

Approaching Tsirelson's bound in a photon pair experiment

Hou Shun Poh, Siddarth K. Joshi, Alessandro Cerè, Adán Cabello, and Christian Kurtsiefer

We present an experimental test of the CHSH Bell inequality in which we observed a value $S = 2.82759 \pm 0.00051$. This constitutes the tightest experimental test of Tsirelson's bound ever reported.

John Bell¹ showed that the results of measurements on quantum systems cannot be explained by local theories, since they violate certain inequalities among the correlations between the outcomes of measurements on two distant locations A and B . The simplest of these Bell inequalities is the one by Clauser, Horne, Shimony, and Holt (CHSH)², which can be written as $|S| \leq 2$, where the parameter S is a combination of correlations $E(a_i, b_j)$ defined as

$$S = E(a_0, b_0) - E(a_0, b_1) + E(a_1, b_0) + E(a_1, b_1), \quad (1)$$

where $a_{0,1}$ and $b_{0,1}$ are measurement settings in A and B , respectively, and each measurement has two possible outcomes, $+1$ or -1 . The correlations $E(a_i, b_j)$ are defined from the joint probabilities P for outcomes $++$, $+-$, $-+$, and $--$ as

$$E(a_i, b_j) = P(++) - P(+ -) - P(- +) + P(--). \quad (2)$$

Boris Tsirelson³ showed that, according to quantum theory, $|S|$ has an upper bound of $2\sqrt{2} \approx 2.82843$.

On the other hand, Alexei Grinbaum⁴ predicted that the violation of the Bell CHSH inequality is upper bounded by $2.82537(2)$, slightly smaller than the Tsirelson bound. This is done in an attempt to address issues surrounding the cut between the observer and the observed system⁵ that exist in the quantum theory.

Not being able to exceed Grinbaum's limit would support that quantum theory is only an effective description of a more fundamental theory⁴, and thus have a deep impact in physics and quantum information processing.

The violation of Bell's inequality has been observed in many experiments. Many of these experiments are based on the generation of correlated photon pairs using cascade decays in

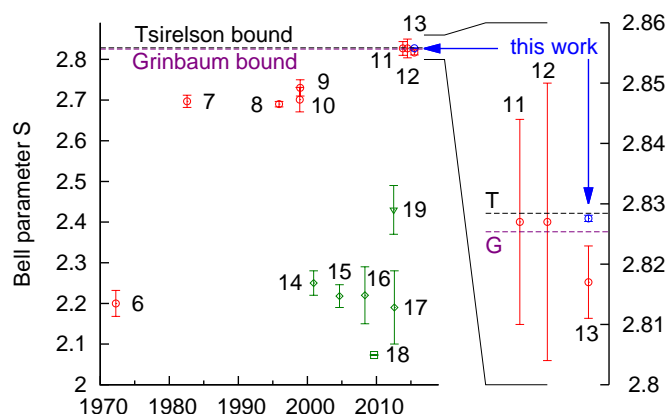


Figure 1. Selected experimental tests of the CHSH Bell inequality with results close to the Tsirelson (T) and Grinbaum (G) bounds in photonic system (circles), atoms and ions (diamonds), Josephson junctions (square), and nitrogen-vacancy centers in diamond (triangle). Numbers represent the references. Figure adapted from an earlier publication²³.

atoms^{6,7}, or exploiting non-linear optical processes⁸⁻¹². Other successful demonstrations were based on internal degrees of freedom of ions¹³⁻¹⁵ and neutral atoms¹⁶, Josephson junctions¹⁷, and nitrogen-vacancy centers in diamond¹⁸. Figure 1 summarizes the result obtained for the Bell parameter and the corresponding uncertainty. So far the Grinbaum bound consistent with all the available experimental results⁶⁻¹⁹.

Motivated by the status quo, we attempt to experimentally approach the Tsirelson Bound, allowing direct observations to be compared with the prediction of the Grinbaum model. An experimental search for the maximal violation of a Bell inequality¹ also tests the principles that predict Tsirelson's bound²⁰⁻²² as possible explanations of all natural limits of correlations.

Our experimental setup is shown in Fig. 2. The output of a laser diode (LD, central wavelength 405 nm) is used to pump

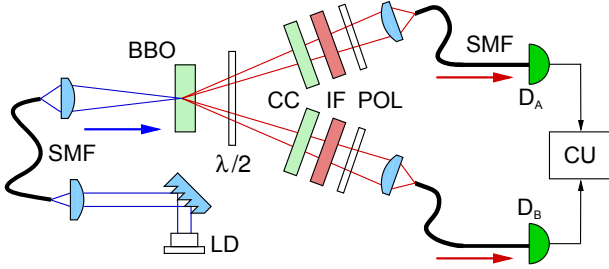


Figure 2. Schematic of the experimental set-up. Polarization correlations of entangled-photon pairs are measured by film polarizers (POL) placed in front of the collection optics. All photons are detected by silicon avalanche photodetectors D_A and D_B , and registered in a coincidence unit (CU). Figure adapted from an earlier publication²³.

a crystal (BBO) cut for type-II phase-matching. Spontaneous parametric down-conversion (SPDC) in a slightly non-collinear configuration generates photon pairs which consists of a horizontally (H) and vertically (V) polarized photon at a wavelength of 810 nm .

Two single mode fibers (SMFs) define two spatial modes matched to the pump mode to optimize the collection²⁴. A half-wave plate ($\lambda/2$) and a pair of compensation crystals (CC) take care of the temporal and transversal walk-off⁸, and allow to adjust the phase between the two decay components to obtain the singlet state $|\Psi^-\rangle = 1/\sqrt{2}(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$.

Film polarizers perform the basis choice and polarization projection. Photons are detected by avalanche photo diodes (APDs), and corresponding detection events from the same pair identified by a coincidence unit (CU).

To arrive at a very clean singlet state, we carefully align the photon pair collection to balance the contributions $|HV\rangle$ and $|VH\rangle$. Their relative phase is adjusted with the CC. Higher order parametric conversion processes²⁵ are minimized by restricting the pump power. The rate of coincidence events depends on the orientation of the polarizers, as expected, and, in our measurements, ranges from a minimum of 26 s^{-1} to a maximum of 217 s^{-1} .

We observe high visibilities of $99.9 \pm 0.1\%$ both in the $\pm 45^\circ$ and in the natural H/V linear polarization basis for the polarization correlations, indicating a high quality of polarization entanglement.

To compensate for the imperfections in the state generation and errors in the setting of the polarizers, we optimized the angular settings of the polarizers in order to observe the largest possible violation.

Correlations E in Eq. (2) are estimated from coincidence

counts N between A and B for settings of the polarizers,

$$E = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_{++} + N_{+-} + N_{-+} + N_{--}}. \quad (3)$$

In order to acquire the necessary statistics, we collect coincidence events for each of the 16 settings required to evaluate S for 1 minute, and then repeat each set of measurement 312 times, registering a total of 33,184,329 pair events in the process. As a result, we obtain in this experiment, via Eqs. (1) and (3), a value of $S = 2.82759 \pm 0.00051$ (result shown in Figure 1), or a separation of $2\sqrt{2} - S = 0.00084 \pm 0.00051$ from the Tsirelson bound.

The result of our experiment violates Grinbaum's bound by 4.3 standard deviations and constitutes the tightest experimental test of Tsirelson's bound ever reported. Therefore, it falsifies the thesis that quantum theory is only an effective version of a deeper theory, and reinforces the thesis that quantum theory is fundamental and that Tsirelson bound is a natural limit. This conclusion strengthens the potential value of those principles that predict Tsirelson's bound²⁰⁻²² for explaining the natural limits of correlations in all scenarios. The possibility of experimentally touching Tsirelson's bound also has important consequences for quantum information and certification of a variety of physical properties.

Details about this work can be found in an earlier publication²³.

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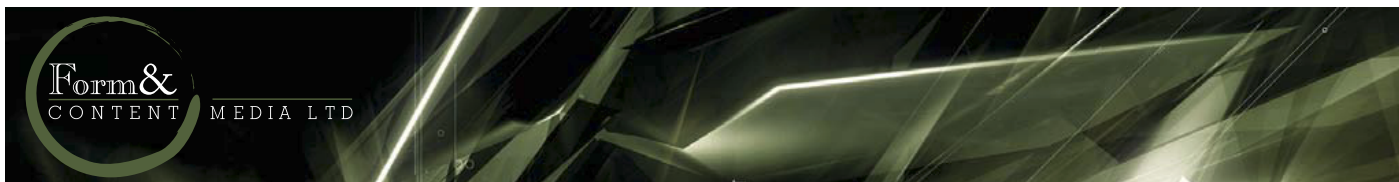
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