Manypairs

(Dated: November 22, 2016)

Abstract

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I. THEORY

We consider the many-box scenario, in which n bipartite boxes are distributed to two parties, and each party only observes the sum of all his boxes' outputs. We write this sum $a_{xy} = \sum_i a^i_{xy}$ for Alice and $b_{xy} = \sum_i b^i_{xy}$ for Bob, when inputs x and y are used by the respective parties. For boxes with two binary measurements, this scenario involves 2 settings for each party, each with $n+1$ possible outcomes.

The correlations observed in this scenario are nonlocal iff they violate a [(n+1, n+1) (n+1, n+1)] Bell inequality. Some of these Bell inequalities are liftings from simpler scenarios, such as the [(2, 2) (2, 2)] scenario. Liftings can be seen as inequalities that are tested on the statistics obtained after performing a binning of the $n+1$ outputs into fewer values. We consider here two particular binnings.

A. Majority voting

Here we consider the binning obtained by comparing the observed output to a fixed threshold $t = n/2$. If the outcome is larger than t, we produce '+1', otherwise we produce '-1', i.e.

$$a'_{xy} = \mathrm{sign}(a_{xy} - t). \tag{1}$$

This particular binning was already considered in several studies [**?**]. In particular, numerical evidences showed that it lead to a violation of the CHSH inequality for large n.

In this case, the CHSH expression can be written as ...

B. Parity binning

Let us now consider the parity binning which consists in only remembering the parity of the observed outcome. This brings us to the 2-outcomes scenario, for which the only Bell inequality of interest for our matter is CHSH.

Denoting by a^i_{xy} and b^i_{xy} the outputs of box i upon measurement x and y, we can write the binned outcomes of Alice as

$$a'_{xy} = (-1)^{\sum_i a^i_{xy}}. \tag{2}$$

The bipartite correlator after binning, and in presence of n boxes, can therefore be written as

$$\begin{aligned} E_{xy}^{(n)} &= \langle(-1)^{\sum_i a_{xy}^i} \times (-1)^{\sum_i b_{xy}^i}\rangle \\ &= \langle\prod_i(-1)^{a_{xy}^i + b_{xy}^i}\rangle \\ &= (E_{xy})^n, \end{aligned} \tag{3}$$

where E_{xy} is the 1-box correlator (which is identical for all boxes). The CHSH inequality for n boxes can thus be conveniently written as

$$S_n = (E_{11})^n + (E_{12})^n + (E_{21})^n - (E_{22})^n \leq 2. \tag{4}$$

Obviously, the usual CHSH statistics don't violate this inequality for large n. Let us however consider measurement of a singlet according to

$$\begin{aligned} A_1 &= \sigma_z, & A_2 &= \cos(2\beta)\sigma_z + \sin(2\beta)\sigma_x \\ B_1 &= \cos\beta\ \sigma_z + \sin\beta\ \sigma_x, & B_2 &= \cos\beta\ \sigma_z - \sin\beta\ \sigma_x. \end{aligned} \tag{5}$$

This gives

$$S_n = 3\cos^n\beta - \cos^n(3\beta). \tag{6}$$

Choosing $\beta = \frac{\beta_0}{\sqrt{n}}$, we find

$$S_n \xrightarrow{n\to\infty} 3e^{-\beta_0^2/2} - e^{-9\beta_0^2/2} = 8\cdot 3^{-9/8} \simeq 2.32 \tag{7}$$

for $\beta_0 = \sqrt{\log(3)/2} \simeq 0.524$.

Hence, the CHSH inequality with parity binning can be violated substantially even in the macroscopic limit of large number of pairs n. This contrasts with the majority voting case, for which evidence suggests that the maximal violation decreases as a function n.

C. Werner states

Let us now analyse the situation we can expect if the state of each box is slightly noisy. Here we assume that it is of the form

$$\rho = V|\psi^-\rangle\langle\psi^-| + (1-V)\mathbb{I}/4, \tag{8}$$

with almost perfect visibility $V = 1 - \epsilon$.

It is straightforward to see that in this case, the CHSH after parity binning is simply rescaled as

$$S_n(\epsilon) = V^n \cdot S_n(\epsilon = 0). \tag{9}$$

Therefore, whenever $V < 1$, the violation disappears in the macroscopic limit : $S_n(\epsilon) \xrightarrow{n\to\infty} 0 < 2$.

Assuming a fixed small ϵ, the value of CHSH for large n and the above settings is

$$\begin{aligned} S_n &\simeq 8 \cdot 3^{-9/8}(1-\epsilon)^n \\ &\simeq 8 \cdot 3^{-9/8}(1-n\epsilon) \end{aligned} \tag{10}$$

This value is two for the critical number of pairs

$$n_c \simeq \frac{1 - 3^{9/8}/4}{\epsilon} \simeq \frac{0.14}{\epsilon} \tag{11}$$

In other words, the critical visibility required to see a violation with n pairs is

$$V_c \simeq 1 - \frac{1 - 3^{9/8}/4}{n} \simeq 1 - \frac{0.14}{n} \tag{12}$$

Also show the plot of the violation as a function of n!

1. *Comparison with the majority binning*

In comparison, for the majority voting strategy, the violation decreases much faster as a function of n. Therefore a large violation for large n is not expected. The violation tail, however is quite long. In fact, it is conjectured that the critical visibility (for a Werner state again) is

$$V_c \simeq 1 - \frac{0.64}{n}. \tag{13}$$

Therefore, the critical number of pairs where a violation can still be observed (for the same noise ϵ) is about 3∼4 times the one for the parity binning.

II. EXPERIMENT

A. Experimental setup

In our experiment (see figure 1), the output of a grating-stabilized laser diode (LD, central wavelength 405 nm) passes through a single mode optical fiber (SMF) for spatial

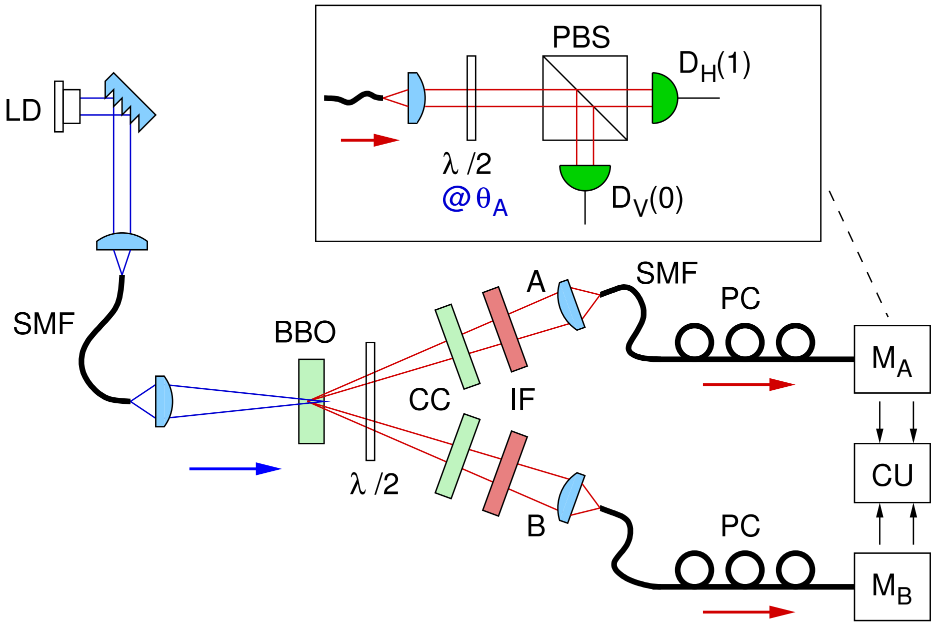

FIG. 1: Schematic of the experimental set-up. Polarization correlations of entangled-photon pairs are measured by the polarization analyzers M_A and M_B, each consisting of a half wave plate ($\lambda/2$) followed by a polarization beam splitter (PBS). All photons are detected by Avalanche photodetectors D_H and D_V, and registered in a coincidence unit (CU).

mode filtering, and is focused to a beam waist of 80 μm into a 2 mm thick BBO crystal cut for type-II phase- matching. There, photon pairs are generated via spontaneous parametric down- conversion (SPDC) in a slightly non-collinear configuration. A half-wave plate ($\lambda/2$) and a pair of compensation crystals (CC) take care of the temporal and transversal walk-off [1]. Two spatial modes (A, B) of down- converted light, defined by the SMFs for 810 nm, are matched to the pump mode to optimize the collection [2]. In type-II SPDC, each down-converted pair consists of an ordinary and extraordinarily polarized photon, corresponding to horizontal (H) and vertical (V) in our setup.

We use a pair of polarization controllers (PC) to minimize the polarization rotation caused by the SMFs to the collected modes.

To arrive at an approximate singlet Bell state, the phase ϕ between the two decay possibilities in the polarization state $|\psi\rangle = 1/\sqrt{2}\left(|H\rangle_A|V\rangle_B + e^{i\phi}|V\rangle_A|H\rangle_B\right)$ is adjusted to $\phi = \pi$ by tilting the CC.

In the polarization analyzers (inset of figure 1), photons from SPDC are projected onto arbitrary linear polarization by $\lambda/2$ plates, set to half of the analyzing angles $\theta_{A(B)}$, and polarization beam splitter (extinction ratio 1/2000 and 1/200 respectively for transmitted and reflected arm) in each analyzer. Photons are detected by avalanche photo diodes (APD), and corresponding detection events from the same pair identified by a coincidence unit if they arrive within $\approx \pm 3$ ns of each other.

The quality of polarization entanglement is tested by probing the polarization correlations in a basis complementary to the intrinsic HV basis of the crystal. With interference filters (IF) of 5 nm bandwidth (FWHM) centered at 810 nm, in the 45° linear polarization basis we observe a visibility $V_{45} = 98.68 \pm 0.20\%$. The visibility in the natural H/V basis of the type-II down- conversion process reaches $V_{\mathrm{HV}} = 99.67 \pm 0.12\%$.

The non-ideal visibility is due to the non-perfect neutralization of the polarization rotation caused by the SM fibers.

B. Measurement and Post-processing

In the realization of this proof of principle experiment, we did not intend to provide a loophole-free demonstration. Due to the limited efficiency of the APD detectors, we assume that the fraction of the photon we detected is a fair representation of the entire ensemble (fair sampling assumption). Similarly, even if Alice and Bob are not space-like separated, we assume that no communication happens between the two measurements. Moreover, the basis choice is not random, as expected in an ideal Bell-like experiment. We instead set the basis and record the number of events in a fixed time. We are assuming that the state generated by the source, and all the other parameters of the experiment, do not change between experimental runs.

The basic measurement lasts 60 s, during which we record an average of 16×10^3 two-fold coincidences between detectors at Alice and Bob. A detection event at the transmitted output of each PBS is associated with 0, reflected one with 1. Three- and four-fold coincidences, as well as two-fold coincidences between detectors belonging to the same party, correspond to multiple pairs of photos generated within the coincidence time window. The rate of these events is negligible, therefore we discarded them.

In order to avoid biases due to the asymmetries in detector efficiencies, to measure one basis (A_j, B_k) we also measure three complementary basis: $(A_j + 45°, B_k)$, $(A_j, B_k + 45°)$, and $(A_j + 45°, B_k + 45°)$. A rotation by 45° effectively swaps the roles of the transmitted and reflected detectors. Each party, when measuring on the rotated basis, needs to apply a NOT operation to the measurement outcome. We repeat the measurement for a range of β, and the corresponding four bases defined by Eq. (5).

To replicate the many-box scenario, for every set of measurement angles we organize the sequence of results into clusters of size n. For each cluster we calculate the majority (parity) binning using Eq. (1) (Eq. (2)). Following the procedure described by Eqs. (3) and (4), we obtain a value of S_n for every n of interest. To evaluate the error associated to every S_n, the same procedure is repeated 1000 times, shuffling the order of the results every time before the clustering.

III. DISCUSSION

Matching between theory and data mismatch due to rotation caused by the SMFs

[1] P. G. Kwiat, K. Mattle, H. Weinfurter, A. Zeilinger, A. V. Sergienko, and Y. Shih, Phys. Rev. Lett. **75**, 4337 (1995), URL `http://link.aps.org/doi/10.1103/PhysRevLett.75.4337`.

[2] C. Kurtsiefer, M. Oberparleiter, and H. Weinfurter, Phys. Rev. A **64**, 023802 (2001).

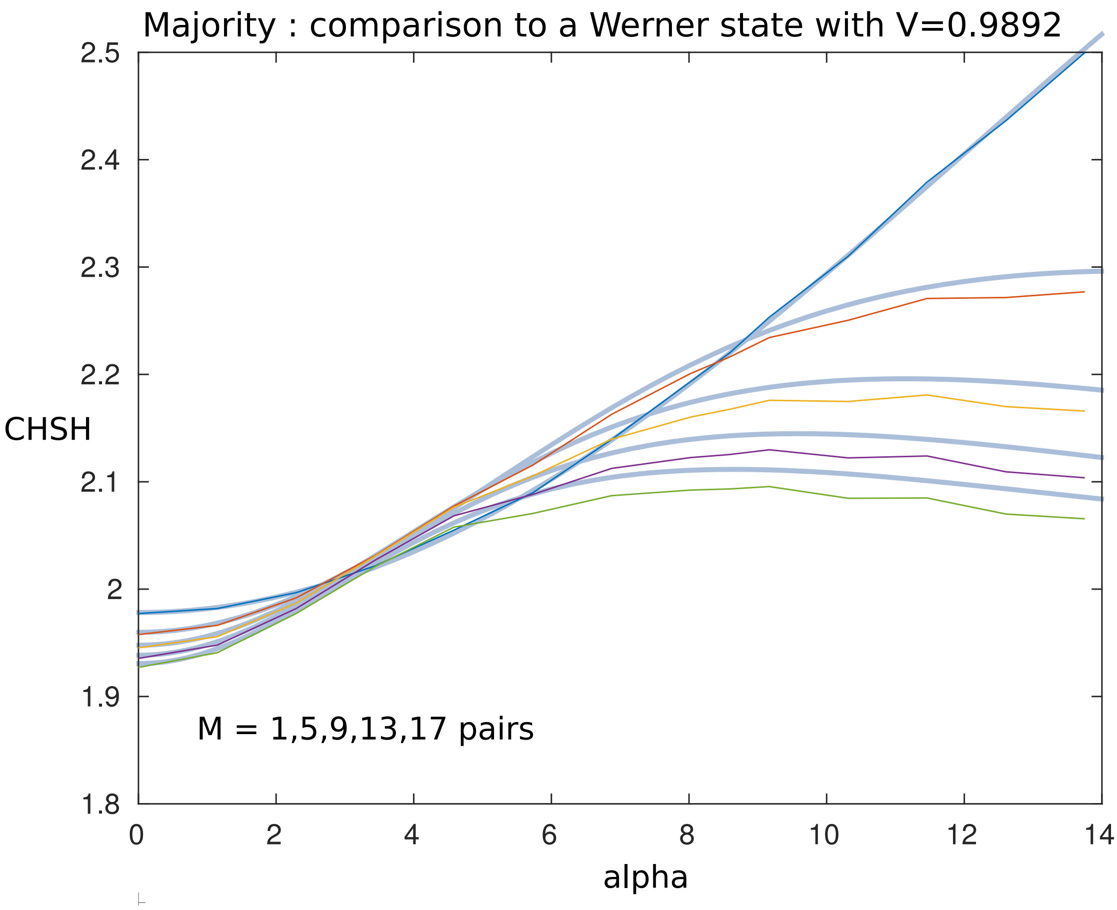

FIG. 2: Majority processing applied to the data.

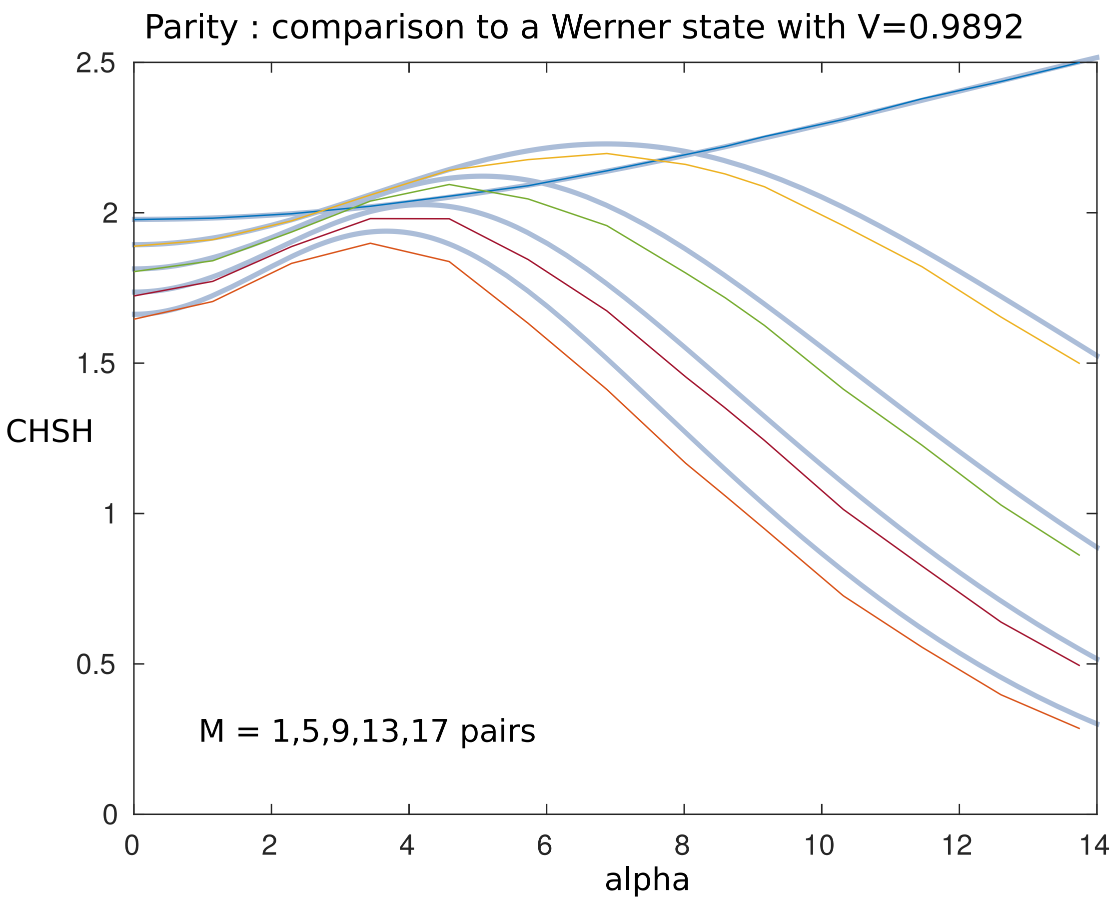

FIG. 3: Parity processing applied to the data.