Closely approaching Tsirelson's bound in a photon pair experiment

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The Clauser-Horne-Shimony-Holt (CHSH) Bell inequality is a common measure to probe non-classical correlations, as it is violated in quantum systems with suitable measurement settings. The maximal violation allowed by quantum theory is given by the Tsirelson bound. Finding experimentally a close proximity to this bound necessitates complementary principles to recent alternative explanations for nonlocal correlations. We present a CHSH test on a maximally entangled polarization state of photon pairs, and find a violation of $S = 2.8276 \pm 0.00082$, or a distance of $S - 2\sqrt{2} = 0.0008 \pm 0.00082$ to the Tsirelson bound.

INTRODUCTION

Giving a quantitative form to concerns about "elements of reality" in early quantum physics, the Bell inequality [1] allowed for an experimental comparison with more traditional descriptions of the physical world. Bell inequalities, in particular in the form of Clauser-Horne-Shimony-Holt (CHSH) [2], have been violated in many experiments, substantially strengthening the confidence in quantum theory. With the concept of entanglement finding its way in new computing and secure communication schemes, interest in explaining this aspect of quantum physics via information-related principles has developed.

Already in 1969, CHSH [2] generalized the Bell theorem and noticed that quantum physics predicts a maximal violation of a Bell inequality in a bipartite system, each with two measurement settings, and two measurement outcomes. In 1980 Tsirelson proved that this maximal violation is independent of the dimensionality of the quantum system [3].

The Tsirelson bound is a fundamental limit not only in quantum theory, but in any theory that satisfies any one of the following principles: information causality (IC) [4], macroscopic locality (ML) [5], and exclusivity (E) [6]. In contrast, other physical principles allow for values beyond the Tsirelson bound: non-signaling [7], non-triviality of communication complexity [8, 9], and local orthogonality [10]. If the Tsirelson bound can be surpassed, quantum theory would be wrong and neither IC, nor ML, nor E would hold in nature. If the Tsirelson bound can be reached but not surpassed, a fundamental prediction of quantum theory would be confirmed and those principles which exactly single out the Tsirelson bound could be the basis for a better understanding of quantum theory from fundamental physical principles. If the Tsirelson bound would be unreachable, fundamentally different principles and theories should be considered.

For any theory satisfying the E principle, the maximum violation of the CHSH Bell inequality implies a similar upper bound for the violation of an, a priori, completely unrelated inequality [11] involving different sets of three sequential sharp measurements [12]. If the maximum violation of the CHSH Bell inequality is actually the Tsirelson bound, then the bound implied by the E principle on this other inequality is exactly the maximum violation predicted by quantum theory (and only reached with quantum systems of dimension five or higher [12]). Reciprocally, experimentally reaching the quantum maximum for this other inequality could be taken as an evidence of the impossibility of violations of the CHSH Bell inequality beyond the Tsirelson bound [12].

Continuous experimental progress made it possible to probe the Tsirelson bound with decreasing uncertainty. Here, we report on a an experiment with entangled photon pairs that pushes this uncertainty by another order of magnitude compared to previous measurements.

CHSH INEQUALITY

To introduce our notation, we briefly recall the results from the CHSH paper [2]. We consider the usual configuration formed by a source of entangled photons and two spatially separated polarization analyzers A and B with possible outcomes $+$ and $-$. The CHSH Bell inequality can be written as $|S| \leq 2$, where the parameter S is a combination of polarization correlations E,

$$S = E(a_0, b_0) - E(a_0, b_1) + E(a_1, b_0) + E(a_1, b_1)\,, \quad (1)$$

where a and b are the experimental settings, i.e., the angle of the polarizer axes with respect to a reference. The correlations E can be calculated from probabilities P_{++}, P_{+-} etc. of obtaining a coincidence event with setting a, b, $a + 90°$, and $b + 90°$, respectively:

$$E = P_{++} - P_{+-} - P_{-+} + P_{--}\,. \quad (2)$$

For settings a, b separated by $\theta = 22.5°$, the maximal value of $S = 2\sqrt{2}$ should be reached according to quantum physics [2], corresponding to the Tsirelson bound.

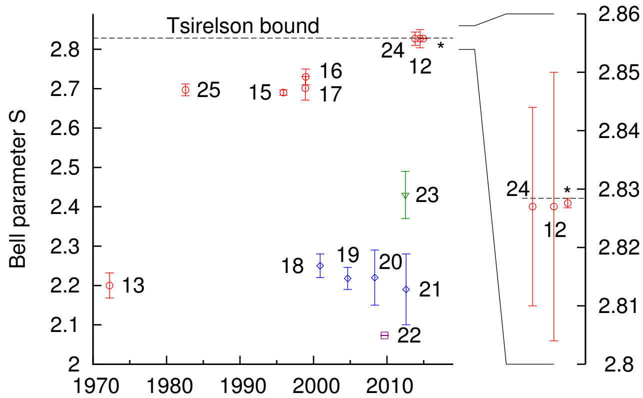

FIG. 1. Selected experimental tests of a CHSH inequality approaching the Tsirelson bound in photonic systems (circles), atoms and ions (diamonds), Josephson junctions (square), and NV centers in diamond (triangle). Numbers represent the references; * corresponds to this work.

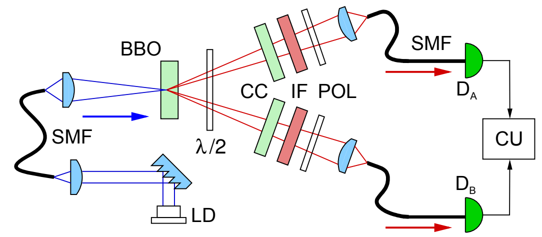

FIG. 2. Schematic of the experimental set-up. Polarization correlations of entangled-photon pairs are measured by film polarizers (POL) placed in front of the collection optics. All photons are detected by silicon avalanche photodetectors D_A and D_B, and registered in a coincidence unit (CU).

Experimentally, we estimate E from the statistical frequency of coincidence counts N between A and B,

$$E = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_{++} + N_{+-} + N_{-+} + N_{--}} \,. \tag{3}$$

PRIOR ART

The violation of Bell's inequality has been observed in many experiments with exceedingly high statistical significance, many of which based on the generation of correlated photon pairs, using cascade decays in atoms [13, 14], or exploiting non linear optical processes as in [12, 15–17]. Other successful demonstrations were based on internal degrees of freedom of ions [18–20] and neutral atoms [21], Josephson Junctions [22], and NV centers in diamond [23]. Figure 1 summarizes the obtained Bell parameter and corresponding uncertainty of several experimental tests.

THIS EXPERIMENT

Our experiment follows the concept in [15] and is shown in Fig. 2. The output of a grating-stabilized laser diode (LD, central wavelength 405 nm) passes through a single mode optical fiber (SMF) for spatial mode filtering, and is focused to a beam waist of 80 μm into a 2 mm thick BBO crystal.

In the crystal, cut for type-II phase-matching, spontaneous parametric down-conversion (SPDC) in a slightly non-collinear configuration generates photon pairs. Each down-converted pair consists of an ordinary and extraordinarily polarized photon, corresponding to horizontal (H) and vertical (V) in our setup. Two SMFs for 810 nm define two spatial modes matched to the pump mode to optimize the collection [26]. A half-wave plate ($\lambda/2$) and a pair of compensation crystals (CC) take care of the temporal and transversal walk-off [15], and allow to adjust the phase between the two decay components to obtain a singlet state $|\Psi^-\rangle = 1/\sqrt{2}\,(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$.

Film polarizers (specified extinction ratio 10^4) perform the basis choice and polarization projection. Photons are detected by avalanche photo diodes (APDs, quantum efficiency $\approx$40%), and corresponding detection events from the same pair identified by a coincidence unit (CU) if they arrive within $\approx \pm 1.2$ ns of each other.

To arrive at a very clean singlet state, we carefully align the photon pair collection to balance the two photon pair contributions $|HV\rangle$ and $|VH\rangle$, and adjust their relative phase with the CC. Furthermore, we minimize contributions from higher order parametric conversion processes [27] by restricting the pump power below 7 mW, leading to average detection rates of 5016 s^{-1} and 4051 s^{-1} at the two the detectors, (both uncorrected for dark counts), resulting in a detected photon pair rate of about 567 s^{-1}. The detectors exhibit dark count rates of 91.7 s^{-1} and 106.2 s^{-1}, respectively. This results in an accidental coincidence rate of 0.0067$\pm$0.0025 s^{-1}, determined by looking at coincidences in two time windows shifted by 10 ns and 25 ns from where the "true" coincidence were appearing.

We test the quality of polarization entanglement by measuring the polarization correlations in the $\pm 45°$ linear polarization basis. With interference filters (IF) of 5 nm bandwidth (FWHM) centered at 810 nm, we observe a visibility $V_{45} = 99.9 \pm 0.1\%$. The visibility in the natural H/V basis of the type-II down-conversion process also reaches $V_{\mathrm{HV}} = 99.9 \pm 0.1\%$. This indicates a high quality of polarization entanglement; the uncertainties in the visibilities are obtained from propagated Poissonian counting statistics.

Due to imperfections in the state generation and errors in the setting of the polarizers, the setting $\theta = 22.5°$ may not yield the maximum possible violation. In order to observe the largest possible violation, and get as close as possible to the Tsirelson bound, we optimized the angular settings of the polarizers.

The optimization starts by setting $a = 0°$. This provides the initial reference axis and corresponds to a_0. Rotating b and recording the rate of coincidences, we identify the angles b'_0 and b'_1 that better match the expected correlation values. Setting $b = b'_0$, we repeat a similar procedure for a, obtaining a'_0 and a'_1. This procedure converged to the resolution of the rotation motors (verified repeatability/resolution 0.1°). For our experimental demonstration the optimal angles are $a'_0 = 1.9°$, $b'_0 = 22.9°$, $a'_1 = 46.8°$, and $b'_1 = 67.7°$.

For evaluating how close we can come with the Bell test to the Tsirelson bound with a known uncertainty, we need to integrate for a sufficiently long time to acquire the necessary counting statistics, assuming we have the usual Poissonian statistics implied by the time invariance of our experiment. We collect coincidence events for each of the 16 settings required to evaluate S for 1 minute, and then repeat again the whole set. Within 312 such complete sets, we registered a total of 33,184,329 pair events. As a result, we obtain in this experiment via Eqs. (1),(3) a value of $S = 2.8276 \pm 0.00082$, or a separation of $S - 2\sqrt{2} = 0.0008 \pm 0.00082$ from the Tsirelson bound. The uncertainty is obtained only by propagating Poissonian counting statistics on the individual pair detections into the expression for S, as any systematical errors (attacks on detectors excluded [28], i.e., under the fair sampling assumption) would only lower the degree of violation.

CONCLUSION

Our search for the maximal violation of a CHSH-type Bell inequality indicates that the Tsirelson bound can be reached, but not surpassed – with very low uncertainty, consistent with the conventional quantum physics description. At least for the physical system we investigated, this suggests that alternative descriptions based on non-signaling, non-triviality of communication complexity, and local orthogonality principles [7–9] that allow to exceed the Tsirelson bound need to be complemented with components that explain why the bound can not be overcome.

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