Approaching Tsirelson's bound in a photon pair experiment

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We present an experimental test of the CHSH Bell inequality on photon pairs in a maximally entangled state of polarization in which a value $S = 2.82759 \pm 0.00051$ is observed. This value comes close to the Tsirelson bound of $|S| \leq 2\sqrt{2}$, with $S - 2\sqrt{2} = 0.00084 \pm 0.00051$. It also violates the bound $|S| \leq 2.82537$ introduced by Grinbaum by 4.3 standard deviations. This violation allows us to exclude that quantum mechanics is only an effective description of a more fundamental theory.

Introduction — Bell [1] showed that the results of measurements on quantum systems cannot be explained by local theories, since they violate certain inequalities among the correlations between the outcomes of measurements on two distant locations A and B. The simplest of these Bell inequalities is the one by Clauser, Horne, Shimony, and Holt (CHSH) [2], which can be written as $|S| \leq 2$, where the parameter S is a combination of correlations $E(a_i, b_j)$ defined as

$$S = E(a_0, b_0) - E(a_0, b_1) + E(a_1, b_0) + E(a_1, b_1)\,, \quad (1)$$

where $a_{0,1}$ and $b_{0,1}$ are measurement settings in A and B, respectively, and each measurement has two possible outcomes, $+1$ or -1. The correlations $E(a_i, b_j)$ are defined from the joint probabilities P for outcomes $++$, $+-$, $-+$, and $--$ as

$$E(a_i, b_j) = P(++) - P(+-) - P(-+) + P(--)\,. \quad (2)$$

Tsirelson [3] showed that, according to quantum theory, $|S|$ has an upper bound of $2\sqrt{2} \approx 2.82843$. Popescu and Rohrlich [4] demonstrated that values up to $S = 4$ were compatible with the no-signaling principle that prevents superluminal communication. This difference stimulated the search for principles singling out Tsirelson's bound as part of the effort for understanding quantum theory from fundamental principles. So far, the following principles have been identified that enforce Tsirelson's bound: information causality [5], macroscopic locality [6], and exclusivity [7]. Other principles, such as non-signaling [4] and nontriviality of communication complexity [8, 9], allow for higher values.

On the other hand, quantum theory introduces a cut between the observer and the observed system [10], but does not provide a definition of what is an observer [11]. To address this problem, Grinbaum has recently tried to integrate the observer into the theory [12]. For this purpose, he introduces a mathematical framework based on algebraic coding theory [13] that provides a general model for communication, and enables an information-theoretic definition of an observer. This definition involves a limit on the complexity of the strings the observer can store and handle. These strings contain all descriptions of states allowed by quantum theory, but may also contain information not interpretable in terms of preparations and measurements. The language dynamics of these strings leads to a continuous model in the critical regime that, when restricted to measurements on bipartite systems in a three-dimensional Euclidean space, predicts that the violation of the Bell CHSH inequality is upper bounded by 2.82537(2). This prediction holds under the assumption that the number of strings with the same complexity after uncomputable Kolmogorov reordering is 6 and some assumptions on the mappings between certain metric spaces (see [12], Sec. V). It further uses the most precise determination available of a critical exponent in three-dimensional Ising conformal field theory [14].

The value predicted by Grinbaum is slightly smaller than the Tsirelson bound, and is so far consistent with all the available experimental results [15–28]. Not being able to exceed Grinbaum's limit would support that quantum theory is only an effective description of a more fundamental theory [12], and would have a deep impact in physics and quantum information processing. This has important consequences for cryptographic security [29], randomness certification [30], characterization of physical properties in device-independent scenarios [31, 32], and certification of quantum computation [33].

An interesting aspect of Grinbaum's work is the prediction that Tsirelson's bound is *experimentally unreachable*, while quantum physics does not impose such a limit. The model can thus be compared with direct observations in nature.

From a more general perspective, an experimental search for the maximal violation of a Bell inequality [1] tests the principles that predict Tsirelson's bound [5–7] as possible explanations of all natural limits of correlations.

Prior work — The violation of Bell's inequality has been observed in many experiments with exceedingly high statistical significance. Many of these experiments are based on the generation of correlated photon pairs using cascade decays in atoms [15, 16], or exploiting non-linear optical processes [17–21]. Other successful demonstrations were based on internal degrees of freedom of ions [23–25] and neutral atoms [26], Josephson junc-

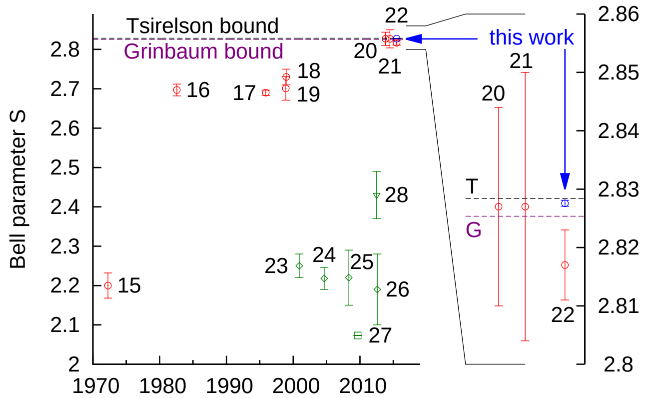

FIG. 1: Selected experimental tests of the CHSH Bell inequality with results close to the Tsirelson (T) and Grinbaum (G) bounds in photonic systems (circles), atoms and ions (diamonds), Josephson junctions (square), and nitrogen-vacancy centers in diamond (triangle). Numbers represent the references.

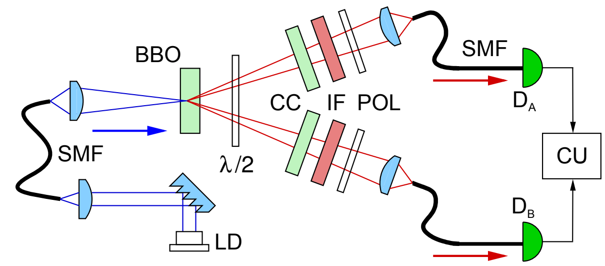

FIG. 2: Schematic of the experimental set-up. Polarization correlations of entangled-photon pairs are measured by film polarizers (POL) placed in front of the collection optics. All photons are detected by silicon avalanche photodetectors D_A and D_B, and registered in a coincidence unit (CU).

tions [27], and nitrogen-vacancy centers in diamond [28]. Figure 1 summarizes the result obtained for the Bell parameter and the corresponding uncertainty of several experimental tests.

While continuous experimental progress has made it possible to approach Tsirelson's bound with decreasing uncertainty, predictions such as Grinbaum's, which would imply a radical departure from standard quantum theory, are compatible with all existing results.

Here, we report on an experiment with entangled photon pairs that pushes the uncertainty in the Bell parameter by another order of magnitude compared to previous experiments.

Our experiment follows the concept in [17] and is shown in Fig. 2. The output of a grating-stabilized laser diode (LD, central wavelength 405 nm) passes through a single mode optical fiber (SMF) for spatial mode filtering, and is focused to a beam waist of 80 μm into a 2 mm thick BBO crystal.

In the crystal, cut for type-II phase-matching, spontaneous parametric down-conversion (SPDC) in a slightly non-collinear configuration generates photon pairs. Each down-converted pair consists of an ordinary and extraordinarily polarized photon, corresponding to horizontal (H) and vertical (V) in our setup. Two SMFs for 810 nm define two spatial modes matched to the pump mode to optimize the collection [34]. A half-wave plate ($\lambda/2$) and a pair of compensation crystals (CC) take care of the temporal and transversal walk-off [17], and allow to adjust the phase between the two decay components to obtain a singlet state $|\Psi^-\rangle = 1/\sqrt{2}\,(|H\rangle_A|V\rangle_B - |V\rangle_A|H\rangle_B)$.

Film polarizers (specified extinction ratio 10^4) perform the basis choice and polarization projection. Photons are detected by avalanche photo diodes (APDs, quantum efficiency ≈40%), and corresponding detection events from the same pair identified by a coincidence unit (CU) if they arrive within $\approx \pm 1.2$ ns of each other.

To arrive at a very clean singlet state, we carefully align the photon pair collection to balance the two photon pair contributions $|HV\rangle$ and $|VH\rangle$, and adjust their relative phase with the CC. Furthermore, we minimize contributions from higher order parametric conversion processes [35] by restricting the pump power below 7 mW, leading to average detection rates of $(4.84 \pm 0.20) \cdot 10^3\,\mathrm{s}^{-1}$ and $(3.45 \pm 0.25) \cdot 10^3\,\mathrm{s}^{-1}$ at the two the detectors (uncorrected for dark counts), resulting in an accidental coincidence rate of $0.020 \pm 0.017\,\mathrm{s}^{-1}$. The rate of coincidence events depends on the orientation of the polarizers, as expected, and, in our measurements, ranges from a minimum of $26\,\mathrm{s}^{-1}$ to a maximum of $217\,\mathrm{s}^{-1}$. The detectors exhibit dark count rates of $91.7\,\mathrm{s}^{-1}$ and $106.2\,\mathrm{s}^{-1}$, respectively.

We test the quality of polarization entanglement by measuring the polarization correlations in the $\pm 45°$ linear polarization basis. With interference filters (IF) of 5 nm bandwidth (FWHM) centered at 810 nm, we observe a visibility $V_{45} = 99.9 \pm 0.1\%$. The visibility in the natural H/V basis of the type-II down-conversion process also reaches $V_{\mathrm{HV}} = 99.9 \pm 0.1\%$. This indicates a high quality of polarization entanglement; the uncertainties in the visibilities are obtained from propagated Poissonian counting statistics.

Due to imperfections in the state generation and errors in the setting of the polarizers, the setting $\theta = 22.5°$ may not yield the maximum possible violation. In order to observe the largest possible violation, and get as close as possible to the Tsirelson bound, we optimized the angular settings of the polarizers.

The optimization starts by setting $a = 0°$. This provides the initial reference axis and corresponds to a_0. Rotating b and recording the rate of coincidences, we identify the angles b'_0 and b'_1 that better match the expected correlation values. Setting $b = b'_0$, we repeat a similar procedure for a, obtaining a'_0 and a'_1. This procedure converged to the resolution of the rotation motors (verified repeatability/resolution 0.1°). For our experiment the optimal angles are $a'_0 = 1.9°$, $b'_0 = 22.9°$, $a'_1 = 46.8°$, and $b'_1 = 67.7°$.

Each of the correlations E in (2) is estimated from coincidence counts N between A and B,

$$E = \frac{N_{++} - N_{+-} - N_{-+} + N_{--}}{N_{++} + N_{+-} + N_{-+} + N_{--}}\,. \tag{3}$$

For evaluating how close we can come with the test of the CHSH Bell inequality to the Tsirelson bound with a known uncertainty, we need to integrate for a sufficiently long time to acquire the necessary counting statistics, assuming we have the usual Poissonian statistics implied by the time invariance of our experiment. We collect coincidence events for each of the 16 settings required to evaluate S for 1 minute, and then repeat again the whole set. Within 312 such complete sets, we registered a total of 33,184,329 pair events. As a result, we obtain in this experiment, via Eqs. (1) and (3), a value of $S = 2.82759 \pm 0.00051$, or a separation of $2\sqrt{2} - S = 0.00084 \pm 0.00051$ from the Tsirelson bound.

The uncertainty we report on this quantity has several contributions. In the following, we go through those we could identify.

Counting statistics – The parametric down conversion process delivers detection events randomly without any specific dynamics. Therefore, the uncertainties in the coincidence events N entering the correlation functions E via (3) show a Poissonian statistics. The contribution from this, propagated through (3) and (1), is $\Delta S_P = 4.9 \cdot 10^{-4}$.

Detector efficiency – It is reasonable to assume that the quantum efficiency of Silicon APDs remains stable over the time necessary for each measurement of a correlation E, approximatively 10 minutes. Single event rates detected for this experiment, approximatively 5000 s^{-1}, are low enough so that the response of the detector is, effectively, linear. Thus, we do not assign any uncertainty in S to any efficiency drift in the detectors.

Detector dead time – The passively quenched Silicon APDs we used have a dead time of approximately 1.6 μs. Fluctuations in the total acquisition time due to the dead time are proportional to the statistical fluctuations in count rate, i.e., the square root of the number of single detection events. Propagating this uncertainty to the calculated value of S, we obtain an uncertainty $\Delta S_D = 5.4 \cdot 10^{-7}$.

Timing uncertainty – The counting intervals of 60 s are defined by a hardware clock in a microcontroller, with a maximum time uncertainty of 100 ns. This time jitter contributes an uncertainty $\Delta S_T = 4.7 \cdot 10^{-11}$. The temperature dependence of the reference clock is also a source of timing uncertainty. The maximum frequency drift of this clock we measured in a similar thermal profile against a Rubidium-stabilized reference oscillator is in less than 0.1 ppm (part per million), leading to an uncertainty of $\Delta S_C = 2.8 \cdot 10^{-9}$.

Angular position of polarizers – From the angular uncertainty of 0.1 degrees of the polarizer rotation stages, we estimate a contribution $\Delta S_R = 1.2 \cdot 10^{-4}$.

The resulting uncertainty quoted above is obtained via $\Delta S = (\Delta S_P^2 + \Delta S_D^2 + \Delta S_T^2 + \Delta S_R^2)^{1/2}$. This analysis suggests that ΔS is dominated by counting statistics, i.e., the total number of registered count events. Our experiment has certainly systematic uncertainties - for example, we do observe an effective setting-dependent variation of the detection efficiency due to small wedge errors in the film polarizers in front of the single mode optical fiber collection optics on the order of a few percent. However, any bias of this kind lowers the value of S (attacks on detectors excluded [36], i.e., under the fair sampling assumption).

Conclusion — The result of our experiment violates Grinbaum's bound by 4.3 standard deviations and constitutes the tightest experimental test of Tsirelson's bound ever reported. Therefore, it shows no evidence in favor of the thesis that quantum theory is only an effective version of a deeper theory and reinforces the thesis that quantum theory is fundamental and that the Tsirelson bound is a natural limit that can be reached. This conclusion strengthens the potential value of those principles that predict Tsirelson's bound [5–7] for explaining the natural limits of correlations in all scenarios. The possibility of experimentally touching Tsirelson's bound as predicted by quantum theory also has important consequences for cryptographic security, since a necessary and sufficient condition for certifying probability distributions independent of the results of an eavesdropper in a device-independent scenario [37] is that the observed probabilities are exactly the ones corresponding to the Tsirelson bound [29]. It is also important for the certification of a variety of physical properties based solely on the assumption of non-signaling (i.e., without making assumptions on the initial state of the system or the inner working of the measurement devices). In this respect, the degree of violation of the CHSH Bell inequality can be used to certify the amount of randomness [30]. The higher the violation, the larger the amount of certified randomness. Reaching the Tsirelson bound can also be used to certify that the state being measured is a maximally entangled state and/or that the local measurements are of the type represented in quantum theory by anti-commutating operators [31, 38]. This can be adapted to practical methods to estimate the fidelity of the maximally entangled estates [32]. Finally, saturating the Tsirelson bound can be used to certify that a general quantum computation was actually performed [33].

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