,t)\text{ and }I(\mathbf{x}_2,t)\text{ fixed}} \\
\left\langle\cdots\right\rangle_\phi=\int\phi\left(\{v\}\right)\left(\cdots\right)d\{v\}
\end{gathered} \tag{2.12}$$

$\left\langle n_1^r n_2^s\right\rangle_I$ may be considered as the conditional expectation of $n_1^r n_2^s$ given particular distributions for the functions $I\left(\mathbf{x}_1,t\right)$ and $I\left(\mathbf{x}_2,t\right)$, with independent Poisson distributions for n_1 and n_2. Then $\left\langle n_1^r n_2^s\right\rangle_I$ is a functional of $I\left(\mathbf{x}_1,t\right)$ and $I\left(\mathbf{x}_2,t\right)$ with $W_k=\alpha c\int_{S_k}d^2x_k\int_{t-T_d}^{t}dt'\,\mathbf{V}^*\left(\mathbf{x}_k,t\right)*\mathbf{V}\left(\mathbf{x}_k,t\right)$. Therefore, the operation $\left\langle\cdots\right\rangle_\phi$ is the ensemble average over the statistics of the functions $\mathbf{V}\left(\mathbf{x}_k,t\right)$, $k=1,2$, all of which are jointly Gaussianly distributed.

This manner of evaluating moments is particularly convenient due to the properties of Poisson distributions. In particular, define the r^{th} *factorial moment* of $n\left(\mathfrak{I}\right)$ as:

$$\left\langle n_k^{(r)}\right\rangle_I=\left\langle n_k\left(n_k-1\right)\left(n_k-2\right)\ldots\left(n_k-r+1\right)\right\rangle_I \tag{2.13}$$

We can determine moments of any order by using the elegant result that:

$$\left\langle n_1^{(r)}n_2^{(s)}\right\rangle_{I(\mathbf{r},t)}=\left(\left\langle n_1\right\rangle_I\right)^r\left(\left\langle n_2\right\rangle_I\right)^s \tag{2.14}$$

where the average of n_k conditional upon $I\left(\mathbf{x}_1,t\right)$ and $I\left(\mathbf{x}_2,t\right)$ is given by:

$$\left\langle n_k\left(t,T_d\right)\right\rangle_I=W_k=\alpha c\int_{S_k}d^2x_k\int_{t-T_d}^{t}dt'\,I\left(\mathbf{x}_k,t'\right)\quad\left(k=1,2\right) \tag{2.15}$$

The phase space averaging generally involves evaluation of *cross-correlation functions of order 2M*:

$$\Gamma^{(2M)}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2\right)=\left\langle I\left(\mathbf{r}_1,t_1\right)I\left(\mathbf{r}_2,t_2\right)\ldots I\left(\mathbf{r}_M,t_M\right)\right\rangle \tag{2.16}$$

Or, substituting $I\left(\mathbf{x}_k,t\right)=\mathbf{V}^{\dagger}\left(\mathbf{x}_k,t\right)*\mathbf{V}\left(\mathbf{x}_k,t\right)$:

$$\Gamma^{(2M)}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right) = \left\langle V^*\left(\mathbf{r}_1,t_1\right)V^*\left(\mathbf{r}_2,t_2\right)\ldots V^*\left(\mathbf{r}_M,t_M\right)V\left(\mathbf{r}_1,t_1\right)V\left(\mathbf{r}_2,t_2\right)\ldots V\left(\mathbf{r}_M,t_M\right)\right\rangle \tag{2.17}$$

Since $V\left(\mathbf{r},t\right)$ is a Gaussian process, the Gaussian moment theorem implies that:

$$\Gamma^{(2M)}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right) = \sum_{\pi}\Gamma\left(\mathbf{r}_{i_1},t_{i_1};\mathbf{r}_{j_1},t_{j_1}\right)\Gamma\left(\mathbf{r}_{i_2},t_{i_2};\mathbf{r}_{j_2},t_{j_2}\right)\ldots\Gamma\left(\mathbf{r}_{i_M},t_{i_M};\mathbf{r}_{j_M},t_{j_M}\right)$$
$$\Gamma\left(\mathbf{r}_i,t_i;\mathbf{r}_j,t_j\right) = \left\langle V^*\left(\mathbf{r}_i,t_i\right)V\left(\mathbf{r}_j,t_j\right)\right\rangle \tag{2.18.a,b}$$

where the subscripts i_p and j_q, $\left(1\le i_p\le M, 1\le j_q\le M\right)$ are integers and $\sum_{\pi}$ denotes summation over all the $M!$ possible permutations of the subscripts.

Equations (2.16)-(2.18) pertain to scalar fields. Extension to vector fields is straightforward. Let the z-axis be the direction of propagation at the space-time point $\left(\mathbf{r},t\right)$. Then the instantaneous intensity is:

$$I\left(\mathbf{r},t\right) = I_x\left(\mathbf{r},t\right) + I_y\left(\mathbf{r},t\right)$$
$$I_x\left(\mathbf{r},t\right) = V_x^*\left(\mathbf{r},t\right)V_x\left(\mathbf{r},t\right) \tag{2.19.a-c}$$
$$I_y\left(\mathbf{r},t\right) = V_y^*\left(\mathbf{r},t\right)V_y\left(\mathbf{r},t\right)$$

Here, $V_x\left(\mathbf{r},t\right)$ and $V_y\left(\mathbf{r},t\right)$ are the x- and y-axis components of the analytic vector field representing the electric field. For an unpolarized source, these are zero-mean, uncorrelated, and identically distributed Gaussian processes. In particular the 2nd order correlation function for x and y components vanishes:

$$\Gamma^{(2)}_{x,y}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2\right) = \left\langle V_x^*\left(\mathbf{r}_1,t_1\right)V_y\left(\mathbf{r}_2,t_2\right)\right\rangle = 0 \tag{2.20}$$

Moreover, from the point of view of their correlation properties, the x- and y-components of the field are indistinguishable:

$$\Gamma^{(2M)}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right) = \Gamma^{(2M)}_{x,x}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right) + \Gamma^{(2M)}_{y,y}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right)$$
$$\Gamma^{(2M)}_{x,x}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right) = \Gamma^{(2M)}_{y,y}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right)$$
$$\left\langle I_x\left(\mathbf{r},t\right)\right\rangle = \left\langle I_y\left(\mathbf{r},t\right)\right\rangle = \tfrac{1}{2}\bar{I}\left(\mathbf{r},t\right)$$

(2.21.a,b)

Where $\bar{I}\left(\mathbf{r},t\right)$ denotes $\left\langle I\left(\mathbf{r},t\right)\right\rangle$.

In view of the above relations, one can see that the counterpart of (2.18) for un-polarized light is:

$$\Gamma^{(2M)}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2;\ldots;\mathbf{r}_M,t_M\right) = \frac{1}{2^M}\sum_{\pi}\Gamma^{(2)}\left(\mathbf{r}_{i_1},t_{i_1};\mathbf{r}_{j_1},t_{j_1}\right)\Gamma^{(2)}\left(\mathbf{r}_{i_2},t_{i_2};\mathbf{r}_{j_2},t_{j_2}\right)\ldots\Gamma^{(2)}\left(\mathbf{r}_{i_M},t_{i_M};\mathbf{r}_{j_M},t_{j_M}\right) \tag{2.22}$$

where, again, $\sum_{\pi}$ denotes summation over all the $M!$ possible permutations of the subscripts.

Finally, we also assume the light is cross-spectrally pure. Under this assumption, the normalized cross-correlation, defined by:

$$\gamma\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2\right)=\frac{\Gamma^{(2)}\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2\right)}{\sqrt{\bar{I}\left(\mathbf{r}_1,t_1\right)\bar{I}\left(\mathbf{r}_2,t_2\right)}} \tag{2.23}$$

takes the form (M&W, Section 4.5.1):

$$\gamma\left(\mathbf{r}_1,t_1;\mathbf{r}_2,t_2\right)=\gamma\left(\mathbf{r}_1,\mathbf{r}_2,0\right)\gamma\left(t_2-t_1\right) \tag{2.24}$$

where $\gamma\left(t_2-t_1\right)$ is the normalized autocorrelation function of the field, or the Fourier transform of the normalized spectral density. Then a natural measure of the coherence time of the light is the integral:

$$T_c=\int_{-\infty}^{\infty}\left|\gamma\left(\tau\right)\right|^2 d\tau \tag{2.25}$$

3. THE MEAN VALUE OF $\left\langle \Delta_{T_a}J_1\left(t\right)\Delta_{T_a}J_2\left(t\right)\right\rangle_{T_a}$ - THE BROWN-TWISS EFFECT

In this section, we determine the mean value of $\left\langle \Delta_{T_a}J_1\left(t\right)\Delta_{T_a}J_2\left(t\right)\right\rangle_{T_a}$ and thereby derive the basic relationship discovered by Brown and Twiss. Using (2.2), (2.3) and (2.7), we have:

$$\left\langle \Delta_{T_a}J_1\left(t\right)\Delta_{T_a}J_2\left(t\right)\right\rangle_{T_a}=\kappa^2\left[\frac{1}{T_a}\int_{t-T_a}^{t}n_1\left(\xi,T_d\right)n_2\left(\xi,T_d\right)d\xi-\left\langle n_1\left(t,T_d\right)\right\rangle_{T_a}\left\langle n_2\left(t,T_d\right)\right\rangle_{T_a}\right] \tag{3.1}$$

Now we commence to determine the mean value of this quantity. Taking the conditional expectation of both sides in (3.1) and using (2.14) and (2.15):

$$\begin{aligned}\left\langle\left\langle \Delta_{T_a}J_1\left(t\right)\Delta_{T_a}J_2\left(t\right)\right\rangle_{T_a}\right\rangle&=\left\langle\kappa^2\left[\frac{1}{T_a}\int_{t-T_a}^{t}\left\langle n_1\left(\xi,T_d\right)\right\rangle_I\left\langle n_2\left(\xi,T_d\right)\right\rangle_I d\xi-\left\langle\left\langle n_1\left(t,T_d\right)\right\rangle_I\right\rangle_{T_a}\left\langle\left\langle n_2\left(t,T_d\right)\right\rangle_I\right\rangle_{T_a}\right]\right\rangle_\phi\\&=\kappa^2\alpha^2c^2\int_{S_1}d^2x_1\int_{S_2}d^2x_2\frac{1}{T_a}\int_{t-T_a}^{t}d\xi\left[\begin{array}{l}\int_{\xi-T_d}^{\xi}dt_1'\int_{\xi-T_d}^{\xi}dt_2'\left\langle I\left(\mathbf{x}_1,t_1'\right)I\left(\mathbf{x}_2,t_2'\right)\right\rangle_\phi\\-\frac{1}{T_a}\int_{t-T_a}^{t}d\xi_1\int_{\xi-T_d}^{\xi}dt_1'\int_{\xi_1-T_d}^{\xi_1}dt_2'\left\langle I\left(\mathbf{x}_1,t_1'\right)I\left(\mathbf{x}_2,t_2'\right)\right\rangle_\phi\end{array}\right]\end{aligned} \tag{3.2}$$

Using the Gaussian moment theorem, we have:

$$\begin{aligned}\left\langle I\left(\mathbf{x}_1,t_1'\right)I\left(\mathbf{x}_2,t_2'\right)\right\rangle_\phi&=\left\langle\left(V_{x1}^*V_{x1}+V_{y1}^*V_{y1}\right)\left(V_{x2}^*V_{x2}+V_{y2}^*V_{y2}\right)\right\rangle_\phi\\&=\left\langle V_{x1}^*V_{x1}V_{x2}^*V_{x2}\right\rangle_\phi+\left\langle V_{x1}^*V_{x1}V_{y2}^*V_{y2}\right\rangle_\phi+\left\langle V_{y1}^*V_{y1}V_{x2}^*V_{x2}\right\rangle_\phi+\left\langle V_{y1}^*V_{y1}V_{y2}^*V_{y2}\right\rangle_\phi\\&=\bar{I}\left(\mathbf{x}_1\right)\bar{I}\left(\mathbf{x}_2\right)\left[1+\frac{1}{2}\left|\gamma\left(\mathbf{x}_1,t_1';\mathbf{x}_2,t_2'\right)\right|^2\right]\end{aligned} \tag{3.3}$$

Substituting this into (3.2) and employing the cross-spectrally pure assumption and stationarity gives:

$$\begin{aligned}&\left\langle\left\langle \Delta_{T_a}J_1\left(t\right)\Delta_{T_a}J_2\left(t\right)\right\rangle_{T_a}\right\rangle\\&=\kappa^2\alpha^2c^2\int_{S_1}d^2x_1\int_{S_2}d^2x_2\frac{1}{T_a}\int_{t-T_a}^{t}d\xi\left[\begin{array}{l}\int_{\xi-T_d}^{\xi}dt_1'\int_{\xi-T_d}^{\xi}dt_2'\left\langle I\left(\mathbf{x}_1,t_1'\right)I\left(\mathbf{x}_2,t_2'\right)\right\rangle_\phi\\-\frac{1}{T_a}\int_{t-T_a}^{t}d\xi_1\int_{\xi-T_d}^{\xi}dt_1'\int_{\xi_1-T_d}^{\xi_1}dt_2'\left\langle I\left(\mathbf{x}_1,t_1'\right)I\left(\mathbf{x}_2,t_2'\right)\right\rangle_\phi\end{array}\right]\\&=\frac{1}{2}\kappa^2\alpha^2c^2T_cT_dG\int_{S_1}d^2x_1\int_{S_2}d^2x_2\bar{I}\left(\mathbf{x}_1\right)\bar{I}\left(\mathbf{x}_2\right)\left|\gamma\left(\mathbf{x}_1,\mathbf{x}_2,0\right)\right|^2\end{aligned} \tag{3.4}$$

where:

$$G\,\square\,\frac{1}{T_aT_cT_d}\int_{t-T_a}^{t}d\xi\left[\int_{\xi-T_d}^{\xi}dt_1'\int_{\xi-T_d}^{\xi}dt_2'\left|\gamma\left(\left|t_2'-t_1'\right|\right)\right|^2-\frac{1}{T_a}\int_{t-T_a}^{t}d\xi_1\int_{\xi-T_d}^{\xi}dt_1'\int_{\xi_1-T_d}^{\xi_1}dt_2'\left|\gamma\left(\left|t_2'-t_1'\right|\right)\right|^2\right] \tag{3.5}$$

Considering the first integral in G, as long as neither t_1' nor t_2' are within several multiples of T_c of their integration limits, the limits of the inner integral may be taken from minus to plus infinity, with an error of order $T_c/T_d << 1$:

$$\int_{\xi-T_d}^{\xi} dt_1' \int_{\xi-T_d}^{\xi} dt_2' \left|\gamma\left(\left|t_2'-t_1'\right|\right)\right|^2 \cong \int_{\xi-T_d}^{\xi} dt_1' \int_{-\infty}^{\infty} d\tau \left|\gamma(\tau)\right|^2 = T_d T_c \tag{3.6.a}$$

where use has been made of (2.25). For the second integral, similar reasoning leads to:

$$\int_{t-T_a}^{t} d\xi \frac{1}{T_a} \int_{t-T_a}^{t} d\xi_1 \int_{\xi-T_d}^{\xi} dt_1' \int_{\xi_1-T_d}^{\xi_1} dt_2' \left|\gamma\left(\left|t_2'-t_1'\right|\right)\right|^2 \cong 2T_d^2 T_c \tag{3.6.b}$$

Hence, substituting (3.6) into (3.5), we find $G = 1 + 2T_d/T_a + O\left(T_c/T_d\right)$, and aside from terms of order $T_c/T_d << 1$, (3.4) becomes:

$$\left\langle \left\langle \Delta_{T_a} J_1(t) \Delta_{T_a} J_2(t) \right\rangle_{T_a} \right\rangle = \frac{1}{2} \kappa^2 \alpha^2 c^2 T_c T_d \left(1 - 2T_d/T_a\right) \int_{S_1} d^2 x_1 \int_{S_2} d^2 x_2 \bar{I}(\mathbf{x}_1) \bar{I}(\mathbf{x}_2) \left|\gamma(\mathbf{x}_1, \mathbf{x}_2, 0)\right|^2 \tag{3.7}$$

This is the basic expression of the Hanbury Brown-Twiss effect [7].

4. Standard Deviation of $\left\langle \Delta_{T_a} J_1(t) \Delta_{T_a} J_2(t) \right\rangle_{T_a}$ - SNR Calculation

Using (2.7):

$$\begin{aligned} \left\langle \left(\Delta\left[\left\langle \Delta_{T_a} J_1(t) \Delta_{T_a} J_2(t) \right\rangle_{T_a} \right] \right)^2 \right\rangle = \kappa^4 \frac{1}{T_a^2} \Bigg\langle \int_{t-T_a}^{t} d\xi_1 \int_{t-T_a}^{t} d\xi_2 & \begin{bmatrix} \left(n_1(\xi_1, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\xi_1, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \\ - \left\langle \left(n_1(\xi_1, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\xi_1, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \right\rangle \end{bmatrix} \\ & \times \begin{bmatrix} \left(n_1(\xi_2, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\xi_2, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \\ - \left\langle \left(n_1(\xi_2, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\xi_2, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \right\rangle \end{bmatrix} \Bigg\rangle \end{aligned} \tag{4.1}$$

Next, change the variables of integration to:

$$\tau = \xi_2 - \xi_1, \quad \zeta = \tfrac{1}{2}\left(\xi_1 + \xi_2\right) \tag{4.2}$$

Then:

$$\begin{aligned} & \left\langle \left(\Delta\left[\left\langle \Delta_{T_a} J_1(t) \Delta_{T_a} J_2(t) \right\rangle_{T_a} \right] \right)^2 \right\rangle \\ & = \kappa^4 \frac{1}{T_a^2} \Bigg\langle \int_{t-T_a}^{t} d\zeta \int_{2\min[\zeta-t, t-\zeta-T_a]}^{2\max[t-\zeta, \zeta-t+T_a]} d\tau \begin{bmatrix} \left(n_1(\zeta - \tfrac{1}{2}\tau, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\zeta - \tfrac{1}{2}\tau, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \\ - \left\langle \left(n_1(\zeta - \tfrac{1}{2}\tau, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\zeta - \tfrac{1}{2}\tau, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \right\rangle \end{bmatrix} \\ & \qquad \times \begin{bmatrix} \left(n_1(\zeta + \tfrac{1}{2}\tau, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\zeta + \tfrac{1}{2}\tau, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \\ - \left\langle \left(n_1(\zeta + \tfrac{1}{2}\tau, T_d) - \langle n_1(t, T_d) \rangle_{T_a} \right) \left(n_2(\zeta + \tfrac{1}{2}\tau, T_d) - \langle n_2(t, T_d) \rangle_{T_a} \right) \right\rangle \end{bmatrix} \Bigg\rangle \end{aligned} \tag{4.3}$$

But we now note that if $|\tau| \geq T_d + lT_c$ for some multiple, l, of correlation times, $n_1\left(\zeta - \tfrac{1}{2}\tau, T_d\right)$ comprises detection events that are statistically independent of those events contributing to $n_1\left(\zeta + \tfrac{1}{2}\tau, T_d\right)$. The same remarks apply to $n_2\left(\zeta - \tfrac{1}{2}\tau, T_d\right)$ and $n_2\left(\zeta + \tfrac{1}{2}\tau, T_d\right)$. Thus when $|\tau| \geq T_d + lT_c$, the two main factors in the integrand are nearly statistically independent. Since the average of each factor separately is zero, it follows that the integrand is negligible when $|\tau| \geq T_d + lT_c$. Then except for contributions of order $T_c/T_d << 1$, we have:

$$\left\langle\left(\Delta\left[\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right]\right)^2\right\rangle=\kappa^4\frac{1}{T_a^2}\left\langle\int_{t-T_a}^{t}d\zeta\int_{\max\{-T_d,2\min[\zeta-t,t-\zeta-T_a]\}}^{\min\{T_d,2\max[t-\zeta,\zeta-t+T_a]\}}d\tau\begin{bmatrix}\left(n_1\left(\zeta-\tfrac{1}{2}\tau,T_d\right)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2\left(\zeta-\tfrac{1}{2}\tau,T_d\right)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\\-\left\langle\left(n_1\left(\zeta-\tfrac{1}{2}\tau,T_d\right)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2\left(\zeta-\tfrac{1}{2}\tau,T_d\right)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\right\rangle\end{bmatrix}\times\begin{bmatrix}\left(n_1\left(\zeta+\tfrac{1}{2}\tau,T_d\right)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2\left(\zeta+\tfrac{1}{2}\tau,T_d\right)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\\-\left\langle\left(n_1\left(\zeta+\tfrac{1}{2}\tau,T_d\right)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2\left(\zeta+\tfrac{1}{2}\tau,T_d\right)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\right\rangle\end{bmatrix}\right\rangle \tag{4.4}$$

Moreover, the terms $\max\left[t-\zeta,\zeta-t+T_a\right]$ and $\min\left[\zeta-t,t-\zeta-T_a\right]$ to the upper and lower limits of the τ integration produce contributions of order $T_d/T_a<<1$. Neglecting these and invoking stationarity, we get:

Next, invoking stationarity, we find:

$$\left\langle\left(\Delta\left[\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right]\right)^2\right\rangle=\kappa^4\frac{2}{T_a}\int_0^{T_d}d\tau\left\langle\begin{bmatrix}\left(n_1(0,T_d)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2(0,T_d)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\\-\left\langle\left(n_1(0,T_d)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2(0,T_d)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\right\rangle\end{bmatrix}\times\begin{bmatrix}\left(n_1(\tau,T_d)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2(\tau,T_d)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\\-\left\langle\left(n_1(\tau,T_d)-\left\langle n_1(t,T_d)\right\rangle_{T_a}\right)\left(n_2(\tau,T_d)-\left\langle n_2(t,T_d)\right\rangle_{T_a}\right)\right\rangle\end{bmatrix}\right\rangle+O\left(\frac{T_d}{T_a},\frac{T_c}{T_d}\right) \tag{4.5}$$

Note that $n_k(0,T_d)=n_k(\tau-T_d,\tau)+n_k(0,\tau-T_d)$ and $n_k(\tau,T_d)=n_k(\tau,\tau)+n_k(0,T_d-\tau)$:

$$\begin{aligned}&\left\langle\left(\Delta\left[\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right]\right)^2\right\rangle\\&=\kappa^4\frac{2}{T_a}\int_0^{T_d}d\tau\left\langle\begin{bmatrix}\left(n_1(\tau-T_d,\tau)-\left\langle n_1(\tau-T_d,\tau)\right\rangle_{T_a}\right)\left(n_2(\tau-T_d,\tau)-\left\langle n_2(\tau-T_d,\tau)\right\rangle_{T_a}\right)-\left\langle\left(n_1(\tau-T_d,\tau)-\left\langle n_1(\tau-T_d,\tau)\right\rangle_{T_a}\right)\left(n_2(\tau-T_d,\tau)-\left\langle n_2(\tau-T_d,\tau)\right\rangle_{T_a}\right)\right\rangle\\+\left(n_1(\tau-T_d,\tau)-\left\langle n_1(\tau-T_d,\tau)\right\rangle_{T_a}\right)\left(n_2(0,\tau-T_d)-\left\langle n_2(0,\tau-T_d)\right\rangle_{T_a}\right)-\left\langle\left(n_1(\tau-T_d,\tau)-\left\langle n_1(\tau-T_d,\tau)\right\rangle_{T_a}\right)\left(n_2(0,T_d-\tau)-\left\langle n_2(0,T_d-\tau)\right\rangle_{T_a}\right)\right\rangle\\+\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(\tau-T_d,\tau)-\left\langle n_2(\tau-T_d,\tau)\right\rangle_{T_a}\right)-\left\langle\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(\tau-T_d,\tau)-\left\langle n_2(\tau-T_d,\tau)\right\rangle_{T_a}\right)\right\rangle\\+\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(0,T_d-\tau)-\left\langle n_2(0,T_d-\tau)\right\rangle_{T_a}\right)-\left\langle\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(0,T_d-\tau)-\left\langle n_2(0,T_d-\tau)\right\rangle_{T_a}\right)\right\rangle\end{bmatrix}\right.\\&\qquad\left.\times\begin{bmatrix}\left(n_1(\tau,\tau)-\left\langle n_1(\tau,\tau)\right\rangle_{T_a}\right)\left(n_2(\tau,\tau)-\left\langle n_2(\tau,\tau)\right\rangle_{T_a}\right)-\left\langle\left(n_1(\tau,\tau)-\left\langle n_1(\tau,\tau)\right\rangle_{T_a}\right)\left(n_2(\tau,\tau)-\left\langle n_2(\tau,\tau)\right\rangle_{T_a}\right)\right\rangle\\+\left(n_1(\tau,\tau)-\left\langle n_1(\tau,\tau)\right\rangle_{T_a}\right)\left(n_2(0,T_d-\tau)-\left\langle n_2(0,T_d-\tau)\right\rangle_{T_a}\right)-\left\langle\left(n_1(\tau,\tau)-\left\langle n_1(\tau,\tau)\right\rangle_{T_a}\right)\left(n_2(0,T_d-\tau)-\left\langle n_2(0,T_d-\tau)\right\rangle_{T_a}\right)\right\rangle\\+\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(\tau,\tau)-\left\langle n_2(\tau,\tau)\right\rangle_{T_a}\right)-\left\langle\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(\tau,\tau)-\left\langle n_2(\tau,\tau)\right\rangle_{T_a}\right)\right\rangle\\+\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(0,T_d-\tau)-\left\langle n_2(0,T_d-\tau)\right\rangle_{T_a}\right)-\left\langle\left(n_1(0,T_d-\tau)-\left\langle n_1(0,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(0,T_d-\tau)-\left\langle n_2(0,T_d-\tau)\right\rangle_{T_a}\right)\right\rangle\end{bmatrix}\right\rangle\end{aligned} \tag{4.6}$$

We can see that contributions from $n_k(\tau-T_d,\tau),n_k(\tau,\tau)$, and $n_k(0,T_d-\tau)$ are approximately mutually statistically independent. In consequence, we have:

$$\begin{aligned}&\left\langle\left(\Delta\left[\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right]\right)^2\right\rangle\\&=\kappa^4\frac{2}{T_a}\int_0^{T_d}d\tau\left\langle\left[\left(n_1(T_d,T_d-\tau)-\left\langle n_1(T_d,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(T_d,T_d-\tau)-\left\langle n_2(T_d,T_d-\tau)\right\rangle_{T_a}\right)\right]^2-\left\langle\left(n_1(T_d,T_d-\tau)-\left\langle n_1(T_d,T_d-\tau)\right\rangle_{T_a}\right)\left(n_2(T_d,T_d-\tau)-\left\langle n_2(T_d,T_d-\tau)\right\rangle_{T_a}\right)\right\rangle^2\right\rangle\end{aligned} \tag{4.7}$$

where we again invoked stationarity, shifting the time arguments by T_d. Now we are ready to use (2.13) and (2.14) to evaluate the integrand above. Dropping the time arguments for the moment:

$$\left\langle\left\langle\left[\left(n_1-\langle n_1\rangle_{T_a}\right)\left(n_2-\langle n_2\rangle_{T_a}\right)\right]^2\right\rangle_I\right\rangle_\phi-\left\langle\left(n_1-\langle n_1\rangle_{T_a}\right)\left(n_2-\langle n_2\rangle_{T_a}\right)\right\rangle^2$$

$$\begin{aligned}
&=\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi-2\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi\left(\langle n_1\rangle+\langle n_2\rangle\right)+\left\langle\langle n_1\rangle_I\langle n_2\rangle_I^2\right\rangle_\phi+\left\langle\langle n_1\rangle_I^2\langle n_2\rangle_I\right\rangle_\phi \\
&+4\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi\langle n_1\rangle\langle n_2\rangle-\left(\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi\right)^2-2\left\langle\langle n_1\rangle_I\langle n_2\rangle_I^2\right\rangle_\phi\langle n_1\rangle-2\left\langle\langle n_1\rangle_I^2\langle n_2\rangle_I\right\rangle_\phi\langle n_2\rangle \\
&+\left\langle\langle n_1\rangle_I^2\langle n_2\rangle_I^2\right\rangle_\phi+\langle n_1\rangle^2\left\langle\langle n_2\rangle_I^2\right\rangle_\phi+\langle n_2\rangle^2\left\langle\langle n_1\rangle_I^2\right\rangle_\phi+2\langle n_1n_2\rangle\langle n_1\rangle\langle n_2\rangle \\
&+\langle n_1\rangle^2\langle n_2\rangle+\langle n_1\rangle\langle n_2\rangle^2-4\langle n_1\rangle^2\langle n_2\rangle^2
\end{aligned} \tag{4.8}$$

According to (2.15): $\left\langle n_k\left(T_d,T_d-\tau\right)\right\rangle_I=\alpha c\int_{S_k}d^2x_k\int_\tau^{T_d}dt'\,I\left(\mathbf{x}_k,t'\right)\quad(k=1,2)$. Now consider the term $\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi$:

$$\begin{aligned}
\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi&=\alpha^2c^2\int_{S_1}d^2x_1\int_{S_2}d^2x_2\int_\tau^{T_d}dt_1'\int_\tau^{T_d}dt_2'\left\langle I\left(\mathbf{x}_1,t_1'\right)I\left(\mathbf{x}_2,t_2'\right)\right\rangle \\
&=\langle n_1\rangle\langle n_2\rangle+\alpha^2c^2\int_{S_1}d^2x_1\int_{S_2}d^2x_2\int_\tau^{T_d}dt_1'\int_\tau^{T_d}dt_2'\left[\left\langle I\left(\mathbf{x}_1,t_1'\right)I\left(\mathbf{x}_2,t_2'\right)\right\rangle-\overline{I}\left(\mathbf{x}_1\right)\overline{I}\left(\mathbf{x}_2\right)\right]
\end{aligned} \tag{4.9}$$

Using (3.3) and (2.24):

$$\begin{aligned}
&\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi-\langle n_1\rangle\langle n_2\rangle \\
&=\tfrac{1}{2}\alpha^2c^2\int_{S_1}d^2x_1\int_{S_2}d^2x_2\overline{I}\left(\mathbf{x}_1\right)\overline{I}\left(\mathbf{x}_2\right)\left|\gamma\left(\mathbf{x}_1,\mathbf{x}_2,0\right)\right|^2\int_\tau^{T_d}dt_1'\int_\tau^{T_d}dt_2'\left|\gamma\left(t_2'-t_1'\right)\right|^2 \\
&=\tfrac{1}{2}\alpha^2c^2\int_{S_1}d^2x_1\int_{S_2}d^2x_2\overline{I}\left(\mathbf{x}_1\right)\overline{I}\left(\mathbf{x}_2\right)\left|\gamma\left(\mathbf{x}_1,\mathbf{x}_2,0\right)\right|^2\left[\left(T_d-\tau\right)\int_0^{T_d-\tau}d\tau'\left|\gamma\left(\tau'\right)\right|^2-\int_0^{T_d-\tau}d\tau'\,\tau'\left|\gamma\left(\tau'\right)\right|^2\right]
\end{aligned} \tag{4.10}$$

The first term in brackets is clearly of order T_c and the second term is of order $T_c^{\,2}$. Thus, we obtain:

$$\left\langle\langle n_1\rangle_I\langle n_2\rangle_I\right\rangle_\phi=\langle n_1\rangle_I\langle n_2\rangle_I+O\left(T_c\right) \tag{4.11}$$

Similarly, for the other terms in (..), we obtain:

$$\left\langle\langle n_1\rangle_I\langle n_2\rangle_I^2\right\rangle_\phi=\langle n_1\rangle\langle n_2\rangle^2+O\left(T_c\right),\quad\left\langle\langle n_1\rangle_I^2\langle n_2\rangle_I\right\rangle_\phi=\langle n_2\rangle\langle n_1\rangle^2+O\left(T_c\right),\quad\left\langle\langle n_1\rangle_I^2\langle n_2\rangle_I^2\right\rangle_\phi=\langle n_2\rangle^2\langle n_1\rangle^2+O\left(T_c\right)$$

(4.12.a-c)

Considering our assumption that $T_c<<T_d<<T_a$, the $O\left(T_c\right)$ terms make negligible contributions to the integral for $\left\langle\left(\Delta\left[\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right]\right)^2\right\rangle$ in (4.7). Then (4.8) yields:

$$\begin{aligned}
\left\langle\left\langle\left[\left(n_1-\langle n_1\rangle_{T_a}\right)\left(n_2-\langle n_2\rangle_{T_a}\right)\right]^2\right\rangle_I\right\rangle_\phi-\left\langle\left(n_1-\langle n_1\rangle_{T_a}\right)\left(n_2-\langle n_2\rangle_{T_a}\right)\right\rangle^2&\cong\langle n_1\rangle\langle n_2\rangle \\
&=\alpha^2c^2\left(T_d-\tau\right)^2\int_{S_1}d^2x_1\overline{I}\left(\mathbf{x}_1\right)\int_{S_2}d^2x_2\overline{I}\left(\mathbf{x}_2\right)
\end{aligned} \tag{4.13}$$

Substitution of this into (4.7) gives:

$$\left\langle\left(\Delta\left[\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right]\right)^2\right\rangle=\kappa^4\alpha^2c^2\frac{2}{3T_a}T_d^3\int_{S_1}d^2x_1\overline{I}\left(\mathbf{x}_1\right)\int_{S_2}d^2x_2\overline{I}\left(\mathbf{x}_2\right) \tag{4.14}$$

Thus, defining the signal-to-noise ratio associated with the correlation measurement, denoted by $SNR_{\Delta J}$, as the ratio of the average of $\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}$ to the standard deviation of this quantity, we have:

$$SNR_{\Delta J}=\sqrt{\tfrac{3}{4}}\alpha cT_c\sqrt{\frac{T_a}{T_d}}\left\{\int_{S_1}d^2x_1\int_{S_2}d^2x_2\overline{I}\left(\mathbf{x}_1\right)\overline{I}\left(\mathbf{x}_2\right)\left|\gamma\left(\mathbf{x}_1,\mathbf{x}_2,0\right)\right|^2\Big/\sqrt{\int_{S_1}d^2x_1'\overline{I}\left(\mathbf{x}_1'\right)\int_{S_2}d^2x_2'\overline{I}\left(\mathbf{x}_2'\right)}\right\} \tag{4.15}$$

It is convenient to recognize that the quantity $cT_c\overline{I}\left(\mathbf{x}\right)$ is the average number of photon arrivals at location $\mathbf{x}$ per second, per Hertz per unit area. This parameter, which we denote by $\overline{N}$, is dependent only upon the physical condition and location of the source. Also, let us express results in terms of the detector bandwidth, $\Delta\nu_d \approx 1/T_d$. Then our results for the mean and SNR associated with $\left\langle \Delta_{T_a} J_1(t)\Delta_{T_a} J_2(t)\right\rangle_{T_a}$ become:

$$\left\langle\left\langle \Delta_{T_a} J_1(t)\Delta_{T_a} J_2(t)\right\rangle_{T_a}\right\rangle = \frac{1}{2}\kappa^2\alpha^2\frac{\Delta\nu_c}{\Delta\nu_d}\left(1-\tfrac{2}{T_a\Delta\nu_d}\right)\sqrt{\int_{S_1} d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2} d^2x_2'\bar{N}(\mathbf{x}_2')}\int_{S_1} d^2x_1\int_{S_2} d^2x_2\bar{\mu}(\mathbf{x}_1)\bar{\mu}(\mathbf{x}_2)\left|\gamma(\mathbf{x}_1,\mathbf{x}_2,0)\right|^2$$

$$SNR_{\Delta J} = \sqrt{\tfrac{3}{4}}\alpha\sqrt{T_a\Delta\nu_d}\left(1-\tfrac{2}{T_a\Delta\nu_d}\right)\sqrt{\int_{S_1} d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2} d^2x_2'\bar{N}(\mathbf{x}_2')}\left\{\int_{S_1} d^2x_1\int_{S_2} d^2x_2\,\bar{\mu}(\mathbf{x}_1)\bar{\mu}(\mathbf{x}_2)\left|\gamma(\mathbf{x}_1,\mathbf{x}_2,0)\right|^2\right\}$$

(4.16.a,b)

where:

$$\bar{\mu}(\mathbf{x}_k) = \bar{N}(\mathbf{x}_k)\Big/\sqrt{\int_{S_1} d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2} d^2x_2'\bar{N}(\mathbf{x}_2')} \qquad (4.17)$$

While these expressions are useful, the usual practice is not to work with $\left\langle \Delta_{T_a} J_1(t)\Delta_{T_a} J_2(t)\right\rangle_{T_a}$ directly but rather with the correlation coefficient:

$$C_{T_a} = \frac{\left\langle \Delta_{T_a} J_1(t)\Delta_{T_a} J_2(t)\right\rangle_{T_a}}{\sqrt{\left\langle\left[\Delta_{T_a} J_1(t)\right]^2\right\rangle_{T_a}\left\langle\left[\Delta_{T_a} J_2(t)\right]^2\right\rangle_{T_a}}} \qquad (4.18)$$

Where $\left\langle\left[\Delta_{T_a} J_k(t)\right]^2\right\rangle_{T_a}$ is the observed time-averaged mean square of the output fluctuations of detector *k*. Proceeding as before, we find:

$$\begin{aligned}\left\langle\left\langle\left[\Delta_{T_a} J_k(t)\right]^2\right\rangle_{T_a}\right\rangle &= \kappa^2\alpha\frac{\Delta\nu_c}{\Delta\nu_d}\sqrt{\int_{S_1} d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2} d^2x_2'\bar{N}(\mathbf{x}_2')}\\ &\times\left\{\int_{S_k} d^2x_k\bar{\mu}(\mathbf{x}_k)+\frac{1}{2}\alpha\left(1-\frac{2}{T_a\Delta\nu_d}\right)\sqrt{\int_{S_1} d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2} d^2x_2'\bar{N}(\mathbf{x}_2')}\int_{S_k} d^2x_k\int_{S_k} d^2x_k'\bar{\mu}(\mathbf{x}_k)\bar{\mu}(\mathbf{x}_k')\left|\gamma(\mathbf{x}_k,\mathbf{x}_k',0)\right|^2\right\}\end{aligned} \qquad (4.19)$$

Ignoring the higher order terms that arise from $\left\langle\left[\Delta_{T_a} J_k(t)\right]^2\right\rangle_{T_a} - \left\langle\left[\Delta_{T_a} J_k(t)\right]^2\right\rangle$, the expression for $\left\langle C_{T_a}\right\rangle$ becomes:

$$\left\langle C_{T_a}\right\rangle = \frac{\frac{1}{2}\alpha\left(1-\tfrac{2}{T_a\Delta\nu_d}\right)\int_{S_1} d^2x_1\int_{S_2} d^2x_2\bar{\mu}(\mathbf{x}_1)\bar{\mu}(\mathbf{x}_2)\left|\gamma(\mathbf{x}_1,\mathbf{x}_2,0)\right|^2}{\sqrt{\left\{\begin{aligned}&\int_{S_1} d^2x_1\bar{\mu}(\mathbf{x}_1)+\frac{1}{2}\alpha\left(1-\tfrac{2}{T_a\Delta\nu_d}\right)\sqrt{\int_{S_1} d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2} d^2x_2'\bar{N}(\mathbf{x}_2')}\\ &\qquad\times\int_{S_1} d^2x_1\int_{S_1} d^2x_1'\bar{\mu}(\mathbf{x}_1)\bar{\mu}(\mathbf{x}_1')\left|\gamma(\mathbf{x}_1,\mathbf{x}_1',0)\right|^2\end{aligned}\right\}\left\{\begin{aligned}&\int_{S_2} d^2x_2\bar{\mu}(\mathbf{x}_2)+\frac{1}{2}\alpha\left(1-\tfrac{2}{T_a\Delta\nu_d}\right)\sqrt{\int_{S_1} d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2} d^2x_2'\bar{N}(\mathbf{x}_2')}\\ &\qquad\times\int_{S_2} d^2x_2\int_{S_2} d^2x_2'\bar{\mu}(\mathbf{x}_2)\bar{\mu}(\mathbf{x}_2')\left|\gamma(\mathbf{x}_2,\mathbf{x}_2',0)\right|^2\end{aligned}\right\}}}$$

(4.20)

Then if we ignore the noise contributions from $\left\langle\left[\Delta_{T_a} J_k(t)\right]^2\right\rangle_{T_a} - \left\langle\left[\Delta_{T_a} J_k(t)\right]^2\right\rangle$, the signal-to-noise ratio associated with C_{T_a} is identical to the expression for $SNR_{\Delta J}$, i.e., $SNR_C \cong SNR_{\Delta J}$

5. Output Statistics for Multi-Channel Correlators

Suppose that $\bar{N}$ and the coherence are approximately constant over a broad frequency band, say $\nu\in[\nu_1,\nu_2]$, so that they characterize the light in the entire band. It would be desirable to make combined measurements over $\nu\in[\nu_1,\nu_2]$ in order to improve the signal-to-noise ratio. To accomplish this, let us divide the light received at each telescope into M_c equal and contiguous frequency bands. For each band we have a separate photodetector and we correlate the output fluctuations associated with the same band separately and then average the results for a given pair of telescopes over all M_c bands. The width of each band is $\Delta\nu_c = (\nu_2 - \nu_1)/M_c$. M_c is chosen so that quasi-monochromicity is satisfied for each band, i.e., $\Delta\nu_c << \nu_1$. In addition, we must ensure that the slow detector assumption is satisfied for each band:

$$\Delta\nu_d << \Delta\nu_c = (\nu_2 - \nu_1)/M_c \tag{5.1}$$

Under these assumptions, it is evident that the average of the correlation of the fluctuations over all these frequency bands is given directly by the right-hand side of (4.16.a):

$$\begin{aligned}&\frac{1}{M_c}\sum_{m=1}^{M_c}\left\langle\left\langle\Delta_{T_a}J_1^{(m)}(t)\Delta_{T_a}J_2^{(m)}(t)\right\rangle_{T_a}\right\rangle\\&=\frac{1}{2}\kappa^2\alpha^2\frac{\Delta\nu_c}{\Delta\nu_d}\left(1-\tfrac{2}{T_a\Delta\nu_d}\right)\sqrt{\int_{S_1}d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2}d^2x_2'\bar{N}(\mathbf{x}_2')}\int_{S_1}d^2x_1\int_{S_2}d^2x_2\bar{\mu}(\mathbf{x}_1)\bar{\mu}(\mathbf{x}_2)\left|\gamma(\mathbf{x}_1,\mathbf{x}_2,0)\right|^2\end{aligned} \tag{5.2}$$

Where $\Delta_{T_a}J_k^{(m)}(t)$ is the fluctuation at the k^{th} light collector for the m$^{\text{th}}$ frequency band. However, since the frequency bands are not overlapping, the quantities $\Delta\left[\left\langle\Delta_{T_a}J_1^{(m)}(t)\Delta_{T_a}J_2^{(m)}(t)\right\rangle_{T_a}\right]$ and $\Delta\left[\left\langle\Delta_{T_a}J_1^{(n)}(t)\Delta_{T_a}J_2^{(n)}(t)\right\rangle_{T_a}\right]$ for $m \neq n$ are statistically independent and of zero mean. Consequently, the signal-to-noise ratio in this case is augmented by a factor of $\sqrt{M_c}$:

$$SNR_{\Delta J} = \sqrt{\tfrac{3}{4}}\alpha\sqrt{T_a M_c\Delta\nu_d}\left(1-\tfrac{2}{T_a\Delta\nu_d}\right)\sqrt{\int_{S_1}d^2x_1'\bar{N}(\mathbf{x}_1')\int_{S_2}d^2x_2'\bar{N}(\mathbf{x}_2')}\left\{\int_{S_1}d^2x_1\int_{S_2}d^2x_2\,\bar{\mu}(\mathbf{x}_1)\bar{\mu}(\mathbf{x}_2)\left|\gamma(\mathbf{x}_1,\mathbf{x}_2,0)\right|^2\right\} \tag{5.3}$$

In effect, via this artifice, the bandwidth of the detectors is increased M_c-fold.

6. Coronography Using Partial Coherence

The expression for $\left\langle\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right\rangle$ is not simply proportional to $\left|\gamma(\mathbf{x}_{c1},\mathbf{x}_{c2},0)\right|^2$ where $\mathbf{x}_{c1}$, and $\mathbf{x}_{c2}$ are the positions of the centroids of the collecting apertures unless neither telescope can resolve the object. For the case in which the telescopes can partially resolve the object, Brown and Twiss [7] examined simple geometries consisting of uniformly bright disks and arrived at the "partial coherence factor" as a correction to the SNR results for the unresolved case. Our results for SNR are suited to examine the more general case involving scenes that have both large, resolvable objects and smaller, unresolvable objects. The effect of resolvable elements in the image to be reconstructed is apparent in the factor $\Phi = \int_{S_1}d^2x_1\int_{S_2}d^2x_2\bar{\mu}(\mathbf{x}_1)\bar{\mu}(\mathbf{x}_2)\left|\gamma(\mathbf{x}_1,\mathbf{x}_2,0)\right|^2$ in the expression for $\left\langle\left\langle\Delta_{T_a}J_1(t)\Delta_{T_a}J_2(t)\right\rangle_{T_a}\right\rangle$, (4.16.a). We may usually assume that the intensities falling on the region occupied by the telescopes are nearly constant. Then $\bar{\mu}(\mathbf{x}_k) \cong 1/S$. Also, the normalized degree of coherence is approximately dependent upon the difference in positions, $\mathbf{x}_2 - \mathbf{x}_1$. Thus we are concerned with the quantity:

$$\Phi \cong \frac{1}{S^2}\int_{S_1}d^2x_1\int_{S_2}d^2x_2\left|\gamma(\mathbf{x}_2-\mathbf{x}_1)\right|^2 \tag{6.1}$$

Moreover, under the same assumptions, the intensity image, $M(\boldsymbol{\theta})$, is the inverse Fourier transform of $\bar{I}\gamma(\mathbf{x}_2-\mathbf{x}_1)$:

$$\begin{aligned}M(\boldsymbol{\theta}) &= \bar{I}\int d^2\mathbf{u}\,\exp(-2\pi i\mathbf{u}*\boldsymbol{\theta})\,\gamma(\Delta\mathbf{r}=\mathbf{u}\bar{\lambda})\\ \gamma(\mathbf{u}) &= \frac{1}{\bar{I}}\int d^2\boldsymbol{\theta}\,\exp(2\pi i\mathbf{u}*\boldsymbol{\theta})\,M(\boldsymbol{\theta})\end{aligned} \tag{6.2.a,b}$$

Now consider a case representative of imaging an exo-solar planet next to its planet star, where we model the star as a uniformly luminous disk centered on the origin of the look angle plane:

$$M(\boldsymbol{\theta}) = B_S \begin{Bmatrix} 1, & |\boldsymbol{\theta}| \le \delta_S \\ 0, & |\boldsymbol{\theta}| > \delta_S \end{Bmatrix} + M_P(\boldsymbol{\theta} - \boldsymbol{\theta}_P)$$

$$\gamma(\mathbf{u}) \cong \frac{4}{\pi} jinc(2\delta_S |\mathbf{u}|) + \frac{B_P \delta_P^2}{B_S \delta_S^2} \exp(2\pi i \mathbf{u} * \boldsymbol{\theta}_P) \gamma_P(\mathbf{u}) \qquad (6.3.a\text{-}c)$$

$$\gamma_P(\mathbf{u}) = \frac{1}{\pi B_P \delta_P^2} \int d^2\boldsymbol{\theta} \exp(2\pi i \mathbf{u} * \boldsymbol{\theta}) M_P(\boldsymbol{\theta})$$

Here, the star has intensity B_S, and angular radius δ_S. $M_P(\boldsymbol{\theta})$ is the image of the planet alone, when it is centered at the origin, $\gamma_P(\mathbf{u})$ is the corresponding (approximately normalized) coherence, and $\boldsymbol{\theta}_P$ is the angular position of the planet. B_P is the average intensity over the extent of the planet δ_P is its approximate angular radius. The *jinc* function is defined as $jinc(|\mathbf{u}|) = J_1(\pi |\mathbf{u}|)/2|\mathbf{u}|$ and in normalizing the expression for $\gamma(\mathbf{u})$ we have ignored terms of the order of the ratio of the planet brightness to the star brightness. With these expressions, we have:

$$\Phi \cong \frac{1}{S^2} \int_{S_1} d^2x_1 \int_{S_2} d^2x_2 \begin{bmatrix} \frac{16}{\pi^2} jinc^2(2\delta_S |\mathbf{x}_2 - \mathbf{x}_1|/\lambda) \\ + \frac{8 B_P \delta_P^2}{\pi B_S \delta_S^2} jinc(2\delta_S |\mathbf{x}_2 - \mathbf{x}_1|/\lambda) \mathrm{Re}\left(e^{2\pi i |\mathbf{x}_2 - \mathbf{x}_1| * \boldsymbol{\theta}_P/\lambda} \gamma_P(|\mathbf{x}_2 - \mathbf{x}_1|/\lambda) \right) \\ + \left| \frac{B_P \delta_P^2}{B_S \delta_S^2} \gamma_P(|\mathbf{x}_2 - \mathbf{x}_1|/\lambda) \right|^2 \end{bmatrix} \qquad (6.4.a,b)$$

$$\gamma(\mathbf{u}) \cong \frac{4}{\pi} jinc(2\delta_S |\mathbf{x}_2 - \mathbf{x}_1|/\lambda) + \frac{B_P \delta_P^2}{B_S \delta_S^2} \exp(2\pi i |\mathbf{x}_2 - \mathbf{x}_1| * \boldsymbol{\theta}_P/\lambda) \gamma_P(|\mathbf{x}_2 - \mathbf{x}_1|/\lambda)$$

Now suppose that $\delta_P \cong 10^{-2} \delta_S$, $\delta_S \cong 10^{-2} |\boldsymbol{\theta}_P|$, as is the case for the Earth-Sun system. Let us also consider circular apertures of diameter D_T with their centers distance $\frac{1}{2} X >> D_T$ from the origin along the x-axis, so that the two aperture surfaces have the form:

$$S_1 : \mathbf{x}_1 = \tfrac{1}{2} X\hat{x} + \boldsymbol{\xi}_1 , |\boldsymbol{\xi}_1| \le D_T \qquad (6.5.a,b)$$

$$S_2 : \mathbf{x}_2 = -\tfrac{1}{2} X\hat{x} + \boldsymbol{\xi}_2 , |\boldsymbol{\xi}_2| \le D_T$$

Then:

$$\Phi \cong \frac{1}{S^2} \int_{|\boldsymbol{\xi}_1| \le D_T/2} d^2\boldsymbol{\xi}_1 \int_{|\boldsymbol{\xi}_1| \le D_T/2} d^2\boldsymbol{\xi}_2 \begin{bmatrix} \frac{16}{\pi^2} jinc^2(2\delta_S |X\hat{x} + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2|/\lambda) \\ + \frac{8 B_P \delta_P^2}{\pi B_S \delta_S^2} jinc(2\delta_S |X\hat{x} + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2|/\lambda) \mathrm{Re}\left(e^{2\pi i (X\hat{x} + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2) * \boldsymbol{\theta}_P/\lambda} \gamma_P((X\hat{x} + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)/\lambda) \right) \\ + \left| \frac{B_P \delta_P^2}{B_S \delta_S^2} \gamma_P((X\hat{x} + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2)/\lambda) \right|^2 \end{bmatrix} \qquad (6.6)$$

Further, suppose that the collecting apertures are nearly large enough to resolve the planetary disk, so that $\max|\boldsymbol{\xi}_1 - \boldsymbol{\xi}_2| = 2D_T \cong 0.1\lambda/\delta_P$, which implies $\gamma_P(X\hat{x} + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2) \cong \gamma_P(X\hat{x})$. Also because $\delta_P \cong 10^{-2} \delta_S$ we can use the asymptotic expression for large arguments in evaluating $jinc(2\delta_S |X\hat{x} + \boldsymbol{\xi}_1 - \boldsymbol{\xi}_2|/\lambda)$ to get:

$$\Phi \cong \frac{1}{S^2}\int_{|\xi_1|\le D_T/2} d^2\xi_1 \int_{|\xi_1|\le D_T/2} d^2\xi_2 \left[\begin{array}{l} \frac{1}{\pi^4}\left(\frac{\lambda}{\delta_S X}\right)^3 \cos^2\left(\pi\left(2\delta_S\left|X\hat{x}+\xi_1-\xi_2\right|/\lambda-\frac{3}{4}\right)\right) \\ +\frac{2B_P\delta_P^2}{\pi^2 B_S\delta_S^2}\left(\frac{\lambda}{\delta_S X}\right)^{3/2}\cos\left(\pi\left(2\delta_S\left|X\hat{x}+\xi_1-\xi_2\right|/\lambda-\frac{3}{4}\right)\right)\mathrm{Re}\left(e^{2\pi i\left(X\hat{x}+\xi_1-\xi_2\right)*\boldsymbol{\theta}_P/\lambda}\gamma_P\left(X\hat{x}\right)\right) \\ +\left|\frac{B_P\delta_P^2}{B_S\delta_S^2}\gamma_P\left(X\hat{x}\right)\right|^2 \end{array}\right] \tag{6.7}$$

Using the approximation $\left|X\hat{x}+\xi_1-\xi_2\right| \cong X+\left(\xi_1-\xi_2\right)*\hat{x}$, we have:

$$\begin{aligned}\Phi \cong {} & \frac{1}{2\pi^4}\left(\frac{\lambda}{\delta_S X}\right)^3+\left|\frac{B_P\delta_P^2}{B_S\delta_S^2}\gamma_P\left(X\hat{x}\right)\right|^2+\frac{1}{2\pi^4}\left(\frac{\lambda}{\delta_S X}\right)^3\cos\left(\frac{4\pi\delta_S}{\lambda}X-\frac{3}{2}\pi\right)\left[P_c^2-P_s^2\right] \\ & +\frac{B_P\delta_P^2}{\pi^2 B_S\delta_S^2}\left(\frac{\lambda}{\delta_S X}\right)^{3/2}\mathrm{Re}\left\{\gamma_P\left(X\hat{x}\right)\left(e^{i2\pi\left(\frac{\delta_S}{\lambda}X-\frac{3}{8}+\left(X\hat{x}\right)*\boldsymbol{\theta}_P/\lambda\right)}\left|E_+\right|^2+e^{-i2\pi\left(\frac{\delta_S}{\lambda}X-\frac{3}{8}-\left(X\hat{x}\right)*\boldsymbol{\theta}_P/\lambda\right)}\left|E_-\right|^2\right)\right\}\end{aligned} \tag{6.8}$$

where:

$$\begin{aligned} & P_c=\frac{1}{S}\int_{|\xi|\le D_T/2} d^2\xi\cos\left(\frac{4\pi\delta_S}{\lambda}\xi*\hat{x}\right), \quad P_s=\frac{1}{S}\int_{|\xi|\le D_T/2} d^2\xi\sin\left(\frac{4\pi\delta_S}{\lambda}\xi*\hat{x}\right), \\ & E_+=\frac{1}{S}\int_{|\xi|\le D_T/2} d^2\xi e^{i2\pi\xi\cdot\left(\frac{\delta_S}{\lambda}\hat{x}*\boldsymbol{\theta}_P/\lambda\right)}, \qquad E_-=\frac{1}{S}\int_{|\xi|\le D_T/2} d^2\xi e^{i2\pi\xi\cdot\left(\frac{\delta_S}{\lambda}\hat{x}*\boldsymbol{\theta}_P/\lambda\right)}\end{aligned} \tag{6.9.a,d}$$

Examine the above integrals. First:

$$\begin{aligned} P_c &= \frac{1}{\frac{\pi}{4}D_T^2}\int_0^{2\pi} d\theta\left[\frac{\lambda D_T/2}{4\pi\delta_S\cos\theta}\sin\left(\frac{2\pi\delta_S}{\lambda}D_T\cos\theta\right)+\left(\frac{\lambda}{4\pi\delta_S\cos\theta}\right)^2\left[\cos\left(\frac{2\pi\delta_S}{\lambda}D_T\cos\theta\right)-1\right]\right] \\ &= \frac{4}{\pi}\int_0^{\frac{2\pi\delta_S}{\lambda}D_T} du\frac{1}{\sqrt{\left(\frac{2\pi\delta_S}{\lambda}D_T\right)^2-u^2}}\left[\frac{1}{u}\sin\left(u\right)+\left(\frac{1}{u}\right)^2\left[\cos\left(u\right)-1\right]\right]\approx O\left(\frac{\lambda}{2\pi\delta_S D_T}\right)\end{aligned} \tag{6.10}$$

We can show similarly that:

$$P_s \approx O\left(\frac{\lambda}{2\pi\delta_S D_T}\right), \quad E_+ \approx O\left(\frac{\lambda}{2\pi\left|\boldsymbol{\theta}_P\right| D_T}\right), \quad E_- \approx O\left(\frac{\lambda}{2\pi\left|\boldsymbol{\theta}_P\right| D_T}\right) \tag{6.11.a-c}$$

By virtue of the assumption $\max\left|\xi_1-\xi_2\right| = 2D_T \cong 0.1\lambda/\delta_P$, P_c and P_s are seen to be of order δ_P/δ_S and E_+ and E_- are of order $\delta_P/\left|\boldsymbol{\theta}_P\right|$. Therefore the only appreciable contributions in (6.8) are the first two terms

$$\Phi \cong \frac{1}{2\pi^4}\left(\frac{\lambda}{\delta_S X}\right)^3+\left|\frac{B_P\delta_P^2}{B_S\delta_S^2}\gamma_P\left(X\hat{x}\right)\right|^2+H.O.T. \tag{6.12}$$

Suppose that X is chosen so that the system images the planetary disc with 100 pixels on a side. Then $\frac{\lambda}{X} \cong 10^{-2}\delta_P$. Hence $\frac{1}{2\pi^4}\left(\frac{\lambda}{\delta_S X}\right)^3 \cong \frac{1}{2\pi^4}10^{-12}$. In other words, the effect of partial coherence is to reduce the contribution of the star to the square of the magnitude of the coherence by a factor of 5×10^{-15}. Therefore, even if the ratio of the planet flux to the star flux, $\frac{B_P\delta_P^2}{B_S\delta_S^2}$, is 10^{-6}, the observed coherence magnitude differs from that of the planet alone by a relative error of approximately 2×10^{-3}. The partial coherence effect allows the system to behave as if it were a coronograph.

As a final remark, we note that it would not be necessary to make each light collector so large that it actually just misses resolving the planet. The same effect can be obtained with a formation of small apertures by taking the intensity fluctuations from a set of apertures covering a larger zone and adding them before calculating the cross-correlations among several such zones. The resulting cross-correlation has a partial coherence equivalent to what would be obtained from telescopes as large in spatial extent as the zone containing the small apertures.

7. CONCLUDING REMARKS

In this paper, we derived signal-to-noise statistics for the Brown-Twiss effect within a modern quantum optics framework. While largely in agreement with previous results, the formulae given here provide more precise expressions reflecting certain non-ideal conditions, including a fresh look at the effects of partial coherence. Results are obtained corresponding to a multi-channel correlator and we demonstrate that the signal-to-noise ratio of the coherence estimate can be markedly improved. Further, examined the effects of partial coherence on a scene typical of exoplanet imaging and showed how partial coherence can be used to greatly attenuate the parent star. With appropriate processing, a formation of small light collectors can be used to measure the planet coherence while suppressing the contribution of the star.

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