Coupling Light to The Transverse Modes of a Near-Concentric Optical Cavity

Adrian Nugraha Utama,[1] **Chang Hoong Chow,**[1]**, Chi Huan Nguyen,**[1] **and Christian Kurtsiefer**[1,2,*]

[1] *Centre for Quantum Technologies, 3 Science Drive 2, Singapore 117543*
[2] *Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542*
[*] *phyck@nus.edu.sg*

Abstract: Optical cavities in the near-concentric regime have near-degenerate transverse modes. The tight focusing transverse modes in this regime enable strong coupling with atoms. These features provide an interesting platform to explore multi-mode interaction between atoms and light. Here, we use a phase spatial light modulator (SLM) to shape the incoming light and match the Laguerre-Gaussian (LG) modes of a near-concentric optical cavity. We demonstrate coupling efficiency close to the theoretical prediction for single LG modes and combinations of them, limited mainly by imperfections in the cavity alignment.

1. Introduction

I'm not sure optical channels is a popular term

= Spatial modes

Transverse modes of an optical channel constitute a separate degree of freedom, apart from frequency, wavevector and polarization, in the propagation of electromagnetic waves. It has a wide range of application, such as increasing the information-carrying capacity in free-space [1] and fiber [2, 3] communications, creating smaller focal volumes to achieve superresolution imaging [4], utilizing orbital angular momentum (OAM) to perform quantum key distribution [5], and producing highly-entangled OAM states [6]. One particular area where transverse modes garnered substantial interest is in optical cavities. For example, transverse modes can be used to track atomic position via the observed mode pattern [7–9]. They can also exhibit inter-mode coupling in the near-degenerate configuration [10, 11], and help enhancing the cooling process in atomic ensembles [12–14]. Perhaps the most exotic application of multiple near-degenerate transverse modes (multimode) cavities is in Bose-Einstein condensates (BEC), where they are used to engineer the atomic interaction within the BEC to create regions of crystallized domains [15–18].

S

has

sound a bit subjective

Optical cavities in the near-concentric regime produce highly focused mode with diffraction-limited focal spot, which enable strong interaction between the optical mode and the atoms placed within [19–21]. Similar to confocal cavities [16, 22], the transverse modes of concentric cavities overlap at the critical point. Even though the critical point is only marginally stable, the cavity can support the fundamental mode less than a wavelength away from the critical point [23]. In the near-concentric regime, the frequency spacing of the transverse modes is ~ 10 MHz to ~ 100 MHz, which are on the order of the hyperfine splitting or the magnetic levels of the atoms. This provides an alternative system to explore atomic nonlinearities with multiple modes simultaneously [24] and extend it to various atomic energy levels and spatial distributions via the transverse modes.

frequencies overlap

can be engineered to be 10MHz to 100MHz, as ideally, it can be anything

hfsplitting is usually GHz

In this work, we couple light to the transverse modes of a near-concentric cavity and characterize the mode structure and efficiency of the coupling procedure. To prepare light from a collimated fiber output to match the spatial distribution of the cavity transverse modes, we use a liquid crystal spatial light modulator (SLM), which modifies the spatial phase information of the incoming beam [25]. In previous works, SLMs have been utilized to excite transverse modes of an optical channel: in multimode fibers with a phase SLM [26] and in confocal cavities using a digital

we optimize the coupling of external Gmode to the transverse...

micromirror device [22] – to the best of our knowledge, it has not been done in near-concentric cavities. Furthermore, we extend this mode-matching procedure to a superposition of the modes and obtain the efficiency. We show that the efficiency depends on the relative phases of the modes. Finally, we examine how close to the critical point are the transverse modes still supported.

to generate a superposition of...

2. Theory

2.1. Transverse modes of a cavity

Optical modes of a cylindrically symmetric cavity with spherical mirrors can be described by the standing wave of the Laguerre-Gaussian (LG) beam [27]:

$$U_{m,l}(\rho,\phi,z) = A_{l,m}\frac{w_0}{w(z)}\left(\frac{\rho}{w(z)}\right)^{l}\mathcal{L}_m^l\left(\frac{2\rho^2}{w^2(z)}\right)\exp\left(-\frac{\rho^2}{w^2(z)}\right)\exp\left(i\,\psi_{m,l}(\rho,\phi,z)\right)\,, \tag{1}$$

where m and l are the radial and azimuthal mode numbers of the LG beams, $A_{l,m}$ is the normalization constant, $w(z) = w_0\sqrt{1+(z/z_0)^2}$ is the beam radius with $z_0 = \pi w_0^2/\lambda$ as the Rayleigh range, $\mathcal{L}_m^l(.)$ is the generalized Laguerre polynomial function, and $\psi_{m,l}(\rho,\phi,z)$ is the real-valued phase of the LG beam, given by:

$$\psi_{m,l}(\rho,\phi,z) = -kz - k\frac{\rho^2}{2R(z)} - l\phi + (2m+l+1)\zeta(z)\,, \tag{2}$$

where $R(z) = z + z_0^2/z$ is the radius of the wavefront and $\zeta(z) = \tan^{-1}(z/z_0)$ is the Gouy phase.

Inside a cavity, the LG modes are bounded by the two mirrors of radii R_1 and R_2 spaced L apart. The g-parameters, $g_1 = 1 - L/R_1$ and $g_2 = 1 - L/R_2$ specify the geometry and stability of the cavity mode. In symmetric cavities ($g_1 = g_2 = g$), the concentric mode is obtained when $L = 2R$ and $g = -1$ (critical point) and is only marginally stable. Near-concentric cavity modes depart from this critical point towards the stable region – we define this distance as the critical distance $d = 2R - L$, with $g = -1 + d/R$.

The resonance frequencies of the cavity depend on the transverse mode numbers m and l:

$$\nu_{q,m,l} = \nu_F\left(q + (2m+l+1)\frac{\Delta\zeta}{\pi}\right) \tag{3}$$

where q is the longitudinal mode number of the cavity, $\nu_F = c/2L$ is the cavity free spectral range, and $\Delta\zeta = \zeta(z_{M2}) - \zeta(z_{M1})$ is the Gouy phase difference between the two cavity mirrors. In near-concentric symmetrical cavities, the transverse mode spacing is given by

$$\Delta\nu_{tr} = \nu_{q+1,0,0} - \nu_{q,0,1} = \frac{\nu_F}{\pi}\cos^{-1}\left(1 - \frac{d}{R}\right)\,, \tag{4}$$

where $\Delta\nu_{tr} \to 0$ as $d \to 0$. The critical distance d can thus be estimated by measuring the frequency separation between the transverse modes [23].

Near-concentric cavities produce atom-cavity coupling strength $g_{ac} \propto 1/\sqrt{V_m}$ comparable to short cavities [21], as the effective mode volume $V_m \approx \pi W_0^2 L$ decreases in tighter focal radii W_0. Interestingly, for a particular critical distance, all the radial modes (transverse modes with $l = 0$) has identical effective mode volumes, resulting in coupling strengths which are equally strong across all radial modes. This comes from the normalization relation $\int_0^\infty e^{-u}\left[\mathbb{L}_m^0(u)\right]^2 du = 1$, yielding the same prefactor $A_{0,m}$ for all radial modes in Eq. 1. which can be used to simulate degenerate boson-spin coupling?

microcavities, or cavities with length in um. Our cavity is considered to be short for others.

at a particular critical distance?

2.2. Mode matching to a cavity

The power transmission through a cavity with identical mirrors is given by [28]:

$$T(\omega) = \frac{P_t(\omega)}{P_{in}} = \eta \frac{\kappa_m^2}{(\kappa_m + \kappa_l)^2 + (\omega - \omega_0)^2} , \tag{5}$$

where $P_t(\omega)$ is the light power transmitted through the cavity, P_{in} is the input power, η is the spatial mode matching efficiency, ω_0 is the cavity resonance frequency, and κ_m and κ_l are the cavity decay rates due to the mirror transmission and scattering losses, respectively. On the other hand, the power reflection of the cavity is given by

$$R(\omega) = \frac{P_r(\omega)}{P_{in}} = 1 - \eta \frac{\kappa_m^2 + 2\kappa_m \kappa_l}{(\kappa_m + \kappa_l)^2 + (\omega - \omega_0)^2} , \tag{6}$$

where $P_r(\omega)$ is the light power reflected by the cavity. The finesse of the cavity is given by $F = \pi/\kappa \nu_F$, where $\kappa = \kappa_m + \kappa_l$ is the total cavity decay rate, obtained from fitting Eq. (5) to the transmission spectrum.

From the transmission and reflection spectrum, the spatial mode matching efficiency η can be obtained. First, we define the "effective" transmission coefficient

$$\alpha = \frac{T(\omega_0)}{1 - R(\omega_0)} , \tag{7}$$

where T and R are measured on cavity resonance. By solving Eq. (5) and Eq. (6) on the cavity resonance, we can obtain the mode matching efficiency η:

$$\eta = \frac{(1 + \alpha)^2}{(2\alpha)^2} T(\omega_0) . \tag{8}$$

Similarly, the cavity decay rates are $\kappa_m = 2\kappa\alpha/(1 + \alpha)$ and $\kappa_l = \kappa(1 - \alpha)/(1 + \alpha)$.

2.3. Beam shaping with SLM

A universal transformation is...

A spatial filter which transforms one optical mode to another can be described by a filter function $T(\mathbf{x}) = M(\mathbf{x}) \exp(i\,\Phi(\mathbf{x}))$ which modulates both the amplitude and the phase of the incoming mode. However, liquid crystal SLM only modulates the phase of the incoming beam and hence only provides the transformation $T(\mathbf{x}) = \exp(i\,\Phi(\mathbf{x}))$. To modulate the amplitude of the incoming beam as well, SLM can be operated in a phase-grating configuration – this produces both the carrier and first-order diffraction beams, which amplitude can be varied by the modulation depth [29, 30]. This method typically requires a high-resolution SLM to sufficiently encode the desired beam profile on the phase-grating, however recent works explored other encoding techniques which allows for amplitude modulation using low-resolution SLMs [25, 31–33]. Alternatively, using two SLMs and a polarizer allows to modulate the amplitude and phase of the incoming beam independently [34–36].

To generate LG modes necessary for cavity mode-matching from a collimated Gaussian beam, we utilize a much simpler technique [37–39] which spatially modulates the incoming Gaussian beam with the phase component of the desired LG modes. The SLM phase function is given by

$$\Phi(\rho, \phi) = \arg\left[U_{m,l}(\rho, \phi, 0)\right] = \arg\left[\mathcal{L}_m^l\left(\frac{2\rho^2}{w^2}\right)\right] - l\phi \tag{9}$$

with the incoming Gaussian mode $U_0(\rho) = A_0 \exp\left(-\rho^2/w_0^2\right)$. The ratio w/w_0 can be varied to optimize the mode overlap of the resulting SLM output mode to the particular LG mode – for relatively small m and l mode numbers, the mode overlap is relatively high with low cross-mode overlap (see Table 1). The mode overlap is defined as the dot product between the two mode profiles $\int(d\sigma)U_1(\rho,\phi)U_2(\rho,\phi)$. The modulus square of mode overlap is the mode matching efficiency η as defined in Section 2.2.

Due to its simplicity, this technique is often implemented using physical phase plates [40,41]. By subjecting the resulting beam to the cavity, only the resonant LG mode is transmitted while other LG modes with different resonance frequencies (see Eq. 2) are attenuated by the cavity.

		Mode matching efficiencies					
SLM output	W/W_0	LG_{00}	LG_{10}	LG_{20}	LG_{30}	LG_{40}	LG_{50}
LG_{10}	0.57	0.1%	**81.2%**	0.0%	2.4%	1.3%	0.7%
LG_{20}	0.45	1.3%	0.1%	**76.9%**	0.1%	1.6%	4.5%
LG_{30}	0.39	0.4%	1.2%	0.5%	**74.6%**	0.3%	0.9%
LG_{40}	0.35	0.2%	0.4%	1.2%	0.8%	**73.2%**	0.5%

Some explanations on non-unity efficiences?

Table 1. Calculated values of the mode matching efficiencies (the square of the mode overlap) between the SLM output and the LG modes for l = 0 cases.

3. Experiment

3.1. Experimental setup

The design and construction

The design of the near-concentric cavity is described in previous works [20,23]. The anaclastic lens-mirror design allows highly divergent modes of the near-concentric cavity to be transformed into collimated modes. This simplifies the requirement of the optical components to generate and measure collimated LG beams on the input and output of the cavity (see Figure 1).

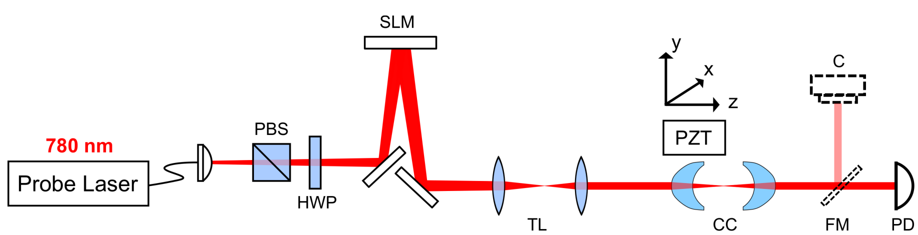


Fig. 1. Optical setup. The SLM (Meadowlark HV 512 DVI) transforms the probe light from the fiber output to match the LG modes of the near concentric cavity (CC). A telescope (TL) is introduced to facilitate mode matching between the SLM beam output and the cavity. The cavity transmission is monitored using either a photodetector (PD) or camera (C), set by the flip mirror (FM).

3.1.1. Mode Generation

As the SLM only modulates light with a particular linear polarization, a sequence of a polarizing beam-splitter (PBS) and a half-wave plate (HWP) prepares the correct polarization to match the SLM polarization axis. Considering the SLM resolution of 512x512 pixels, we minimize the pixelation artifact by using a significant portion of the SLM area (12.8 × 12.8 cm). To achieve this, we create a slightly divergent beam by changing the length of the fiber coupler. The resulting

not neccessary?

We prepare a slightly divergent beam with beam diameter ranges from 3 to 7mm, measured at the SLM beam diameter on the SLM ($1/e^2$ width) ranges from 3 to 7 mm, depending on the desired output beam type and size.

The phase modulation applied on the SLM consists of three components: the LG mode-generating phase pattern as described in Eq. 9, the correction phase pattern provided by the manufacturer, and a quadratic phase pattern which effectively acts as a Fresnel lens with variable focal length. Explicity form? This SLM-generated Fresnel lens helps in filtering out the unmodulated light on the SLM output (more commonly done with a blazed grating pattern [39]). In addition, the combination of the Fresnel lens with a telescope of variable length and magnification creates a collimated LG beam with tunable beam size. The appropriate values for the Fresnel lens and telescope parameters can be obtained with ray-tracing simulations.

was/is

3.1.2. Cavity Alignment

In the cavity design [23], one cavity mirror is placed on 3D piezo translation stage (Figure 1) to allow for both the longitudinal (z direction) and transverse alignment (x and y directions). The longitudinal alignment brings the cavity to be resonant to a particular light frequency. The transverse alignment is necessary to bring the two mirrors to be cylindrically symmetric. Small rotational misalignment on the tip and tilt direction can also be corrected by the transverse alignment, if the mirrors are perfectly spherical. However, this correction might lead to the two anaclastic lens-mirror axes not exactly being aligned with the cavity axis, resulting in slightly asymmetric collimated output modes.

controls the cavity length

The transmission and reflection spectrum of the cavity are obtained by measuring the light intensity with a photodetector while varying the cavity length linearly over time. The detuning from the cavity resonance is expressed correspondingly in units of light frequency – the conversion factor can be determined by measuring the spacing of the frequency sideband generated with an electro-optical modulator. As the cavity is located inside a glass cuvette of the vacuum chamber, we characterize the cuvette transmission loss and apply the correction on the cavity spectra.

3.1.3. Measurement of the Mode Matching Efficiency

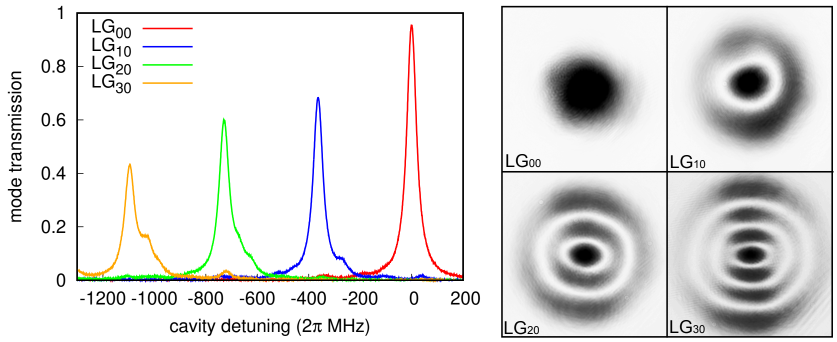
The mode matching efficiency η (Eq. 8) quantifies how well the input mode matches and couples to the cavity mode. It only depends on the power transmission at resonance $T(\omega_0)$ and the "effective" transmission coefficient α (Eq. 7). Nominally, $\alpha = \kappa_m/(2\kappa_l + \kappa_m)$ only depends on the ratio of the decay rates κ_m and κ_l, and thus is a physical property of the cavity under study. properties of the cavity mirrors

We characterize the value of the α by coupling a Gaussian beam (from an aspheric-collimated single mode fiber output mode) into the cavity without the SLM. The transmission and reflection spectrum were recorded, and from the fitting, we obtained $T(\omega_0) = 19.5(1)\%$, $R(\omega_0) = 33.6(2)\%$, and $\kappa = 2\pi \times 24.8(8)$ MHz. From the fitted parameters, we estimated $\alpha = 0.294(2)$, which results in a mode matching efficiency of $\eta = 94(1)\%$ for Gaussian beam, and cavity decay rates of $\kappa_m = 2\pi \times 11.3(4)$ MHz and $\kappa_l = 2\pi \times 13.5(4)$ MHz.

To estimate the mode matching efficiencies for SLM-generated LG modes, we obtain the cavity transmission spectrum $T(\omega)$ and multiply it with $(1+\alpha)^2/(2\alpha)^2$ (the prefactor in Eq. 8) to obtain the mode transmission spectrum $\eta(\omega)$. We fit this spectrum with a Lorentzian profile, and estimate the mode matching efficiency $\eta = \eta(\omega_0)$ from the fit amplitude. The parameters from the ray-tracing simulation helps to start the coupling procedure, and we fine-tune these values further to maximize the mode matching efficiency.

3.2. Mode-matching to single LG modes

We generate a single LG mode using the SLM and couple it to the near concentric cavity. The cavity is located at a critical distance of $d = 4.8(2)\mu$m with $g = -0.99912(4)$, corresponding to a measured transverse mode spacing of $\Delta\nu_{tr} = \nu_F(1 - \Delta\zeta/\pi) = 182(5)$ MHz between adjacent LG modes. The cavity spectra and the camera-captured output modes are depicted on Figure 2



why 2pi here?

Fig. 2. Left: cavity mode transmission spectra of LG modes with no angular momentum ($l = 0$). The detuning is defined with respect to the LG_{00} resonance, and the modes are spaced $2\Delta\nu_{tr}$ apart. Right: the corresponding cavity mode output observed with the camera.

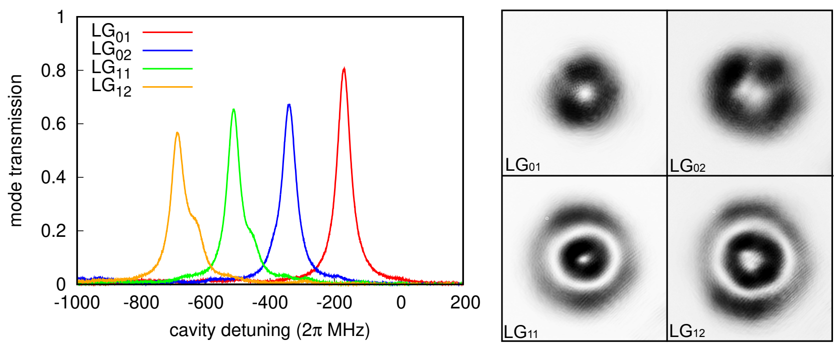


Fig. 3. Left: cavity mode transmission spectra of LG modes with a few units of angular momentum ($l = 1$ and $l = 2$). Right: the corresponding cavity mode output observed with the camera.

Mode	Sim.	Exp.	Mode	Sim.	Exp.
LG_{00}	100%	96(1)%	LG_{01}	93.1%	81(1)%
LG_{10}	81.2%	68(1)%	LG_{02}	84.4%	67(1)%
LG_{20}	76.9%	57(1)%	LG_{11}	81.8%	63(1)%
LG_{30}	74.7%	38(1)%	LG_{12}	79.8%	53(1)%

Table 2. Comparison of mode matching efficiencies between the simulation and the experiment for single LG modes.

for LG modes with no angular momentum ($l = 0$), and on Figure 3 for LG modes with angular momentum ($l \neq 0$). The measured mode matching efficiencies are close to the simulated values (see Table 2), although it degrades with higher mode numbers. We attribute this to limited SLM pixel resolution, axial mismatch between the cavity and the anaclastic lens axis due to the tip-tilt misalignment, and mirror surface deviation from a perfect spherical profile. These factors also contribute to some irregularities on the output mode observed by the camera.

3.3. Mode-matching to a superposition of LG modes

We demonstrate the coupling of Gbeam to an arbitrary superposition of LG modes

We couple the SLM-generated beam to more than one LG modes simultaneously. We use the method described in Section 3.1.1, by considering the resultant mode as a superposition of individual LG modes:

$$U_{res} = \sum A_{l,m} \exp\left(i\phi_{l,m}\right) LG_{lm}\,, \tag{10}$$

where $A_{l,m}$ is the amplitude of each constituting LG mode and $\phi_{l,m}$ is the relative phase of the LG mode.

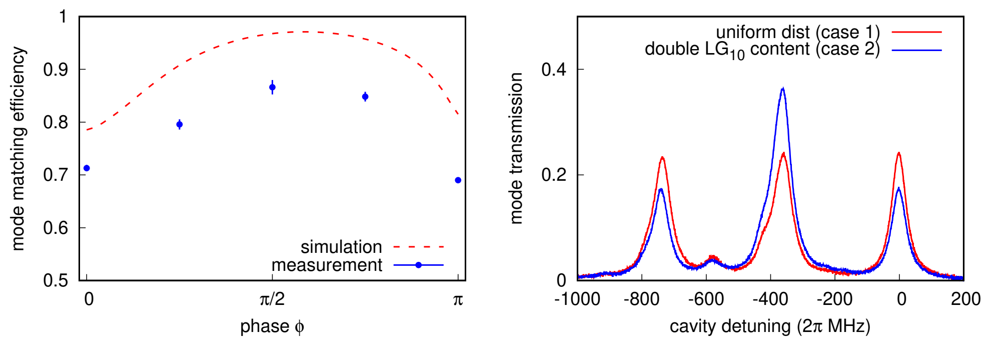


Fig. 4. Left: Coupling to equal parts of LG_{00} and LG_{10} modes while varying their phase difference. Right: Coupling to a superposition of LG_{00}, LG_{10} and LG_{20} modes.

Figure 4 (left) depicts the mode matching efficiency in coupling the superposed mode $U_{\{00,10\}} = \left(LG_{00} + e^{i\phi} LG_{10}\right)/\sqrt{2}$ with varying relative phase angle ϕ. To obtain a balanced distribution of LG_{00} and LG_{10}, we introduce a mode amplitude A_{10} to the SLM spatial phase pattern:

$$\Phi = \arg\left[U_{\{00,10\}}\right] = \arg\left[\frac{LG_{00} + A_{10} e^{i\phi} LG_{10}}{\sqrt{1 + A_{10}^2}}\right], \tag{11}$$

and vary the amplitude A_{10} and w/w_0, maximising the mode matching efficiency subject to the balanced distribution constraint. The mode matching efficiency is obtained by adding the mode transmission amplitudes of both the LG_{00} and LG_{10} modes, while ensuring that they are balanced up to $\sim 1\%$ error. The measured values follow a similar trend with the simulated values, with some offset ($\sim 10\%$) attributable to the SLM pixel size and the mirror irregularities as described previously. The highest mode matching efficiency occurs around $\phi = \pi/2$, as the SLM encodes the LG_{00} and LG_{10} modes into the in-phase and quadrature component of the mode and increases its efficiency.

shows? of a superposition of three modes

Figure 4 (right) depicts the mode transmission spectra in coupling three modes simultaneously. The modes LG_{00}, LG_{10}, and LG_{20} are superposed with a phase difference of $2\pi/3$ between each mode, as to distribute the phases evenly on the complex plane. The SLM spatial pattern is given by:

$$\Phi = \arg\left[U_{\{00,10,20\}}\right] = \arg\left[\frac{LG_{00} + A_{10}e^{i2\pi/3}LG_{10} + A_{20}e^{i4\pi/3}LG_{20}}{\sqrt{1 + A_{10}^2 + A_{20}^2}}\right] \tag{12}$$

where A_{10}, A_{20} and w/w_0 are parameters to be varied to obtain the desired mode distribution and the efficiency. Two examples are illustrated in the figure: (1) equally distributed modes, i.e. $U_{\{00,10,20\}} = \left(LG_{00} + e^{i2\pi/3}LG_{10} + e^{i4\pi/3}LG_{20}\right)/\sqrt{3}$, and (2) LG_{10} content double the content of the other modes, i.e. $U_{\{00,10,20\}} = \left(LG_{00} + \sqrt{2}e^{i2\pi/3}LG_{10} + e^{i4\pi/3}LG_{20}\right)/2$. The mode matching efficiencies predicted with the simulation under optimized parameters are 95.6% and 97.2% for case (1) and (2), while the measured efficiencies are 71(1)% and 70(1)%, respectively.

The theoretical predicted efficiencies with optimized params are ...

3.4. Mode-matching at different critical distances

The critical distance $d = 2R - L$ characterizes how far the cavity is away from the concentric configuration ($L = 2R$). Small critical distances provide strong field focusing and a small mode volume. In addition, the frequency spacing of the transverse modes decreases with smaller critical distances, leading to the mode degeneracy at the critical point [23].

which was proved to be important for some applications[cite]

We study how the mode matching of a single LG mode performs at different critical distances. We use the SLM to couple to LG_{00}, LG_{10}, and LG_{20} modes of the cavity, and obtain the cavity transmission spectra. The linewidth of the cavity spectra increases for smaller critical distances, while the mode transmission decreases. This is due to the diffraction losses as the cavity approaches the critical point. Figure 5 shows the mode transmission amplitude and cavity linewidth for various critical distances. Here, the mode transmission amplitude is evaluated in the same way as the mode matching efficiency η, but with the same α value as measured in Section 3.1.3 without accounting for the diffraction loss. Thus, it describes the mode-matching efficiency weighted by a factor associated with the diffraction loss. The critical distance for one particular cavity length is estimated from the transverse mode spacing. By moving the translation stage along the cavity axis and keeping the laser frequency fixed, we obtain neighbouring cavity spectra spaced $\Delta d = \lambda/2$ apart. Figure 6 shows the cavity transmission mode captured with the camera. The diffraction rings become visible at the critical distance where the linewidth increases.

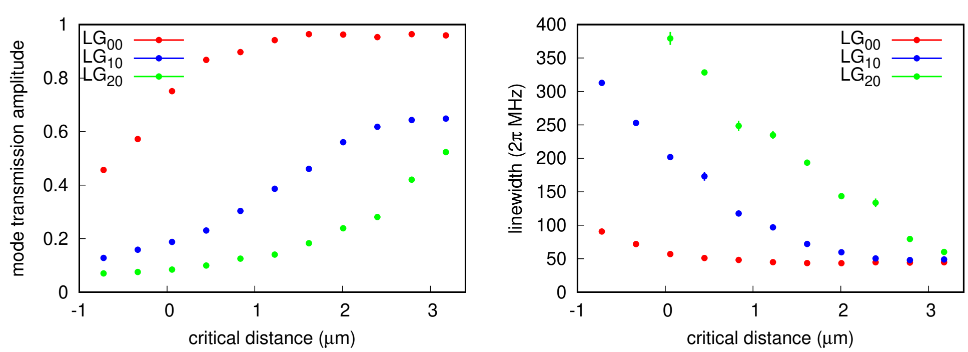
We observed that the linewidth of ...

mode tranX amplitudes and cavity linewidths

the theoretical model to evalue tranmission ampltidue is.. (maybe explicit form here). then you can note that it is similar to 3.1.3

translation stage?
maybe ignore the details of how you move the cavity length here.

spatial profile of cavity transmission mode...



No green on plots, i guess?

No theoretical curves to support?

Fig. 5. Left: Mode transmission amplitude of different LG modes over a range of critical distances. Right: The corresponding linewidth (FWHM) of the modes.

The near-concentric cavity can support LG modes reasonably close (~ a few μm) to the critical point. However, higher order LG modes start to exhibit diffraction losses at larger critical distances, due to larger LG beam sizes. The performance of the cavity mirrors can be

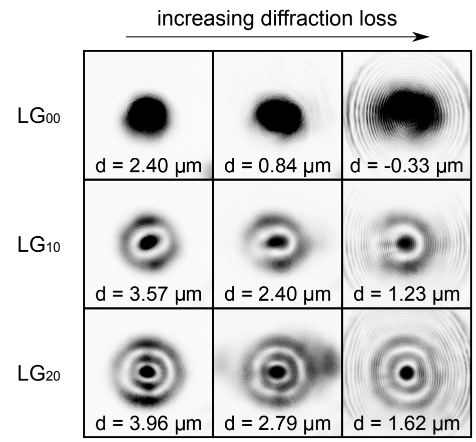


Fig. 6. The cavity modes observed with the camera, before and after the diffraction losses dominate. The diffraction rings are due to the aperture of the anaclastic lens.

characterized with an effective aperture – for every round trip, the cavity mode bounces off a circular aperture with diameter a on the mirror, effectively blocking some outer parts of the beam. As a first order approximation, we assume the LG modes to be unperturbed after subsequent round trips. To estimate the onset of the diffraction loss, we choose an aperture size to block $\sim 1\%$ of the mode (the diffraction loss is $2\kappa_{ap} \sim 2\pi \times 20\,\mathrm{MHz}$), which on the same order as the mirror transmission and scattering losses. From Figure 5 (right), the effective aperture diameter is estimated to be $a_{\mathrm{exp}} = 1.40(6)\,\mathrm{mm}$ with the onset of the diffraction loss at critical distances of $0.46(8)\,\mu\mathrm{m}$ for LG_{00}, $1.8(3)\,\mu\mathrm{m}$ for LG_{10}, and $3.8(6)\,\mu\mathrm{m}$ for LG_{20}.

The estimated effective aperture $a_{\mathrm{exp}} = 1.40(6)\,\mathrm{mm}$ is comparatively lower than the nominal aperture of the anaclastic lens-mirror design $a_{\mathrm{nom}} = 4.07\,\mathrm{mm}$. We suspect this to be due to a combination of the following factors: (1) local aberrations of the mirror surface due to thermo-mechanical stresses [42, 43] and optical surface irregularities, (2) angle-dependent phase variation of the multi-layered coating [44], as the deposition process creates thicker layers near the center of the mirror and thinner layers near the perimeter, (3) the validity of the paraxial approximation [45] for strongly diverging modes, particularly for higher orders. Even though the LG modes are quite far from degeneracy ($\sim 100\,\mathrm{MHz}$) in this regime, it may be possible to reach near-degeneracy either by slightly modifying the mirror shape or the coating layers – this strategy would create a slightly different "effective" cavity length for different LG modes, allowing the modes to come closer or even overlap in frequency.

reductin of reflectivity coating on the outer rim?

very nice point! Can expand it to another theoretical paper/work?

4. Conclusion

I think the conclusion kinda need to be improved

Near-concentric cavity is a promising avenue to explore interaction between atoms and strongly focusing near-degenerate spatial modes. We demonstrate a small step in this direction, as we couple to different LG modes in the near-concentric regime using the SLM, with relatively high mode matching efficiency. Furthermore, the SLM can be adapted to couple to a superposition of LG modes. The cavity supports LG modes up to critical distances of a few μm before the diffraction loss dominates.

We explore/study the coupling to several modes simultaneously of near-con cavities.

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