Direct measurement of coherent light proportion from a ~~practical~~ laser~~source~~

Xi Jie Yeo,[1] Eva Ernst,[1] Alvin Leow,[2] Jaesuk Hwang,[1] Lijiong Shen,[1] Peng Kian Tan,[1] and Christian Kurtsiefer[1, 2]

[1] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543*

[2] *Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117551*

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We present a technique to estimate the proportion of coherent emission in the light emitted by a ~~practical laser source~~ semiconductor laser diode without spectral filtering, ~~using interferometric photon correlation measurements, where photon correlations are measured~~ by measuring photon correlations between the light emitted from the output ports of an asymmetric Mach-Zehnder interferometer. Using this technique, we determine the proportion of coherent emission of the brightest mode in the light emitted by a laser diode, in a ~~statistical~~ mixture of light ~~from stimulated and spontaneous~~ with coherent and incoherent emission, and a ~~statistical~~ mixture of light ~~from stimulated~~ with coherent emission at two distinguishable spectral bands.

I. INTRODUCTION

~~The invention of lasers can be traced to work describing the emission process of the light from an atom to be spontaneous or stimulated [1].~~

In traditional models describing macroscopic lasers, the laser undergo a sharp transition from undergoing spontaneous emission, to dominantly stimulated emission [2–4]. An ensemble of light emitters undergoing stimulated emission will emit coherent light that has a well-defined phase, whereas an ensemble of light emitters undergoing spontaneous emission will emit incoherent light which is randomly phased [5].

~~In traditional models of macroscopic lasers [2–4], the light emitted from a laser is modeled to originate dominantly from stimulated emission. These models also predict a phase transition of the nature of emission at the lasing threshold, separating two operating regimes where light emitted is either spontaneous or stimulated.~~

However, experiments on small lasers ~~have shown~~ suggest that the transition from spontaneous to stimulated emission is not abrupt, but extends over a range ~~for the pump power [6–10].~~ of pump powers [6–14]. Across this range, the light emitted from the laser is a mixture of spontaneous and stimulated emission.

~~In these experiments,~~ Thus, observations of the transition from spontaneous to stimulated emission ~~was characterized by~~ require methods to measure the proportion of coherent light or incoherent light in this mixture. In these experiments, the proportion of coherent and incoherent light emitted by the laser were estimated by measuring the second-order photon correlation $g^{(2)}$, which corresponds to the distribution of timing separations between single photon detection events, using the standard Hanbury-Brown and Twiss scheme [15]. The measurement result can be explained using Glauber's theory of optical coherence [16], where incoherent light from spontaneous emission would exhibit a "bunching" signature with $g^{(2)}(0) > 1$, while coherent light from stimulated emission exhibits a Poissonian distribution with $g^{(2)} = 1$.

~~The "bunching" signature associated with incoherent light has a characteristic timescale inversely related to the~~ The operating conditions where the laser emits dominantly coherent light would then be estimated to be where $g^{(2)} \to 1$.

A limitation of using second-order photon correlation to quantify the proportion of coherent and incoherent light emitted by the laser, is due to situations where incoherent light exhibits $g^{(2)} \approx 1$, indistinguishable from the same measurement of coherent light. For example, when the spectral width of the ~~light, according to the Wiener-Khintchine theorem related through a Fourier transform [17–19]. In a practical measurement,~~ incoherent light is broad, the characteristic timescale of the "bunching" signature is small [17–19], to the extent that it may be smaller than the detector timing resolution. As the amplitude of the "bunching" signature scales with the ratio of characteristic timescale of the light to the timing response of the detectors~~[20]. Thus, when the spectral width of the incoherent light is broad, to~~ [20], the ~~extent that the characteristic timescale of the "bunching" signature is smaller than the detector timing uncertainty~~second-order photon correlations measurment of incoherent light exhibits $g^{(2)} \approx 1$, and fails to distinguish if the laser is emitting coherent light, incoherent light ~~may exhibit $g^{(2)} \approx 1$, like coherent light~~or a mixture of the two.

To overcome the failure to characterise the light emission from $g^{(2)}$ due to limited detector timing ~~uncertainty~~resolution, a narrow band of ~~incoherent~~ light can be prepared with filters from a wide ~~optical spectrum of an incoherent licht source~~light source with a wide optical spectrum [21]. The narrow spectral width of a filtered incoherent light has a correspondingly larger characteristic coherence timescale, which may be long enough to be resolvable by the detectors.

~~However~~Nevertheless, for characterising the transition from spontaneous to stimulated emission of a laser, such spectral filtering presents some shortcomings. First, as spectral filtering discards light outside the transmission window of a filter, a result would be inconclusive for the full emission of the source. Second, spectral filtering requires *a priori* information or an educated guess of the central frequency and bandwidth of stimulated emission. Third, it has been shown that spectral filtering below the Schawlow-Townes linewidth of the laser result in $g^{(2)}(0) > 1$, similar to light from spontaneous emission [22].

~~This motivates for methods quantifying~~ These shortcomings motivate for methods that quantifies the proportion of coherent or incoherent light emitted by a laser source without the need for spectral filtering~~. A method to characterise the stimulated and spontaneous emission from a pulsed laser has been demonstrated before [14].~~, in order to properly characterise the transition of laser diode emission from spontaneous to stimulated.

In this paper, we present a method to quantify ~~bounds for~~ the proportion of coherent light for a continuous wave laser, without the use of spectral filters. Specifically, we investigate the brightest mode of coherent emission from a semiconductor laser diode without spectral filtering, by ~~using interferometric photon correlations . Earlier methods~~ measuring second-order photon correlations between the light emitted from the output ports of an asymmetric Mach-Zehnder interferometer. This measurement, also known as interferometric photon correlation, was originally applied to differentiate between incoherent light and coherent light with amplitude fluctuations [23]. Earlier variations of interferometric photon correlation measurements were used to study spectral diffusion in organic molecules embedded in solid matrix [24, 25]. ~~The method of interferometric photon correlation used in this paper was originally applied to differentiating between incoherent light and coherent light with amplitude fluctuations [23] .~~ In contrast to second-order photon correlations, this method can clearly distinguish between a finite linewidth coherent light and broadband incoherent light~~[26]. We use this~~, and has temporal features related to the characteristic timescale of the coherent light [26], which is typically of interest in contrast to the characteristic timescale of the coherent light in the the temporal features of standard second-order photon correlations. We extend the use of interferometric photon correlation as a method to extract the ~~fraction~~ proportion of coherent light emitted by the laser diode over a range of ~~pump powers~~ operating currents across the lasing threshold~~,~~. ~~and in the lasing regime above threshold where coherent light is emitted~~

Using this method, we were also able to identify a regime above the lasing threshold where the proportion of coherent light significantly reduced over a range of operating currents. Further investigation revealed that the laser was emitting into two distinguishable longitudinal modes ~~.~~ in this regime. If conventional second-order correlation was used, this regime of multimode operation may have been overlooked as $g^{(2)} = 1$ even when these two longitiudinal modes are incoherent with respect to each other [27, 28].

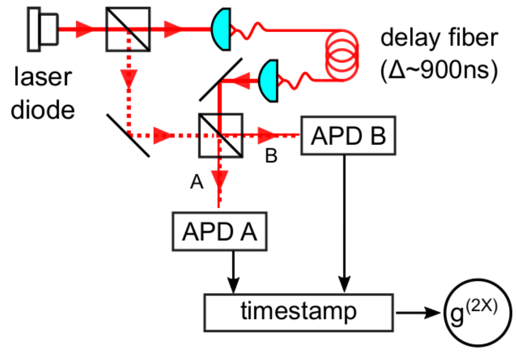

FIG. 1. Experimental setup for measuring interferometric photon correlations. Light from a laser diode enters an asymmetric Mach-Zehnder Interferometer. Singe photon avalanche photodetectors (APD) ~~generate photodetection events~~ at each output port of the interferometer generate photodetection events, which are time-stamped to extract the the correlations numerically.

II. INTERFEROMETRIC PHOTON CORRELATIONS

The setup for an interferometric photon correlation measurement $g^{(2X)}$ is shown in Fig. 1. Light emitted by the laser diode is sent through an asymmetric Mach-Zehnder interferometer, with a long propagation delay Δ between the two paths of the interferometer that exceeds the coherence time of the light.

With a light field $E(t)$ at the input, the light fields at the output ports A, B of the interferometer are

$$E_{A,B}(t) = \frac{E(t) \pm E(t+\Delta)}{\sqrt{2}}\,, \tag{1}$$

with the relative phase shift π acquired by one of the output fields from the beamsplitter.

~~Using these expressions for the electrical fields, the~~ The temporal correlation of photodetection events between the two output ports is given by

$$g^{(2X)}(t_2 - t_1) = \frac{\langle E_A^*(t_1) E_B^*(t_2) E_B(t_2) E_A(t_1)\rangle}{\langle E_A^*(t_1) E_A(t_1)\rangle \langle E_B^*(t_2) E_B(t_2)\rangle}\,\cancel{.}, \tag{2}$$

~~Therein~~therein, $\langle\rangle$ indicates an expectation value~~and/or an ensemble average.~~. Using Eqn. 1, $g^{(2X)}(t_2 - t_1)$ can be grouped in several terms:

$$
\begin{aligned}
&g^{(2X)}(t_2 - t_1) = \\
&= \frac{1}{4}[\langle E^*(t_1)E^*(t_2)E(t_2)E(t_1)\rangle \\
&\quad + \langle E^*(t_1+\Delta)E^*(t_2+\Delta)E(t_2+\Delta)E(t_1+\Delta)\rangle \\
&\quad + \langle E^*(t_1+\Delta)E^*(t_2)E(t_2)E(t_1+\Delta)\rangle \\
&\quad + \langle E^*(t_1)E^*(t_2+\Delta)E(t_2+\Delta)E(t_1)\rangle \\
&\quad - \langle E^*(t_1+\Delta)E^*(t_2)E(t_2+\Delta)E(t_1)\rangle \\
&\quad - \langle E^*(t_1)E^*(t_2+\Delta)E(t_2)E(t_1+\Delta)\rangle] \,.
\end{aligned}
\tag{3}
$$

The first two terms have the form of conventional second-order photon correlation functions $g^{(2)}(t_2 - t_1)$. The next two terms are conventional second-order photon correlation functions, time-shifted forward and backward in their argument by the propagation delay Δ in the interferometer. The last two terms have negative signs and reduce $g^{(2X)}$, leading to a dip at zero time difference ~~$t_2 - t_1$~~$t_2 - t_1 = 0$, with a width given by the coherence time of the light.

The expectation values appearing in Eqn. 3 for $g^{(2X)}$ can be evaluated by using ~~statistical~~ expressions [5] of $E(t)$ for incoherent light and coherent light [26].

For incoherent light, $g^{(2X)}$ exhibits a "bunching" signature peaking at time differences plus and minus the propagation delay, ~~$g^{(2X)}(\pm\Delta) = 1 + 1/4$~~$g^{(2X)}(\pm\Delta) = 1 + (1/4)$. At zero-time difference, the expected "bunching" signature from conventional second-order photon correlation functions in the first two terms of Eqn. 3 and the dip from the last two terms of Eqn. 3 cancel each other, resulting in $g^{(2X)}(0) = 1$ ~~.~~ (see Fig. 3 top).

For coherent light, since the second-order photon correlation function $g^{(2)}$ has a constant value of 1, the $g^{(2X)}$ will show the negative contributions from the last two terms of Eqn. 3, resulting in $g^{(2X)}(0) = 1/2$ ~~.~~ (see Fig. 3 bottom).

III. EXTRACTING FRACTION OF COHERENT LIGHT EMITTED IN A MIXTURE

In order to obtain an interpretation of the nature of the light emitted beyond just presenting the components of $g^{(2X)}$, we consider a light field that is neither completely coherent nor incoherent. We assume that light emitted by the laser is a mixture of coherent light field E_{coh}, and ~~an uncorrelated~~ a light field E_{unc} ~~, which nature~~ uncorrelated to E_{coh}. The nature of E_{unc} can be coherent, incoherent, or a coherent-incoherent mixture. In the following, we try to extract quantitative information about the components from the interferometric photon correlations $g^{(2X)}$, namely the fraction of ~~coherent~~ light intensity of the brightest coherent component in the light field, and a collective treatment of all the rest.

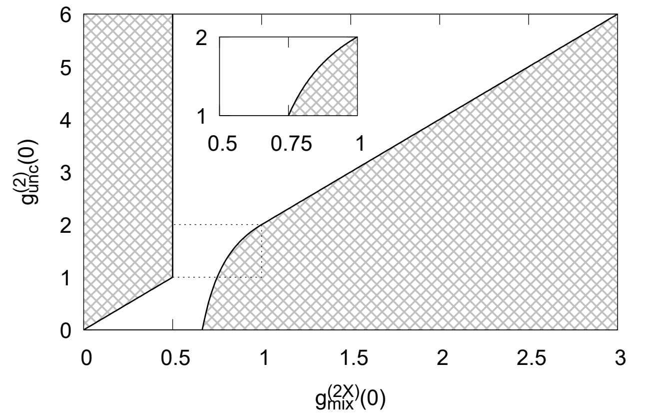

FIG. 2. Combinations of $g^{(2)}_{\mathrm{unc}}(0)$ and $g^{(2X)}_{\mathrm{mix}}(0)$ that correspond to physical and real-valued ρ. In shaded areas, no such solution exist. Inset: Zoom into the region $1 \le g^{(2)}_{\mathrm{unc}}(0) \le 2$, where the uncorrelated light source is assumed to be a mixture of coherent and completely incoherent light, and thermal light.

We model the light field mixture with an electrical field

$$E_{\mathrm{mix}}(t) = \sqrt{\rho}E_{\mathrm{coh}}(t) + \sqrt{1-\rho}E_{\mathrm{unc}}(t)\,, \tag{4}$$

where ρ is the fraction of ~~coherent~~ light intensity of the brightest coherent emission, and the respective light field terms are normalised such that ~~$|E_{\mathrm{res}}| = |E_{\mathrm{coh}}| = |E_{\mathrm{unc}}|$~~$|E_{\mathrm{mix}}| = |E_{\mathrm{coh}}| = |E_{\mathrm{unc}}|$.

Evaluating photon correlation in Eqn. 3 with this light model, and further assuming that ~~(1)~~ first, the propagation delay in the interferometer is significantly longer than the coherence time scale of the light source, and ~~(2)~~ second, the interferometer has good visibility yields

$$g^{(2X)}_{\mathrm{mix}}(0) = 2\rho - \frac{3\rho^2}{2} + \frac{(1-\rho)^2}{2}g^{(2)}_{\mathrm{unc}}(0)\,, \tag{5}$$

at zero time difference, with only two remaining parameters, ρ and $g^{(2)}_{\mathrm{unc}}(0)$, the zero time difference second order photon correlation of the uncorrelated field (see Appendix A).

~~The connection in~~ Using Eqn. 5, together with the physical ~~requirement~~ constraint $0 \le \rho \le 1$ for the coherent light fraction limits the possible combinations of $g^{(2)}_{\mathrm{unc}}(0)$ and $g^{(2X)}_{\mathrm{mix}}(0)$, shown as non-shaded areas in Fig. 2; the exact expressions for the boundaries are given in Appendix B.

We can now further assume that the nature of the uncorrelated light source is some mixture of coherent and completely incoherent light ($g^{(2)}(0) = 1$), and thermal light ($g^{(2)}(0) = 2$). This constrains the second-order photon correlation of the uncorrelated light:

$$1 \le g^{(2)}_{\mathrm{unc}}(0) \le 2. \tag{6}$$

We impose these bounds in Eqn. 5, and extract the bounds to the fraction of ~~coherent light of~~ light intensity in the brightest coherent emission ρ with an upper bound,

$$\rho \leq \sqrt{2 - 2\,g^{(2X)}(0)}, \tag{7}$$

and a lower bound,

$$\rho \geq \begin{cases} \frac{1}{2} + \frac{1}{2}\sqrt{3 - 4\,g^{(2X)}(0)}, & \text{for } \frac{1}{2} \leq g^{(2X)}(0) \leq \frac{3}{4} \\ 2 - 2\,g^{(2X)}_{\text{mix}}(0), & \text{for } \frac{3}{4} \leq g^{(2X)}(0) \leq 1 \end{cases}, \tag{8}$$

with $g^{(2X)}_{\text{mix}}(0)$ ranging from 1/2 for fully coherent light, to 1 for fully incoherent light.

In practice, these two bounds for ρ are quite tight, and allow to extract a fraction of coherent light in an experiment with a small uncertainty.

IV. EXPERIMENT

In our experiment, we measure interferometric photon correlations of light emitted from a temperature-stabilised distributed feedback laser diode with a central wavelength around 780 nm.

The setup is shown in Fig. 1. Interferometric photon correlations are obtained from an asymmetric Mach-Zehnder interferometer, formed by 50-50 fibre beamsplitters and a propagation delay Δ of about 900 ns through a 180 m long single mode optical fibre in one of the arms. Photoevents at each output port of the interferometer were detected with actively quenched silicon single photon avalanche photo diodes (APD). The detected photoevents were time-stamped using a timetagger with a resolution of 2 ns for an integration time T.

The correlation function $g^{(2X)}$ is extracted through histogramming all time differences $t_2 - t_1$ between detection event pairs in the inverval T numerically~~, which allows for a clean normalization.~~. The resulting correlation is fitted to a two-sided exponential function,

$$g^{(2X)}(t_2 - t_1) = 1 - A \cdot \exp\left(-\frac{|t_2 - t_1|}{\tau_c}\right), \tag{9}$$

where τ_c is the characteristic time constant of the coherent light, and A is the amplitude of the dip. The value of $g^{(2X)}(0)$ is the extracted from the fit as $1 - A$. Examples of measured correlation functions and corresponding fits for different laser powers are shown in Fig. 3.

A. Tranisition from incoherent to coherent light

A transition from incoherent to coherent emission is expected as the laser current is increased across the lasing threshold of the laser. We identify the lasing threshold of a laser diode $I_{L,th}$, by measuring the steepest increase of optical power with the laser current (see Fig 4). For our diode, we find $I_{L,th} = 37\,\text{mA}$.

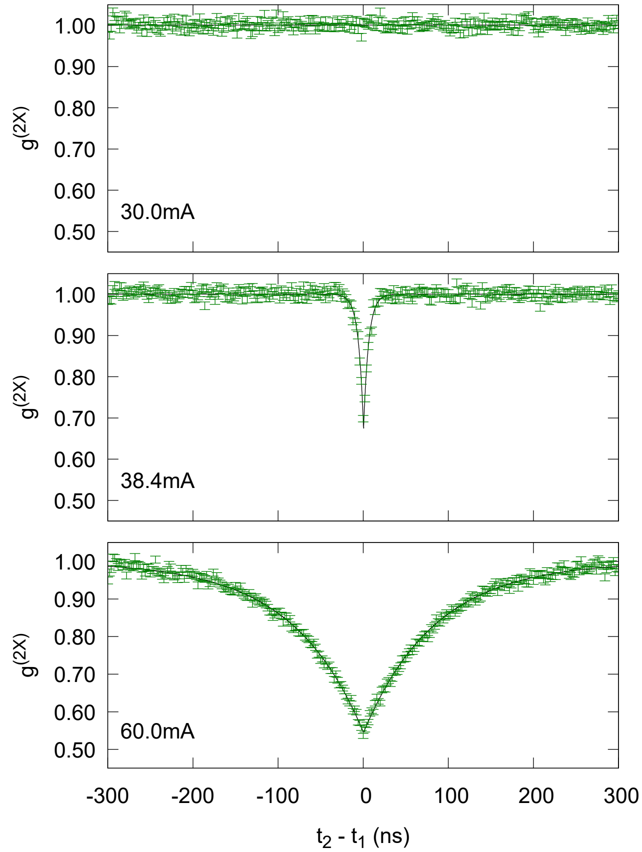

FIG. 3. Interferometric photon correlations $g^{(2X)}$ for different laser currents I_L, extracted from a histogram of photodetector time differences (green symbols). The error range at a specific time bin indicates an expected uncertainty according to a Poissonian counting statistics. The black solid lines show a fit to Eqn. 9, resulting in values for A (from top to bottom) of -0.0006 ± 0.0003, 0.326 ± 0.008, 0.455 ± 0.002, respectively.

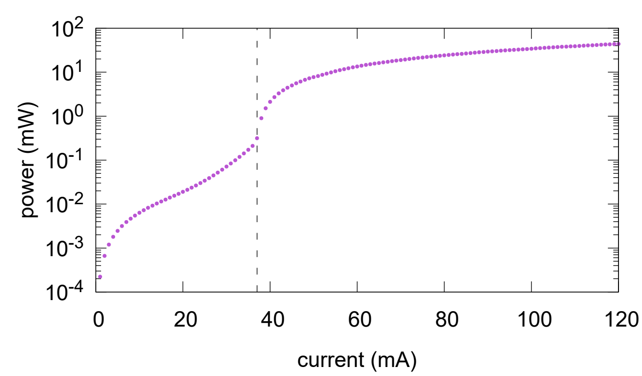

FIG. 4. Measured laser power against laser current I_L. The sharpest change in current was measured at $I_{L,th} = 37\,\text{mA}$, indicating the threshold current (dashed line).

To observe the transition from incoherent to coherent emisssion, we measure the fraction of ~~coherent~~ light intensity of the brightest coherent component ~~in the light field~~ ρ , at different laser current I_L across the lasing threshold, extracted from $g^{(2X)}$ measurements. The interferometric photon correlation measurements as a function of the laser current ~~around the lasing threshold~~ are shown in Fig. 5 (top part). The amplitude of the dip is then extracted by fitting these measurements to Eqn. 9, from which the upper bound and lower bound of the fraction of ~~coherent~~ light from the most dominant coherent emission ρ is extracted (see Fig. 5, middle part).

From the fit, the fraction ρ of coherent emission remains near 0 when operated below threshold. Above the lasing threshold at 37 mA, ρ increases with I_L in a phase-transition manner, reaching $\rho = 0.986$ (90% confidence interval: 0.982 to 0.989) at $I_L = 120$ mA. This agrees with the expectation that the emission of the laser diode is increasingly dominated by stimulated emission past the lasing threshold[29, 30].

Near the lasing threshold, a tight upper and lower bound of ρ is observed, in agreement with the expectation that emission of light with a ~~statistical~~ mixture of coherent and incoherent light from the laser diode is expected when operating it near its lasing threshold [29, 30].

The coherence time of the coherent light τ_c can also be extracted from fitting $g^{(2X)}$ measurements to Eqn. 9 (bottom Fig. 5). We observe that the coherence time increases with the current after the threshold current, before reaching a steady value between 300 to 350 ns. The increase of coherence time correspond to a narrowing of the emission linewidth, agrees with predictions from laser theory, that line narrowing is expected with an increase in pumping of the laser (here an increase in laser current)[30]. An oscillation of the coherence time is also observed starting at ~~the about~~ 66 mA, with a periodicity of about 6 mA.

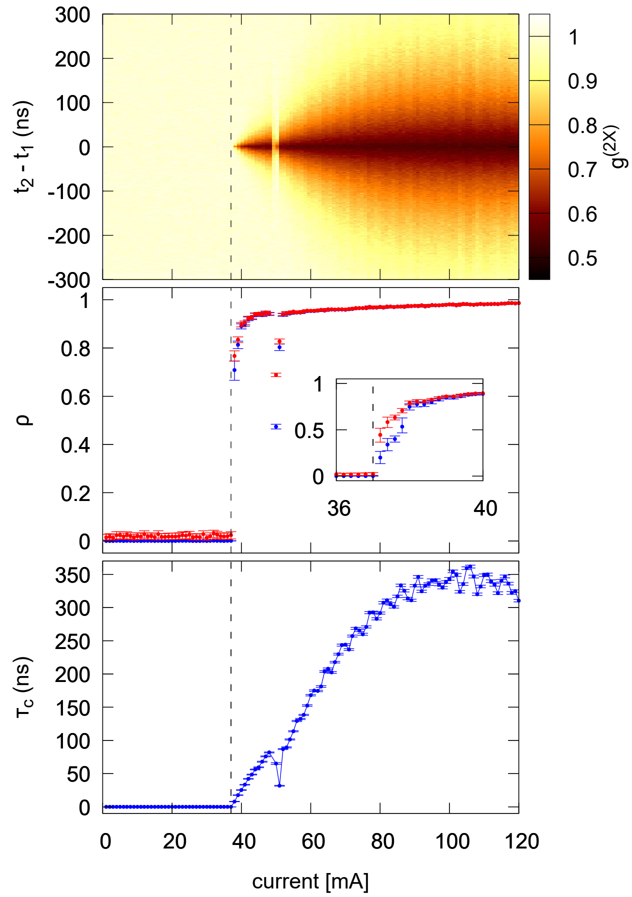

FIG. 5. Top: Interferometric photon corrrelations $g^{(2X)}$ for different ~~drive~~ operating currents I_L. Middle: Corresponding upper bound of fraction ρ of coherent light (red) extracted via Eqn. C1, and the lower bound (blue) extracted via Eqn. C2 from $g^{(2X)}(0)$. The dip in ρ is a result of emission at multiple ~~chop~~ chip modes as explained in Section IV B. The inset shows the extracted bounds for ρ at finer steps of laser current near the lasing threshold. Bottom: Coherence time of coherent light τ_c extracted from $g^{(2X)}$. The dashed line indicates the threshold current $I_{L,th} = 37$ mA.

B. Light statistics near a mode hop

Above the lasing threshold, the laser oscillates at different longitudinal modes for different laser currents. The technique to extract the fraction of coherent and incoherent light allows to investigate the behavior also in the ~~transition regime between~~ regime where oscillation on different longitudinal lasing modes is observed.

~~For this, we~~ In measuring the proportion of coherent light ρ for different operating currents, we observed a reduction in ρ over a range of currents above the lasing threshold (Fig. 5 middle). To further investigate, we measured the spectrum of light emitted by the laser diode ~~at different currents above the lasing threshold~~ over this range of currents with an optical spectrum analyser based on a Michelson interferometer with a spectral resolution of 2 GHz (Bristol 771B-NIR). ~~The laser diode emitted light into two distinct narrow spectral bands with a changing power ratioin a diode current range between~~ At laser currents between 49.0 mA and 52.4 ~~mA~~mA, two distinct narrow spectral bands of light were emitted by the laser diode with a varying power ratio, suggesting emission in two chip modes. Outside this window, only one of the modes was present. Below 49.0 mA, the laser emission was centered around 780.07 nm, above 52.4 mA around 780.34 nm,

The power fractions $r_{\alpha,\beta}$ of these two chip modes α and β emitting respectively at powers $P_{\alpha,\beta}$ near this transition,

$$r_{\alpha,\beta} = \frac{P_{\alpha,\beta}}{P_\alpha + P_\beta}\,, \tag{10}$$

undergo a ~~nearly linear transition~~ transition from a chip mode with emission wavelength centered around

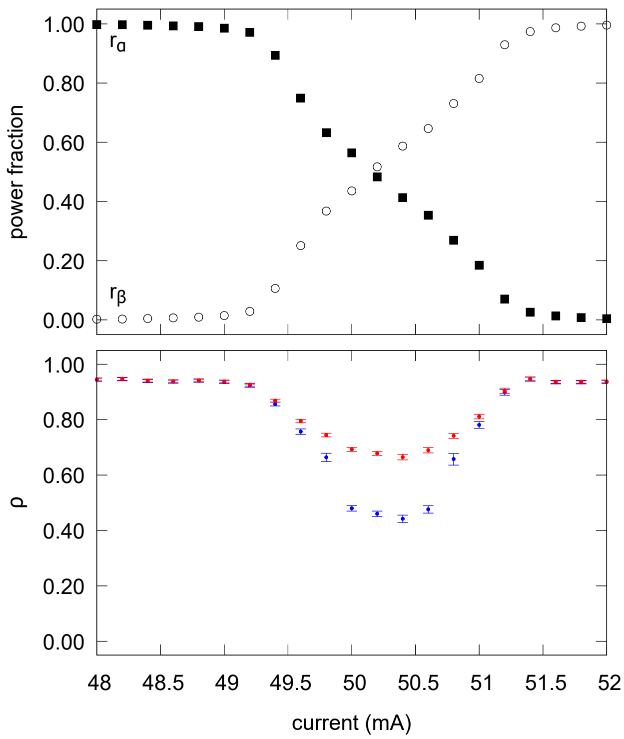

FIG. 6. Different chip modes of the laser diode are excited for different currents, resulting in a reduction of the $g^{(2X)}$ signature in a mode competition regime. Top: Power ratios $r_{\alpha,\beta}$ as a function of current for the chip modes α and β emitting in narrow bands around 780.07 nm (solid squares) and 780.34 nm (hollow circles), respectively. Bottom: Upper bound of fraction ρ of coherent light (red) extracted via Eqn. C1, and the lower bound (blue) extracted via Eqn. C2 from $g^{(2X)}(0)$.

780.07 nm to another chip mode with centered around 780.34 nm (see top traces of Fig. 6).

We measured $g^{(2X)}$ in the same transition regime ~~and extract~~ at finer current steps and extracted the proportion of coherent light from the ~~most dominant~~ brightest coherent emission ρ as described above (see Fig. 6, bottom trace). In the transition regime, the fraction ρ of coherent emission extracted ~~this way~~ from $g^{(2X)}$ decreases, when there is emission at multiple chip modes, and increases again when the emission approaches a single chip mode. ~~This~~ Based on our model of the light field in Eqn. 4, this can be interpreted as the light of one emission band being uncorrelated to the light of the other emission band, although the light in each band is coherent with itself.

V. CONCLUSION

We presented a method to extract the proportion of coherent light emitted by a laser diode without the use of spectral filters, using interferometric photon correlations. As a demonstration, we measured interferometric photon correlations of light emitted from a laser diode over a range of operating currents~~near the lasing threshold,~~ , and extracted the proportion of ~~coherent light~~ light intensity emitted from the brightest coherent emission, showing an increase in proportion of coherent light emission as the operating current was increased past the lasing threshold. We also used this technique to characterize the coherence of emission in a ~~transition regime~~ mode hop between longitudinal modes above the lasing threshold, and find a reduction of the fraction of coherent light there, suggesting that the two longitudinal modes can be viewed as independent and mutually incoherent coherent emissions. Apart from the characterisation of lasers, this method may also be useful in practical applications of some continuous-variable quantum key distribution protocols [31, 32], where the noise of a coherent state source such as a laser, may need to be characterised [33–35].

Appendix A: Interferometric photon-correlation for a mixture of light fields

We show here in further detail the derivation of Eqn.5, by calculating the interferometric photon correlation using the model light field in Eqn. 4.

The evaluation of $g^{(2X)}$ via Eqn. 3 requires the conventional second-order photon correlation function ~~$g^{(2)}(t_1 - t_2) = \langle E^*(t_1)E^*(t_2+\Delta)E(t_2)E(t_1+\Delta)\rangle$~~ $g^{(2)}(t_1 - t_2) = \langle E$ and the last ~~terms (CK: how many?)~~ two terms in Eqn. 3 containing negative signs. For the light field mixture Eqn. 4, its conventional second-order photon correlation function is given by

$$\begin{aligned} g^{(2)}_{\mathrm{mix}}(t_2-t_1) = & \\ =&\rho^2\, g^{(2)}_{\mathrm{coh}}(t_2-t_1) + (1-\rho)^2\, g^{(2)}_{\mathrm{unc}}(t_2-t_1) \\ &+ 2\rho(1-\rho)\left[1 + \Re[g^{(1)}_{\mathrm{coh}}(t_2-t_1)\, g^{(1)*}_{\mathrm{unc}}(t_2-t_1)\right] \end{aligned}, \tag{A1}$$

where $g^{(1)}$ is the first-order field correlation function for the respective component light fields, $g^{(1)*}$ its complex conjugate, and $\Re[\cdots]$ extracts the real part of its argument.

The last terms in Eqn. 3 can be written as

$$\begin{aligned} &\langle E^*_{\mathrm{mix}}(t_1)E^*_{\mathrm{mix}}(t_2+\Delta)E_{\mathrm{mix}}(t_2)E_{\mathrm{mix}}(t_1+\Delta)\rangle \\ =&\rho^2\, |g^{(1)}_{\mathrm{coh}}(t_2-t_1)|^2 + (1-\rho)^2 |g^{(1)}_{\mathrm{unc}}(t_2-t_1)|^2 \\ &+2\rho(1-\rho)\,\Re[g^{(1)}_{\mathrm{coh}}(t_2-t_1)\, g^{(1)*}_{\mathrm{unc}}(t_2-t_1)] \\ &+2\rho(1-\rho)\,\Re[g^{(1)}_{\mathrm{coh}}(\Delta)\, g^{(1)*}_{\mathrm{unc}}(\Delta)] \end{aligned}, \tag{A2}$$

where $g^{(1)}(\Delta) \approx 0$ for our experimental situation of the propagation delay Δ significantly larger than the coherence times of the respective light sources. Note that all terms in Eqn. A2 are real-valued.

With this, the interferometric photon correlation at zero time difference is given by

$$
\begin{aligned}
&g^{(2X)}_{\mathrm{mix}}(0) = \\
&= \frac{1}{4}[g^{(2)}_{\mathrm{mix}}(\Delta) + g^{(2)}_{\mathrm{mix}}(-\Delta) \\
&\qquad + 2(\rho^2\, g^{(2)}_{\mathrm{coh}}(0) + (1-\rho)^2\, g^{(2)}_{\mathrm{unc}}(0) + 2\rho(1-\rho)) \\
&\qquad - 2(\rho^2\, |g^{(1)}_{\mathrm{coh}}(0)|^2 + (1-\rho)^2 |g^{(1)}_{\mathrm{unc}}(0)|^2)]\,.
\end{aligned} \tag{A3}
$$

We make further assumptions that the propagation delay in the interferometer Δ is significantly longer than the coherence time scale of the light source, such that $g^{(2)}_{\mathrm{mix}}(\pm\Delta) \approx 1$, the interferometer has good visibility, such that $\cancel{g^{(1)}(0) \approx 1}$ $|g^{(1)}(0)| \approx 1$, and for the second order photon correlation of the coherent light field is $g^{(2)}_{\mathrm{coh}}(0) = 1$. The evaluation of these assumptions in Eqn. A3 leads to the relationship shown in Eqn. 5.

Appendix B: Boundaries of physically meaningful combinations of interferometric correlations in a mixture

Assuming a binary mixture of the light field as per Eqn. 4, the interferometric correlation of the mixture, $g^{(2X)}_{\mathrm{mix}}(0)$, and the conventional second order correlation of the incoherent light, $g^{(2)}_{\mathrm{unc}}(0)$, at zero time difference are constrained by relation Eqn. 5. Further assuming the physical requirement $0 \leq \rho \leq 1$ for the coherent light fraction gives a lower bound for $g^{(2)}_{\mathrm{unc}}(0)$,

$$
g^{(2)}_{\mathrm{unc}}(0) \geq \begin{cases} 0\,, & g^{(2X)}_{\mathrm{mix}}(0) \leq \frac{2}{3} \\ 3 + \frac{1}{1-2g^{(2X)}_{\mathrm{mix}}(0)}, & g^{(2X)}_{\mathrm{mix}}(0) \in [\frac{2}{3}, 1] \\ 2g^{(2X)}_{\mathrm{mix}}(0) & g^{(2X)}_{\mathrm{mix}}(0) \geq 1 \end{cases}. \tag{B1}
$$

For $g^{(2X)}_{\mathrm{mix}}(0) \in [0, \frac{1}{2})$, there is an upper bound for $g^{(2)}_{\mathrm{unc}}(0)$,

$$
g^{(2)}_{\mathrm{unc}}(0) \leq 2g^{(2X)}_{\mathrm{mix}}(0)\,. \tag{B2}
$$

Appendix C: Error propagation from fitting of $g^{(2X)}$ measurement

Standard techniques in propagation of uncertainties are not used, as the expressions in Eqn. 7-8 would lead to indefinite values of uncertainties at some values of A. We thus extract the upper and lower bounds of ρ by performing a change in variables from the probability density of A.

The probability density of A is assumed to be a normal distribution with a mean value and standard deviation respectively the value and uncertainty of A extracted from the curve fitting of $g^{(2X)}$ to Eqn. 9.

The probability densities describing the upper and lower bounds of ρ is obtained from a change of variable from A by rewriting Eqn. 7-8 in terms of A. The transfomation of the upper bound is

$$
\rho = \sqrt{2A}, \tag{C1}
$$

and the lower bound,

$$
\rho = \begin{cases} 2A, & \text{for } 0 \leq A \leq \frac{1}{4} \\ \frac{1}{2} + \frac{1}{2}\sqrt{4A-1}, & \text{for } \frac{1}{4} \leq A \leq \frac{1}{2} \end{cases}. \tag{C2}
$$

We exclude non-physical values of ρ by setting the probability density outside the domain $0 \leq \rho \leq 1$ to 0~~,~~ ~~to 0.~~ . The probability density is renormalised by dividing over its integral. From these probability densities of the upper bound and lower bound of ρ, we compute the expectation of ρ and its 90% confidence interval, which would be reported respectively as the data points and errorbars in plots which contains measurements of ρ.

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