SUPPLEMENTAL MATERIAL: AMOUNT OF BELL VIOLATION WITH PARITY BINNING

In the main text we discuss the relation between the number of pairs at which a Bell violation can still be observed, for either majority of parity binning, and the quality of the source in terms of visibility V. The amount of Bell violation that is obtained in the many-pair scenario when using a majority binning is described in [11]. Here we analyse how the amount of Bell violation depends on the number of pairs in the case of parity binning and compare it to the majority case. In particular, we show that its decreases more and more slowly as the visibility increases.

To see this, we consider the CHSH expression (Eq. 10 in main text), together with the choice of setting

$$\beta = \frac{\beta_0}{\sqrt{n}}\ , \quad \beta_0 = \frac{\sqrt{\ln(3)}}{2}. \tag{1}$$

As discussed in the main text, these settings give rise to a violation for a number of pairs smaller than

$$n_c(V) = \frac{1 - 3^{9/8}/4}{1 - V}. \tag{2}$$

We then estimate the sensibility of the Bell violation to the number of pairs by computing the amount of violation that can still be observed when the number of pairs is half of the maximum possible number, i.e. $n = n_c/2$. For this, we define the ratio

$$R = \frac{S^n(V, n = n_c(V)/2) - 2}{S^n(V, n = 1) - 2}. \tag{3}$$

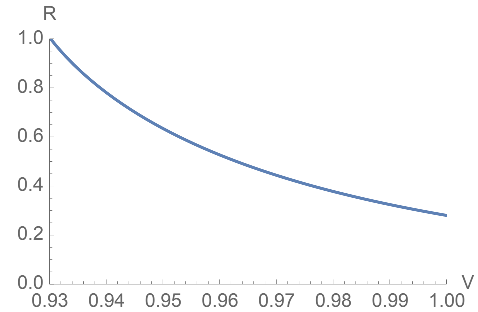

FIG. 1. Amount of Bell violation remaining in the parity case when considering $n = n_c/2$ pairs.

This quantity is represented in Fig. 1. Interestingly, only a fraction of the initial violation is lost independently of the visibility. The decrease in violation is thus linear in n.

Moreover, since the number of pairs considered here increases with the visibility, the Bell violation with parity binning becomes less and less sensitive to the number of pairs as the visibility increases. This contrasts with the case of majority voting, where the violation is upper-bounded by the case $V = 1$, which decays as $\sim 1/\sqrt{n}$.

Given this qualitative difference between the Bell violation provided by the majority and parity binnings, one should expect that the Bell violation provided by the parity binning would outperform the one provided by the majority procedure for a sufficiently large visibility. From Fig. 2, we see that this cross-over occurs around $V = 0.994$.

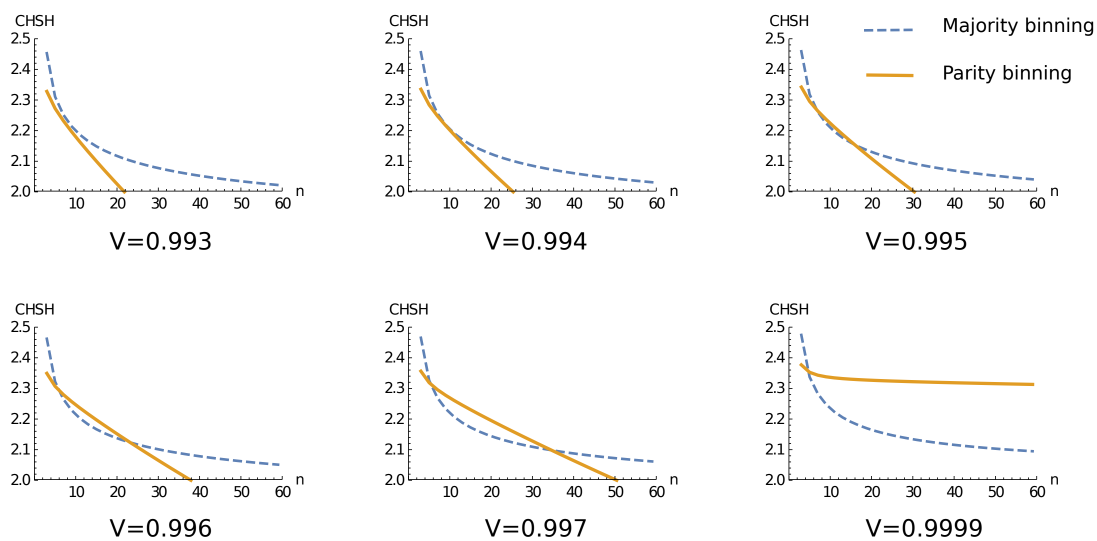

FIG. 2. CHSH violation achieved by the majority and parity binnings as a function of the source visibility V and number of pairs n. For $V < 0.994$, the largest Bell violation is achieved by the majority strategy. For $V > 0.994$, the parity strategy provides a large violation for a range of n.