Direct measurement of coherent light proportion from a practical laser source

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We present a technique to estimate the proportion of coherent emission in the light emitted by a practical laser source without spectral filtering, using interferometric photon correlation measurements, where photon correlations are measured between the light emitted from the output ports of an asymmetric Mach-Zehnder interferometer. Using this technique, we measured the proportion of coherent emission in the light emitted by a laser diode, in a statistical mixture of light from stimulated and spontaneous emission, and a statistical mixture of light from stimulated emission at two distinguishable spectral bands.

I. INTRODUCTION

The theories that lead to the invention of lasers can be traced to the papers describing the emission process of the light from an atom to be spontaneous or stimulated [1]. An ensemble of light emitters undergoing stimulated emission will emit coherent light that has a well-defined phase, whereas an ensemble of light emitters undergoing spontaneous emission will emit incoherent light which is randomly phased [2].

In traditional models of macroscopic lasers [3–5], the light emitted from a laser is modeled to originate dominantly from stimulated emission. These models also predict a phase transition of the nature of emission at the lasing threshold of the laser, separating two operating regimes where light emitted is either spontaneous or stimulated.

However, experiments on small lasers have shown that the transition from spontaneous to stimulated emission is not abrupt, but extends over a range for the pump power [6–10]. Across this range the light emitted from the laser is a mixture of spontaneous and stimulated emission.

In these experiments, the transition from spontaneous to stimulated emission was characterized by measuring the second-order photon correlation $g^{(2)}$, which corresponds to the distribution of timing separations between single photon detection events, using the standard Hanbury-Brown and Twiss scheme [11]. The measurement result can be explained using Glauber's theory of optical coherence [12], where incoherent light from spontaneous emission would exhibit a "bunching" signature with $g^{(2)}(0) > 1$, while coherent light from stimulated emission exhibits a Poissonian distribution with $g^{(2)} = 1$.

The "bunching" signature associated with incoherent light has a characteristic timescale inversely related to the spectral width of the light, according to the Wiener-Khintchine theorem ~~[13, 14]. (Maybe that needs also an optics reference? Is WK as a mathematical argument enough?)~~related through a Fourier transform [13–15]. In a practical measurement, the amplitude of the "bunching" signature scales with the ratio of characteristic timescale of the light to the timing response of the detectors [16]. Thus, when the spectral width of the incoherent light is broad, to the extent that the characteristic timescale of the "bunching" signature is smaller than the detector timing uncertainty, incoherent light may exhibit $g^{(2)} \approx 1$, like coherent light.

To overcome the limited detector timing uncertainty, a narrow band of incoherent light can be prepared with filters from a wide optical spectrum of an incoherent licht source [17]. The narrow spectral width of a filtered incoherent light has a correspondingly larger characteristic coherence timescale, which may be long enough to be resolvable by the detectors.

However, for characterising the transition from spontaneous to stimulated emission of a laser, such spectral filtering presents some shortcomings. First, as spectral filtering discards light outside the transmission window of a filter, a result would be inconclusive for the full emission of the source. Second, spectral filtering requires *a priori* information or an educated guess of the central frequency and bandwidth of stimulated emission. Third, it has been demonstrated that spectral filtering below the Schawlow-Townes linewidth of the laser would result in $g^{(2)}(0) > 1$ similar to light from spontaneous emission [18].

This motivates for methods which quantify the proportion of coherent light emitted from the light source without the need for spectral filtering. A method to characterise the stimulated and spontaneous emission from a pulsed laser has been demonstrated before [19], which prompts for a method applicable to lasers in continuous wave operation.

~~Apart from the characterisation of lasers, a method that quantifies the proportion of coherent light in a light source by sampling the light at low intensities may also be useful in practical applications of some continuous-variable quantum key distribution protocols [20, 21], where the noise of a coherent state source such as a laser, may need to be characterised [22–24].(CK: this para should go in an outlook section)~~

In this paper, we present a method of using interferometric photon correlations to quantify bounds for the proportion of coherent light ~~emitted~~ of the brightest nmode of coherent emission from a semiconductor laser

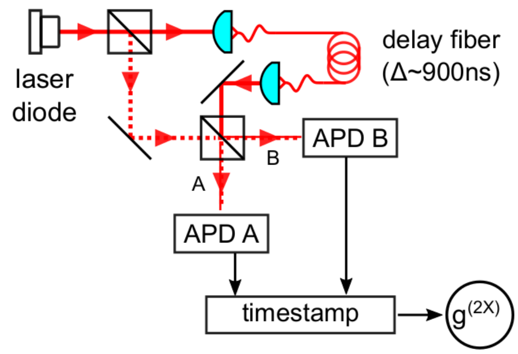


FIG. 1. Experimental setup for measuring interferometric photon correlations.Light from a laser diode enters an asymmetric Mach-Zehnder Interferometer. Singe photon avalanche photodetectors (APD) generate photodetection events at each output port of the interferometer, which are time-stamped to extract the the correlations numerically.

diode without spectral filtering. Interferometric photon correlation measurements were initially used to study spectral diffusion in organic molecules embedded in solid matrix [25, 26], and were also applied to differentiate between incoherent light and coherent light with amplitude fluctuations [27]. In contrast to second-order photon correlations, interferometric photon correlations can clearly distinguish between a finite linewidth coherent light and broadband incoherent light [28]. We use this method to extract the fraction of coherent light emitted by the laser diode over a range of pump powers near the lasing threshold, and in the lasing regime above threshold where coherent light is emitted into two distinguishable longitudinal modes.

II. INTERFEROMETRIC PHOTON CORRELATIONS

The setup for an interferometric photon correlation measurement $g^{(2X)}$ is shown in Fig. 1. Light emitted by the laser diode is sent through an asymmetric Mach-Zehnder interferometer, with a long propagation delay Δ between the two paths of the interferometer that exceeds the coherence time of the light.

With a light field $E(t)$ at the input, the light fields at the output ports A, B of the interferometer are

$$E_{A,B}(t) = \frac{E(t) \pm E(t+\Delta)}{\sqrt{2}}\,, \tag{1}$$

with the relative phase shift π acquired by one of the output fields from the beamsplitter.

Using these expressions for the electrical fields, the temporal correlation of photodetection events between the two output ports is given by

$$\begin{aligned} &g^{(2X)}(t_2 - t_1) \\ &= \frac{\langle E_A^*(t_1) E_B^*(t_2) E_B(t_2) E_A(t_1)\rangle}{\langle E_A^*(t_1) E_A(t_1)\rangle \langle E_B^*(t_2) E_B(t_2)\rangle}\,. \end{aligned} \tag{2}$$

Therein, $\langle\rangle$ indicates an expectation value and/or an ensemble average. Using expressions in Eqn. 1, the correlation $g^{(2X)}(t_2 - t_1)$ can be grouped in several terms:

$$\begin{aligned} &g^{(2X)}(t_2 - t_1) \\ = &\frac{1}{4}[\langle E^*(t_1)E^*(t_2)E(t_2)E(t_1)\rangle \\ &+\langle E^*(t_1+\Delta)E^*(t_2+\Delta)E(t_2+\Delta)E(t_1+\Delta)\rangle \\ &+\langle E^*(t_1+\Delta)E^*(t_2)E(t_2)E(t_1+\Delta)\rangle \\ &+\langle E^*(t_1)E^*(t_2+\Delta)E(t_2+\Delta)E(t_1)\rangle \\ &-\langle E^*(t_1+\Delta)E^*(t_2)E(t_2+\Delta)E(t_1)\rangle \\ &-\langle E^*(t_1)E^*(t_2+\Delta)E(t_2)E(t_1+\Delta)\rangle] \end{aligned} \tag{3}$$

The first two terms have the form of conventional second-order photon correlation functions $g^{(2)}(t_1, t_2)$$g^{(2)}(t_2 - t_1)$. The next two terms are conventional second-order photon correlation functions, time-shifted forward and backward in their argument by the propagation delay Δ in the interferometer. The last two terms have negative signs and reduce $g^{(2X)}$, leading to a dip at zero time difference $t_2 - t_1$, with a width given by the coherence time of the light.

The expectation values appearing in Eqn. 3 for $g^{(2X)}$ can be evaluated by using statistical expressions [2] of $E(t)$ for incoherent light , coherent light , and mixtures thereof and coherent light [28]. [28].

For incoherent light, $g^{(2X)}$ exhibits a "bunching" signature peaking at time differences plus and minus the propagation delay, $g^{(2X)}(\pm\Delta) = 1 + 1/4$. At zero-time difference, the expected "bunching" signature from conventional second-order photon correlation functions in the first two terms of Eqn. 3 and the dip from the last two terms of Eqn. 3 cancel each other, resulting in $g^{(2X)}(0) = 1$.

For coherent light, since the second-order photon correlation function $g^{(2)}$ has a constant value of 1, the $g^{(2X)}$ will show the negative contributions from the last two terms of Eqn. 3, resulting in $g^{(2X)}(0) = 1/2$.

III. EXTRACTING FRACTION OF COHERENT LIGHT EMITTED IN A MIXTURE

In We consider a case where the light field is neither completely coherent nor incoherent. In particular, we consider a mixture of incoherent and coherent light , coherent light field E_{coh}, with an uncorrelated light field E_{unc}, which nature of light emission could be coherent, incoherent, or a coherent-incoherent mixture. From interferometric photon correlations $g^{(2X)}$ would

~~show the features of incoherent and coherent light scaled proportionally to their fraction in the mixture~~, we demonstrate a method to extract information on the fraction of coherent light intensity of the brightest coherent emission, over the mean intensity of the mixture.

The resultant light field of the mixture above can be written as

$$E_{\mathrm{mix}}(t) = \sqrt{\rho}E_{\mathrm{coh}}(t) + \sqrt{1-\rho}E_{\mathrm{unc}}(t), \tag{4}$$

where ρ is the fraction of coherent light intensity of the brightest coherent emission, over the mean intensity of the mixture. The respective light field terms have been normalised such that $|E_{\mathrm{res}}| = |E_{\mathrm{coh}}| = |E_{\mathrm{unc}}|$.

It follows from Eqn.,3 that the evaluation of $g^{(2X)}$ requires the evaluation of the conventional second-order photon correlation function $g^{(2)}$, $\langle E^*(t_1)E^*(t_2+\Delta)E(t_2)E(t_1+\Delta)\rangle$, and its complex conjugate.

For the light field of the mixture described above, the second-order photon correlation function is

$$\begin{aligned} & g^{(2)}_{\mathrm{mix}}(t_2-t_1) \\ =& \rho^2\, g^{(2)}_{\mathrm{coh}}(t_2-t_1) + (1-\rho)^2\, g^{(2)}_{\mathrm{unc}}(t_2-t_1) \\ +& 2\rho(1-\rho)\left[1+\Re[g^{(1)}_{\mathrm{coh}}(t_2-t_1)\, g^{(1)*}_{\mathrm{unc}}(t_2-t_1)]\right] \end{aligned}, \tag{5}$$

where $g^{(1)}$ is the first-order field correlation function for the respective component light fields with $g^{(1)*}$ its complex conjugate, and $\Re[\cdots]$ extracts the real part of its argument.

The second last term in Eqn. 3,

$$\begin{aligned} & \langle E^*_{\mathrm{mix}}(t_1)E^*_{\mathrm{mix}}(t_2+\Delta)E_{\mathrm{mix}}(t_2)E_{\mathrm{mix}}(t_1+\Delta)\rangle \\ =& \rho^2\, |g^{(1)}_{\mathrm{coh}}(t_2-t_1)|^2 + (1-\rho)^2 |g^{(1)}_{\mathrm{unc}}(t_2-t_1)|^2 \\ +& 2\rho(1-\rho)\,\Re[g^{(1)}_{\mathrm{coh}}(t_2-t_1)\, g^{(1)*}_{\mathrm{unc}}(t_2-t_1)]) \\ +& \rho^2\, |g^{(1)}_{\mathrm{coh}}(\Delta)|^2 + (1-\rho)^2 |g^{(1)}_{\mathrm{unc}}(\Delta)|^2 \\ +& 2\rho(1-\rho)\,\Re[g^{(1)}_{\mathrm{coh}}(\Delta)\, g^{(1)*}_{\mathrm{unc}}(\Delta)] \end{aligned}, \tag{6}$$

where $g^{(1)}(\Delta) \approx 0$ when the propagation delay Δ is significantly larger than the coherence times of the respective light sources. We note that since all terms in the above expression are real valued, its complex conjugate equals to itself.

By substituting Eqn.,5-6 into Eqn.,3, we evaluate the interferometric photon correlation at zero-time difference,

$$\begin{aligned} & g^{(2X)}_{\mathrm{mix}}(0) \\ =& \frac{1}{4}[g^{(2)}_{\mathrm{mix}}(\Delta) + g^{(2)}_{\mathrm{mix}}(-\Delta) \\ & +2(\rho^2\, g^{(2)}_{\mathrm{coh}}(0) + (1-\rho)^2\, g^{(2)}_{\mathrm{unc}}(0) + 2\rho(1-\rho)) \\ & -2(\rho^2\, |g^{(1)}_{\mathrm{coh}}(0)|^2 + (1-\rho)^2 |g^{(1)}_{\mathrm{unc}}(0)|^2)] \end{aligned}, \tag{7}$$

Using the fact that $g^{(2)}_{\mathrm{coh}}(0) = 1$ for coherent light, $g^{(1)}(0) \approx 1$ for an interferometer with good visibility, and $g^{(2)}_{\mathrm{mix}}(\Delta) \approx 1$ for Δ longer than the characteristic timescale of the light source, Eqn.,7 reduces to

$$g^{(2X)}_{\mathrm{mix}}(0) = 2\rho - \frac{3\rho^2}{2} + \frac{(1-\rho)^2}{2} g^{(2)}_{\mathrm{unc}}(0). \tag{8}$$

As we consider the scenario where the nature of the uncorrelated light source is coherent, incoherent light, or a coherent-incoherent mixture, the second-order photon correlation of the uncorrelated light is bounded by

$$1 \le g^{(2)}_{\mathrm{unc}}(0) \le 2. \tag{9}$$

We impose these bounds in Eqn.,8, and extract the bounds to the fraction of coherent light of the brightest coherent emission ρ with an upper bound,

$$\rho \le \sqrt{2 - 2\, g^{(2X)}(0)}, \tag{10}$$

and a lower bound,

$$\rho \ge \begin{cases} \frac{1}{2} + \frac{1}{2}\sqrt{3 - 4\, g^{(2X)}(0)}, & \text{for } \frac{1}{2} \le g^{(2X)}(0) \le \frac{3}{4} \\ 2 - 2\, g^{(2X)}(0), & \text{for } \frac{3}{4} \le g^{(2X)}(0) \le 1 \end{cases}. \tag{11}$$

with $g^{(2X)}(0)$ ranging from ~~0.5~~ 1/2 for fully coherent light, to 1 for fully incoherent light. ~~The fraction of coherent light in the mixture,~~

The conditional expressions of Eqn.,11 is due to the minimum allowed value of $g^{(2)}_{\mathrm{unc}}(0)$ such that the solution of Eqn.,8 gives a real valued ρ, ~~can then be extracted from $g^{(2X)}(0)$ via~~

$$\rho = 2\left(1 - g^{(2X)}(0)\right).$$

when $\frac{3}{4} \le g^{(2X)}(0) \le 1$.

IV. EXPERIMENT

In this work, we perform interferometric photon correlation measurements of light emitted from a temperature stabilised distributed feedback laser diode with central wavelength ~~at~~ around 780 nm at different operating currents. The interferometric photon correlation measurements were performed over a range of operating currents, which covers a region where the laser diode is expected to emit a statistical mixture of ~~spontaneous and stimulated emission~~coherent and incoherent light, and a region above where ~~stimulated emission of light~~ coherent light emission at two frequencies were observed.

We construct the asymmetric Mach-Zehnder interferometer using 50-50 fibre beamsplitters for splitting and recombination of the beam, with a propagation delay Δ of about 900 ns through a 180 m long single mode optical fibre.

The photoevents at each output port of the interferometer were detected with actively quenched silicon single photon avalanche photo diodes (APD) with a timing resolution lower than 50 ps. The detected photoevents were time-stamped using a timetagger with a resolution of 2 ns.

The interferometric photon correlations $g^{(2X)}$ are extracted from the time stamps through histogramming all time differences $t_2 - t_1$ over some integration time T between detection event pairs numerically, which allows for a clean normalization. The resulting correlation is fitted to a two-sided exponential function,

$$g^{(2X)}(t_2 - t_1) = 1 - A \cdot \exp\left(-\frac{|t_2 - t_1|}{\tau_c}\right) , \qquad (12)$$

where τ_c is the characteristic time constant of the coherent light, and A is the amplitude of the dip. The value of $g^{(2X)}(0)$ is the extracted from the fit as $1 - A$. ~~From Eqn. ??, the corresponding fraction of coherent light is simply given by~~

$$\rho = 2A \,.$$

Examples of measured correlation functions and corresponding fits for different laser powers are shown in Fig. 2.

We note that the expressions in Eqn.,10-11 would lead to indefinite values of uncertainties at some values of A if standard techniques in propagation of uncertainties are used. We thus extract ρ from A by performing a change in variables of the probability density function of A. To do this, we rewrite Eqn.,10-11 in terms of A, with upper bound,

$$\rho \leq \sqrt{2A}, \qquad (13)$$

and a lower bound,

$$\rho \geq \begin{cases} 2A, & \text{for } 0 \leq A \leq \frac{1}{4} \\ \frac{1}{2} + \frac{1}{2}\sqrt{4A - 1}, & \text{for } \frac{1}{4} \leq A \leq \frac{1}{2} \end{cases} . \qquad (14)$$

We assume the probability density function of A to be a normal distribution with a mean value and standard deviation respectively the value and uncertainty of A extracted from the curve fitting of $g^{(2X)}$ to Eqn.,12. The probability density function of ρ is obtained from a change of variable from A and setting the probability density outside the domain $0 \leq \rho \leq 1$ to 0, to exclude non-physical values of ρ. The probability density function is renormalised by dividing over its integral. From this probability density function of ρ, we compute the expectation of ρ and its 90% confidence interval, which would be reported respectively as the data points and errorbars in plots which contains measurements of ρ.

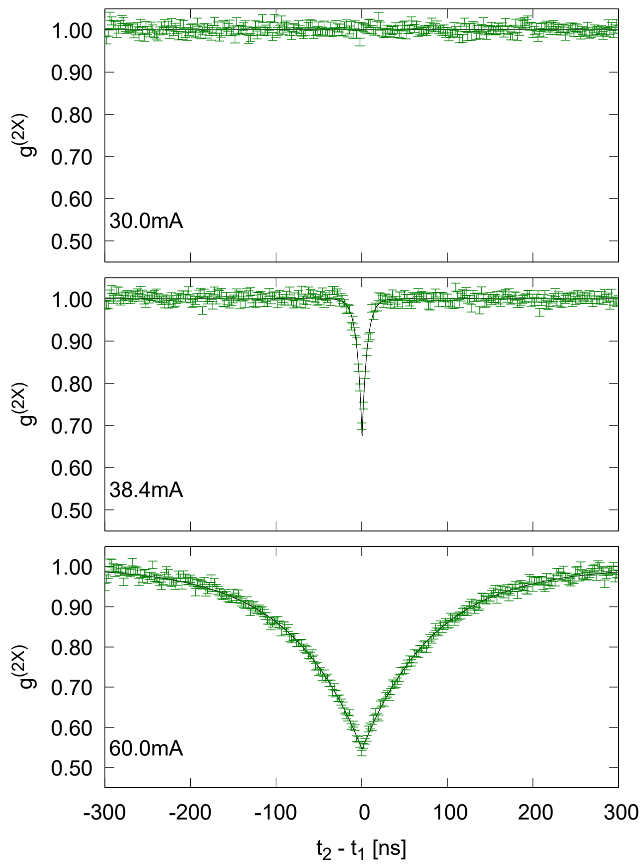


FIG. 2. Interferometric photon correlations $g^{(2X)}$ for different injection currents, extracted from a histogram of photodetector time differences (~~red~~ green symbols)~~;~~. The error range at a specific time bin indicates an ~~expectde~~ expected uncertainty according to a Poissonian counting statistics. The black solid lines show a fit to Eqn.~~12~~refeqn:fit. From the fit, the we find values for ~~$g^{(2X)}(0)$~~ A (from top to bottom) of ~~1.0003 ± 0.0003~~ -0.0006 ± 0.0003, ~~0.727 ± 0.0005~~ 0.326 ± 0.008, ~~0.590 ± 0.0003~~ 0.455 ± 0.002, respectively.

A. Coherent-incoherent light mixture near lasing threshold

Emission of light with a statistical mixture of ~~stimulated and spontaneous~~ coherent and incoherent light emission from the laser diode is expected when operating it near its lasing threshold current[29, 30]. To estimate this threshold, we measure the optical power emitted by the laser diode at different currents, and find the sharpest change ~~at a current of 36.4~~of optical power with respect to current. We measure this threshold to be 37 mA~~(CK: do we have a figure for that?).~~ , as shown in Fig 3.

Sample interferometric photon correlation measurements of the emitted light for ~~laser~~ operating current

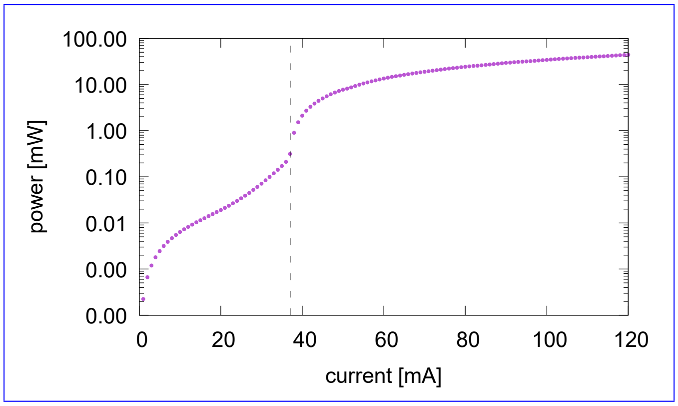


FIG. 3. Purple circles: power against current measurement of the laser diode, taken from 1 to 120 mA in steps of 1 mA.The sharpest change in current was measured at 37 mA, shown in dotted lines.

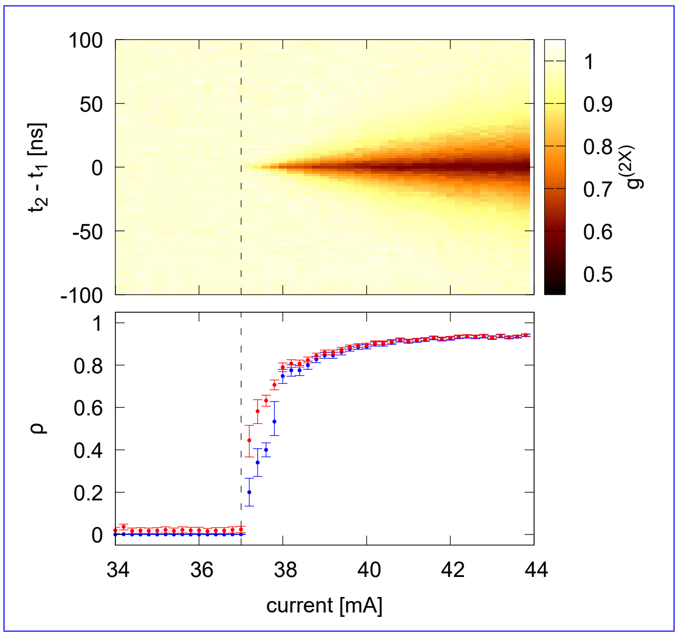


FIG. 4. Top: Interferometric photon corrrelations $g^{(2X)}$, measured for a range of laser currents around the lasing threshold. Bottom: Corresponding upper bound of fraction ρ of coherent light (red) extracted via Eqn. 13, and the lower bound (blue) extracted via Eqn. ~~??~~ 14 from $g^{(2X)}(0)$. The dotted line indicates the lasing threshold of ~~36.4~~37 mA.

above and below the threshold are shown in Fig. 2.

The interferometric photon correlation measurements as a function of the laser current around the lasing threshold are shown in Fig. 4 (top part). The ~~fraction ρ of coherent light~~ amplitude of the dip is then extracted by fitting these measurements to Eqn. 12, from which the upper bound and lower bound of the fraction of coherent light from the most dominant coherent emission ρ is extracted (see Fig. 4, bottom part). We observe an increasing trend in ~~the fraction of coherent light emitted from the laser diode~~ ρ above the threshold at ~~36.4~~37 mA in a phase-transition manner. This agrees with the expectation that the emission of the laser diode is increasingly dominated by stimulated emission past the lasing threshold~~. (CK: There should be some stone age laser theory paper or textbook that shows or predicts this? Mayman? Haken?)~~[29, 30].

B. Mixture of coherent light at two frequencies

Above the lasing threshold, the laser oscillates at different longitudinal modes for different drive currents. The technique to extract the fraction of coherent and incoherent light allows to investigate the behavior also in the transition regime between oscillation on different longitudinal lasing modes.

For this, we measured the spectrum of light emitted by the laser diode at different currents above the lasing threshold with an optical spectrum analyser based on a Michelson interferometer with a spectral resolution of ~~4~~2 GHz (Bristol 771B-NIR). The laser diode emitted light into two distinct narrow spectral bands with a changing power ratio in a diode current range between ~~46.8~~49.0 mA and ~~48.2~~52.4 mA. Outside this window, only one of the modes was present. Below ~~46.8~~49.0 mA, the laser emission was centered around ~~779.88~~780.07 nm, above ~~48.2~~52.4 mA around ~~780.15~~780.34 nm~~. (CK: Do we have some idea about the width, or can we claim an upper bound of the linewidth, ,~~. The linewidths of both chip modes are measured to be about 2 GHz suggesting that the spectral measurements were limited by the ~~spectrometer?)~~spectral resolution of 2 GHz of the spectrum analyser.

The power fractions $r_{\alpha,\beta}$ of these two chip modes α and β near this transition,

$$r_{\alpha,\beta} = \frac{P_{\alpha,\beta}}{P_\alpha + P_\beta}\,, \tag{15}$$

undergo a nearly linear transition (see top traces of Fig.5).

We measured $g^{(2X)}$ in the same transition regime and extract the proportion of coherent light from the most dominant coherent emission ρ as described above (see Fig. 5, bottom trace). In the transition regime, the fraction ρ of coherent emission extracted this way decreases, ~~and reaches a constant value outside the transition region. (CK: Why not 1? is the 0.82 also found elsewhere?)~~when there is emission at multiple chip modes, and increases again when the emission approaches a single chip mode. This can be interpreted as the light of one emission band being ~~incoherent~~ uncorrelated to the light of the other emission band, although the light in each band is coherent with itself. **(CK: Presumably the beat frequency between the two modes is too high to be detected in the transition regime? XJ:**

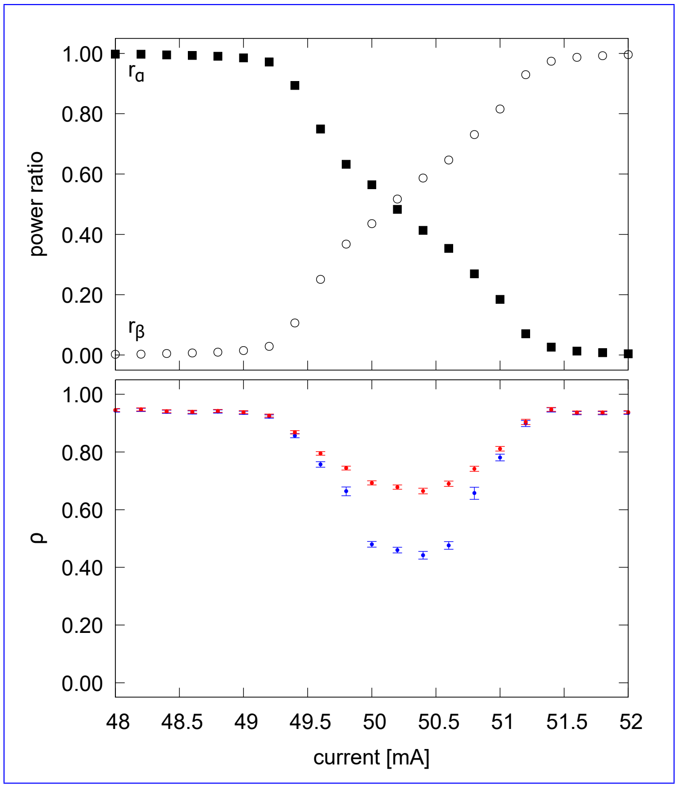
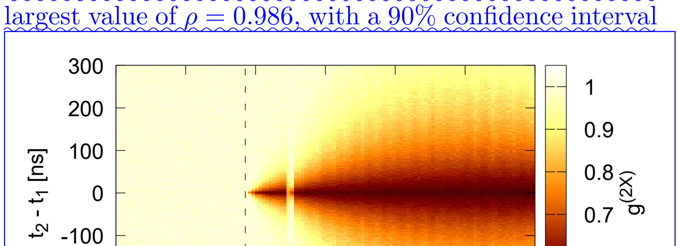


FIG. 5. Different chip modes of the laser diode are excited for different currents, resulting in a reduction of the $g^{(2X)}$ signature in a mode competition regime. Top: Power ratios $r_{\alpha,\beta}$ as a function of current for the chip modes α and β emitting in narrow bands around ~~779.88~~780.07 nm (~~red~~solid squares) and ~~780.15~~780.34 nm (~~blue~~hollow circles), respectively. Bottom: ~~Fraction~~ Upper bound of fraction ρ of coherent light ~~ρ in the same measurement,~~ (red) extracted ~~from $g^{(2X)}(0)$~~via Eqn.~~(CK: Can you use "power ratio" in the top figure label instead~~ 13, and ~~eventually put labels $r_{\alpha,\beta}$ next to~~ the ~~traces? Consider not joining lines in top traces, as there are discrete current steps? Error bars on symbols if easy to obtain?~~lower bound (blue) extracted via Eqn. 14 from $g^{(2X)}(0)$.

The beat frequency appears in the $g^{(2)}$ terms and the negative terms in $g^{(2X)}$ which cancels out each other.)

C. Coherent emission trend below and above lasing threshold

We investigate the general trend in ρ of coherent emission, by performing $g^{(2X)}$ measurements to cover operating current of the laser significantly below and above the lasing threshold. From Fig 6, the measured ρ of coherent emission remains near 0 when operated below threshold. When operating above the lasing threshold, the measured ρ of coherent emission increases, with the largest value of $\rho = 0.986$, with a 90% confidence interval from 0.982 to 0.989 measured at 120 mA.



V. CONCLUSION

In this paper, we presented a method to extract the proportion of coherent light emitted by a laser diode without the use of spectral filters, using interferometric photon correlations. As a demonstration, we measured interferometric photon correlations of light emitted from a laser diode over a range of operating currents near the lasing threshold, and extracted the proportion of coherent light emitted from the brightest coherent emission, showing an increase in proportion of coherent light emission as the operating current was increased past the lasing threshold. We also used this technique to characterize the coherence of emission in a transition regime between longitudinal modes above the lasing threshold, and find a reduction of the fraction of coherent light there, suggesting that the two longitudinal modes can be viewed as independent and mutually incoherent coherent emissions. Apart from the characterisation of lasers, this method may also be useful in practical applications of some continuous-variable quantum key distribution protocols [20, 21], where the noise of a coherent state source such as a laser, may need to be characterised [22–24].

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