Analysis and Improvement of Bandwidth of the Frequency Modulation Spectroscopy

Kenneth Koh Chee Hong

Supervisor: Professor Christian Kurtsiefer

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Department of Physics

National University of Singapore

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Table of Contents

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Abstract

In this experiment, we aim to measure the bandwidth of our existing Frequency Modulation (FM) spectroscopy setup, and thereafter improve its bandwidth by improving selected components of the spectroscopy. Using the frequency at 90° phase lag as a proxy to the bandwidth, our initial measurement of the frequency at 90° phase lag of our existing FM spectroscopy turns out to be around 1.95kHz for current modulation. After implementing some improvements on the current modulation electronic circuit and FM signal processing, we successfully achieved a frequency of around 370kHz at 90° phase lag for our improved FM spectroscopy.

Additionally, we also implemented a Proportional (P) control loop system and measured its frequency at 90° phase lag to be 1.20MHz, which we expect for it to not limit the bandwidth of our improved FM spectroscopy significantly.

This improvement in bandwidth of our FM spectroscopy allows us to eliminate the ongoing technical noise, which is currently dominating at the kilohertz region.

Chapter 1

Introduction

The usage of laser (acronym for "light amplification by stimulated emission of radiation") can be seen in many fields of research and industrial application. In particular, laser is widely used for research on quantum information science, like in Single Photon Technologies etc. Industrial produced lasers have a certain level of frequency stability. However, to be used for precise measurements in research, we have to improve the existing laser's frequency stability. This is where the Frequency Modulation (FM) spectroscopy comes in, where the technique involves the 'locking' of a laser to improve its frequency stability [1]. More details on the FM spectroscopy and the setup we used will be discussed in the next chapter.

In the discussion of FM spectroscopy, we are often interested in the bandwidth of the FM spectroscopy. The bandwidth of our FM spectroscopy tells us how fast our system responds to a change in the input. In this case, when the bandwidth is at a high frequency, the laser intensity noise is reduced towards the shot noise limit and subsequently the signal-to-noise ratio is increased [2]. Hence, it is evident that the higher the bandwidth of our FM spectroscopy, the higher the signal-to-noise ratio we achieve for our system, resulting in more stable laser, which makes it more useful particularly in the quantum information science department. In this project, we will aim to investigate the bandwidth of the FM spectroscopy in the lab and improve our FM spectroscopy by attempting to increase its bandwidth.

As there are many conventions of measuring the bandwidth of the spectroscopy, for this experiment we will do a standardized measure of the frequency at 90° phase lag as the bandwidth of our spectroscopy. Appendix A will provide a reasoning for our choice of the frequency at 90° phase lag. As the term "frequency at 90° phase lag" is quite a mouthful, we will use the term "target frequency" to indicate the same quantity in this paper. The 2 terms "bandwidth" and "target frequency" may be used quite interchangeably in this paper, but we should understand that the target frequency is just a standardized measurement that we use as a proxy to the bandwidth.

For a quick overview, we will have a brief introduction on the FM spectroscopy in Chapter 2, followed by the initial measurement of the target frequency of the FM spectroscopy in Chapter 3. Chapter 4 will focus on the improvement of the bandwidth of the FM spectroscopy via the laser driver and external cavity diode laser (ECDL), while Chapter 5 will do the same for the FM board. Lastly, Chapter 6 will discuss on the bandwidth of the control loop.

Chapter 2

Frequency Modulation (FM) Spectroscopy

We will first give a brief introduction on the mechanism of FM spectroscopy, as well as the setup that we used in the lab.

2.1) Setup

The FM spectroscopy technique we used in this experiment is a modulation transfer spectroscopy in rubidium atoms. The spectroscopy is used to lock a laser to the atomic transition. For our purpose, we focus our spectroscopy on the D2 transitions in ^{85}Rb and ^{87}Rb (at 780.24nm) [3, 4]. Figure B1 and B2 in Appendix B will show the atomic D2 transitions in ^{85}Rb and ^{87}Rb respectively.

Figure 1 shows a schematic diagram of a FM spectroscopy experimental setup and Figure C1 in Appendix C shows an actual image of the FM spectroscopy we used.

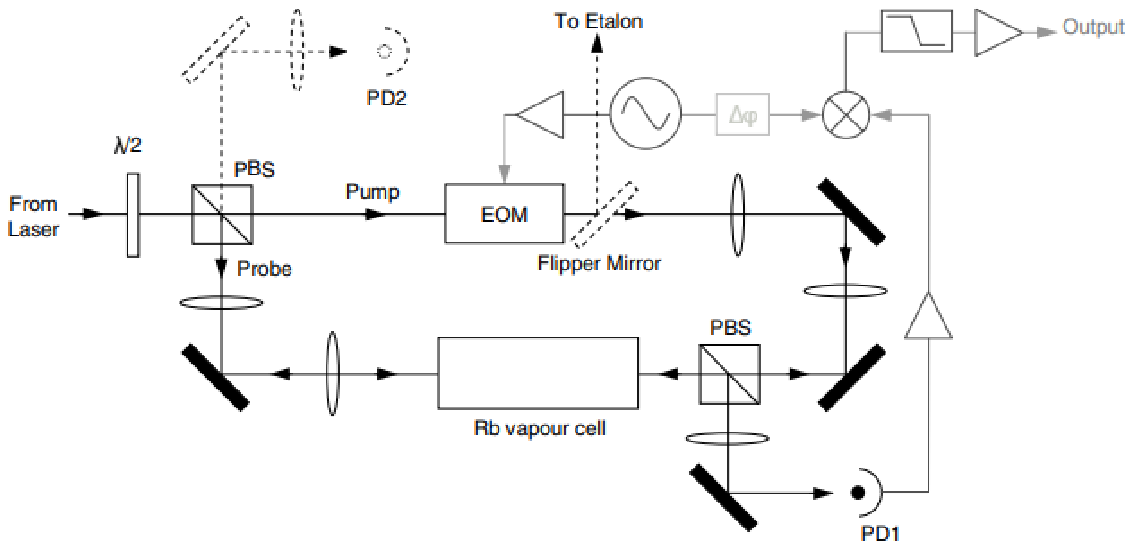


Figure 1. Schematic diagram of the FM Spectroscopy experimental setup [5]. $\lambda/2$: half-wave plate, PBS: polarizing beam splitter, electro-optic modulator (EOM): electro-optic modulator, PD: photodiode.

Running through the setup from the top, we first have the laser. For the laser, we have the laser diode, where we used a near infrared (NIR) diode laser (Model number: GH0781RA2CT). As the linewidth of our laser diode is comparatively wide compared to the rubidium atomic transition linewidth, we would need an external cavity to reduce the linewidth of our laser. Hence, we use an external cavity diode laser (ECDL) which uses the first order diffraction from a grating to feedback on the laser. This results in a laser that is single frequency, narrow linewidth and is continuously tuneable over a wide range [6]. The piezo-electric transducer (PZT) on the grating changes the external cavity length slightly, which allows for fine frequency tuning and locking the laser to the atomic transition. The output of the ECDL is passed through an optical isolator to prevent light from being reflected back into the laser.

The electro-optic modulator (EOM) will modulate the laser's frequency, generating sidebands of 20MHz frequency. The Rb vapour cell contains a mixture of ^{85}Rb and ^{87}Rb, where our laser light will pass through stimulating the D2 transition in the atoms, which will be essential to perform our spectroscopy and locking our laser.

2.2) Mechanism for Laser locking

Having introduced our experimental setup for the FM spectroscopy to lock our laser, we will give a short explanation as to how our FM spectroscopy works for laser locking and frequency stabilization. The idea of the FM spectroscopy is to maintain a frequency stabilization by having a frequency reference through atomic response [7]. In our case, the atomic response will be the D2 transitions in ^{85}Rb and ^{87}Rb.

Firstly, we send our laser through the EOM to produce a pair of weak sidebands at 20MHz. Then, we send a portion of the laser beam, that is enough to saturate the Rb transition, into the Rb vapour cell, achieving a saturated absorption spectroscopy. Afterwards, the laser beam will enter the photodetector (photodiode) to be converted into electrical signal. This electrical signal, together with the 20MHz sideband are mixed and demodulated. We then scan the piezo across the range of voltage to obtain a spectroscopy signal corresponding to the D2 transitions in ^{85}Rb and ^{87}Rb. The deviation from 0V to the resonance is our error signal, as the error signal is defined as the difference between the input variable and the feedback variable [8], which we feed into a control loop feedback system. The control loop feedback then changes the piezo voltage accordingly to maintain its position at resonance, hence locking the laser and giving us the stabilized frequency.

Chapter 3

Measuring Initial Bandwidth of FM Spectroscopy

Now, with an understanding of the FM spectroscopy, we will proceed with the measurement of its bandwidth. From here on, all measurement of the bandwidth will be done in the target frequency. We will break down the entire control loop of our FM spectroscopy into several components to be considered. Figure 2 shows an illustration of the several different components of our control loop.

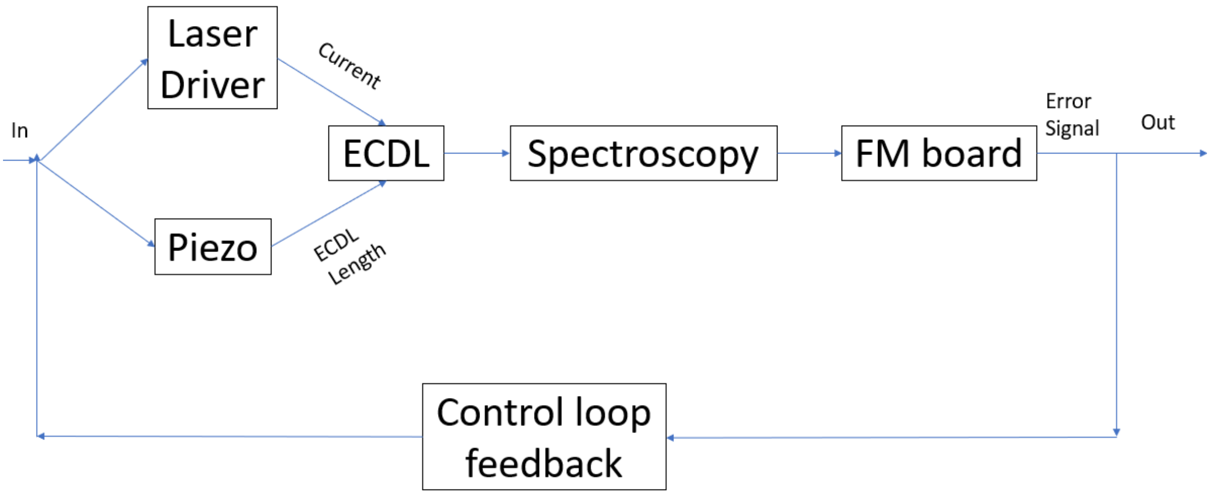


Figure 2. Illustration of the entire control loop of the FM spectroscopy.

For a brief description of Figure 2, we first have our input signal to our control loop, where we have a choice to modulate either the current in the laser driver or the external cavity length using the piezo. The signal then passes through our spectroscopy (shown in Figure 1) and is then sent into our FM board for signal processing with an oscilloscope to look for the error signal. The error signal is then sent through the control loop feedback back to the input (with a negative sign) to lock the laser at our desired frequency.

We first take a measurement of the target frequency of the entire FM spectroscopy, excluding the control loop feedback. This is done by first stabilizing the laser to the atomic transition. A small change in the input would change the laser frequency, which results in a deviation from the atomic frequency in the form of an error signal. This input-output response is measured by a network analyser. The stabilization is only performed slowly to take care mainly of the laser long-term drift. Thus, the response measurement would not be affected greatly by the stabilization.

A network analyser (shown in Figure C2 in the appendix) measures our target frequency by first sending a sinusoidal input signal with a specific amplitude and frequency into the system we are measuring. Depending on the response of the system, the system will output the sinusoidal signal that has a corresponding phase lag. The network analyser will then compare the input and output sinusoidal signals to give us the phase lag of the system we are measuring at the specific amplitude and frequency of the sinusoidal signal.

Figures 3 and 4 show the phase response of the target frequency of the piezo and current collected by the network analyser respectively.

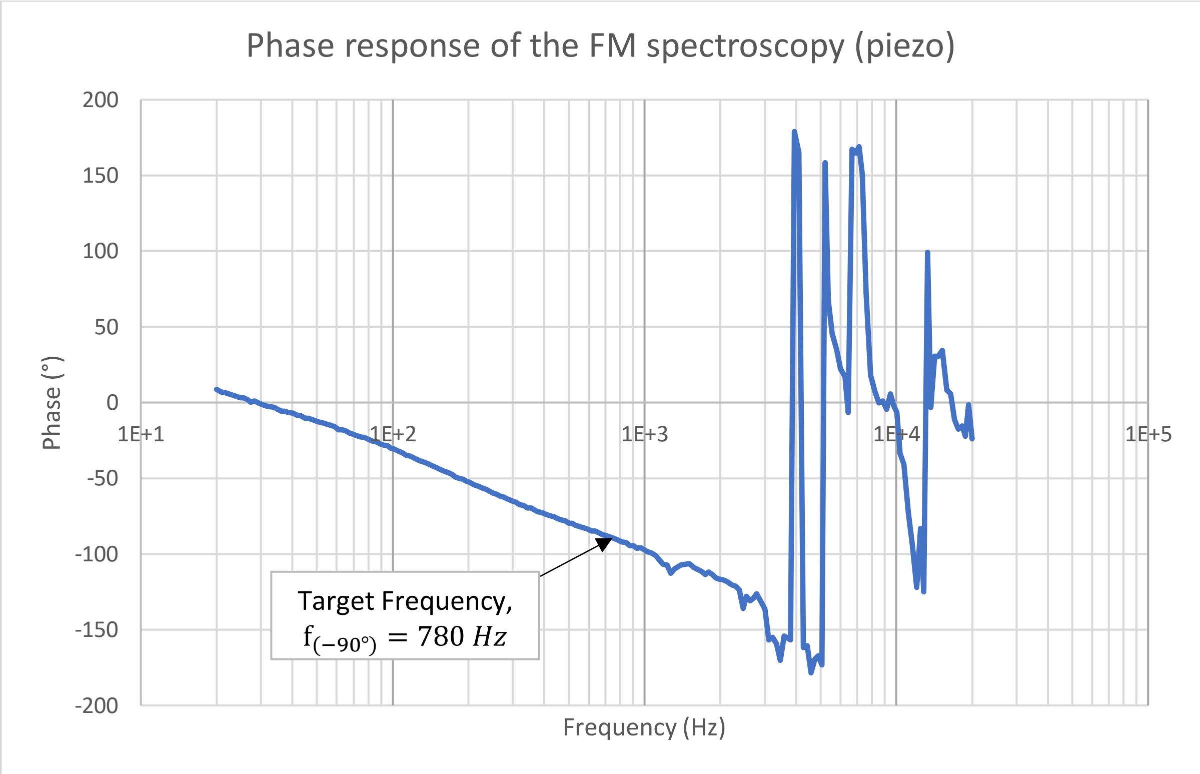


Figure 3. Phase response of the FM Spectroscopy (modulating the piezo). Target Frequency (at 90° phase lag), $f_{(-90°)} = 780Hz$.

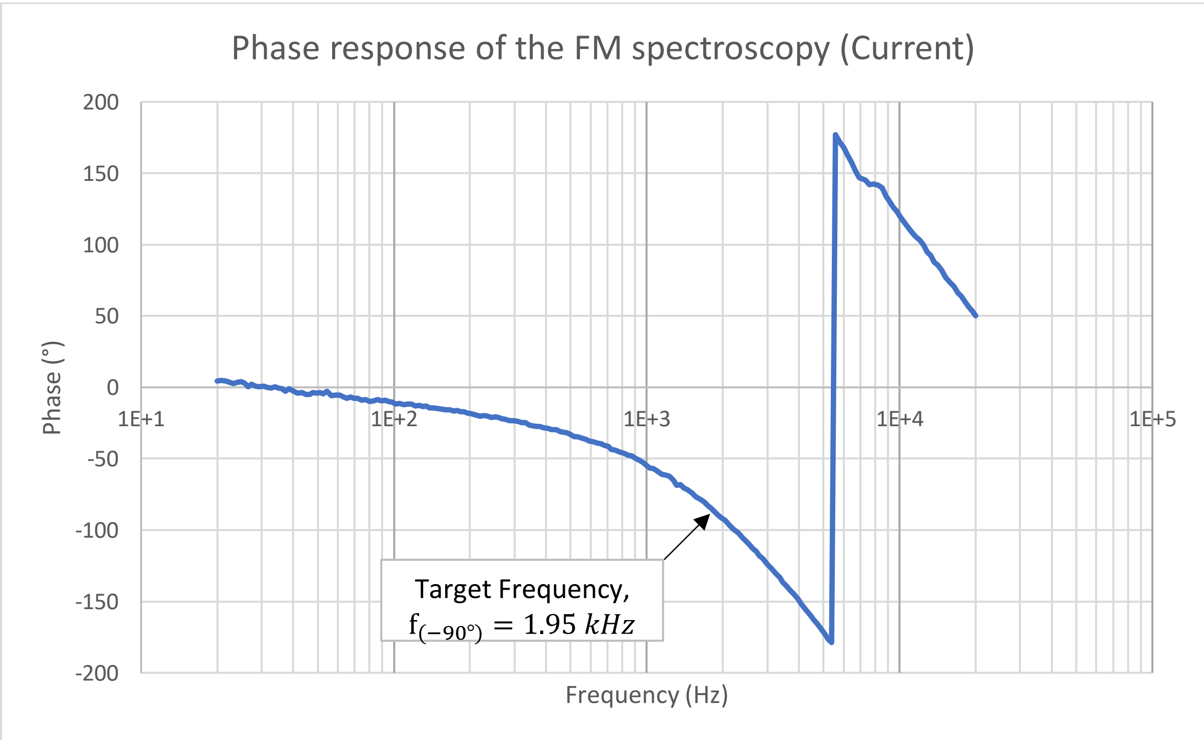


Figure 4. Phase response of the FM Spectroscopy (modulating the current). Target Frequency (at 90° phase lag), $f_{(-90°)} = 1.95kHz$.

As we can see, the target frequency of the FM spectroscopy when displacing the piezo from resonance is extremely low at around $f_{(-90°)} = 780Hz$, while the target frequency when displacing the current from resonance is slightly better at around $f_{(-90°)} = 1.95kHz$, but still not much of an improvement. With this bandwidth, we will not be able to reduce the noise in the kilohertz regime. This will be the baseline target frequency that we will work from for our improvement of the bandwidth of the FM spectroscopy.

Now that we know the initial target frequency of the entire FM spectroscopy, we will measure the target frequency of each component in our FM spectroscopy. This is to find out which component is limiting the target frequency of our FM spectroscopy, resulting in it being at such a low value as seen in Figures 3 and 4.

Chapter 4

Bandwidth of Laser Driver and ECDL

4.1) Measuring Bandwidth of the Laser Driver

We will first attempt to measure the bandwidth of our initial laser driver (Shown in Figure C3 in the Appendix). The laser driver supplies a constant current to the laser. The input into this system is the modulation channel of the laser driver, and it outputs the voltage across the laser diode.

In order to isolate the laser driver and measure only its target frequency to figure out if it's the limiting factor in the low target frequency in our FM spectroscopy, we created a small device, which is a light emitting diode (LED) connected in parallel circuit, to simulate the circuit in the ECDL (image of the device is shown in Figure C4 in the Appendix). An LED is used instead of a laser diode to prevent us from destroying the laser diode due to its higher sensitivity. It is also because LED have similar electrical characteristics as the laser diode. Then, we used the network analyser to directly measure the target frequency of the laser driver. Figure 5 shows the phase response of the laser driver.

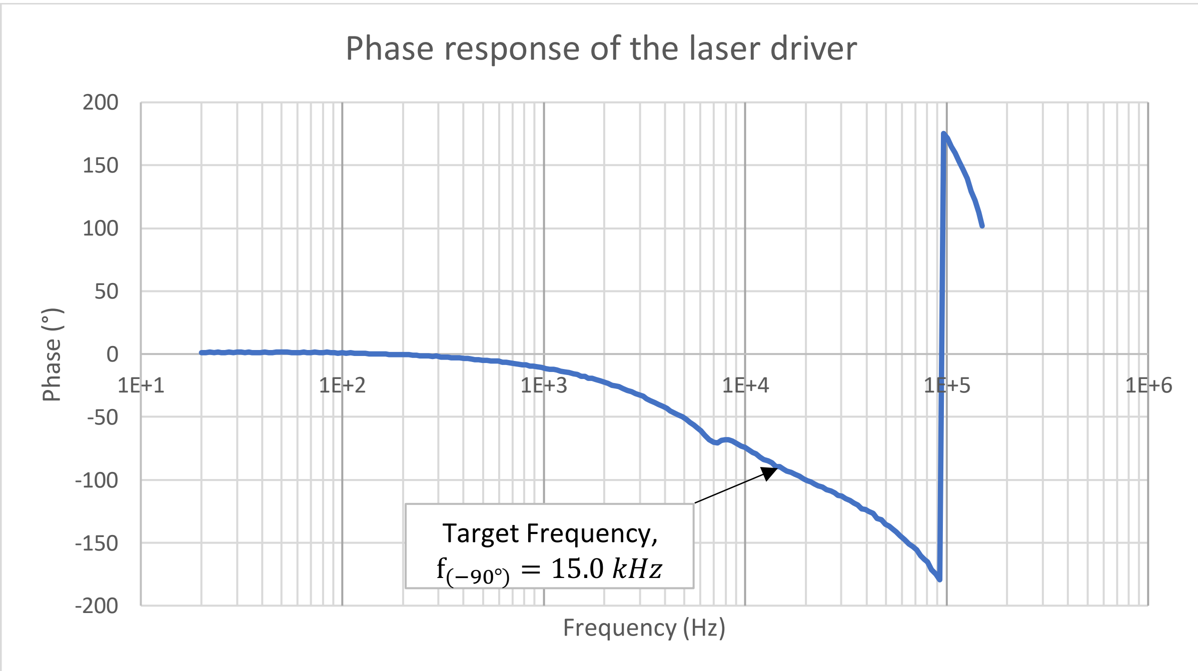


Figure 5. Phase response of the laser driver. Target Frequency (at 90° phase lag), $f_{(-90°)} = 15.0kHz$*.*

As we can see, the target frequency of the laser driver is low as well, at around $f_{(-90°)} = 15.0kHz$ at the 90° phase lag.

4.2) Improving the Bandwidth of Laser Driver and ECDL

4.2.1) Building a New Circuit for Wideband Current Injection into the Laser Diode

To improve the bandwidth of the laser driver and ECDL, we attempt building a circuit for a wideband current injection into the laser diode. Figure 6 shows a schematic diagram of the circuit, and Figure C5 in the Appendix show an image of the actual implementation of our circuit.

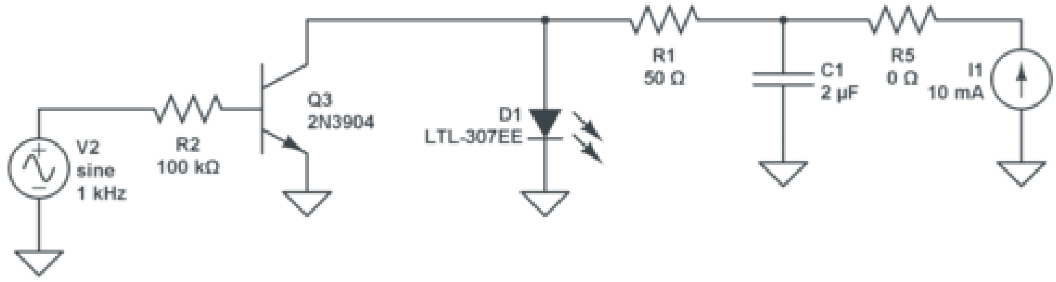


Figure 6. Schematic diagram of circuit for wideband current injection into the laser diode.

By studying Figure 6, we can see that the transistor acts like a “current traffic control”. If we provide a certain voltage through the base terminal, the collector current is amplified, by a factor of approximately 200x as we see later as well as in the datasheet. Therefore, the transistor diverts a bit of current away from the laser diode. The speed of this current modulation/diversion (in other words the frequency bandwidth) is only limited by the transistor. Due to the 50Ω “separating” resistor, it does not really interact with the $2\mu F$ stabilizer capacitor. Hence, this channel can be used for wideband current modulation.

On the other hand, the noise from the current driver can be stabilized with the $2\mu F$ stabilizer capacitor, with a cut off frequency of 1kHz. In summary, the constant current supply from the laser driver and the wide-band current modulation are 2 separate channels.

Now, to implement the circuit, we need to note several things. Firstly, the modulation voltage channel needs to have an offset of around 1V to activate the transistor. Also, the resistor on the base terminal determines the proportional constant of the current modulation – the higher the resistance, the lower the current modulation strength.

4.2.2) Measuring the Bandwidth of Laser Driver and ECDL with New Circuit

After installing the circuit mentioned above into the ECDL for the wideband current injection into the laser diode, we now use the network analyser to measure the target frequency of the laser driver and ECDL with the new circuit. Figure 7 shows the phase response of the laser driver and ECDL with new circuit implemented in the ECDL.

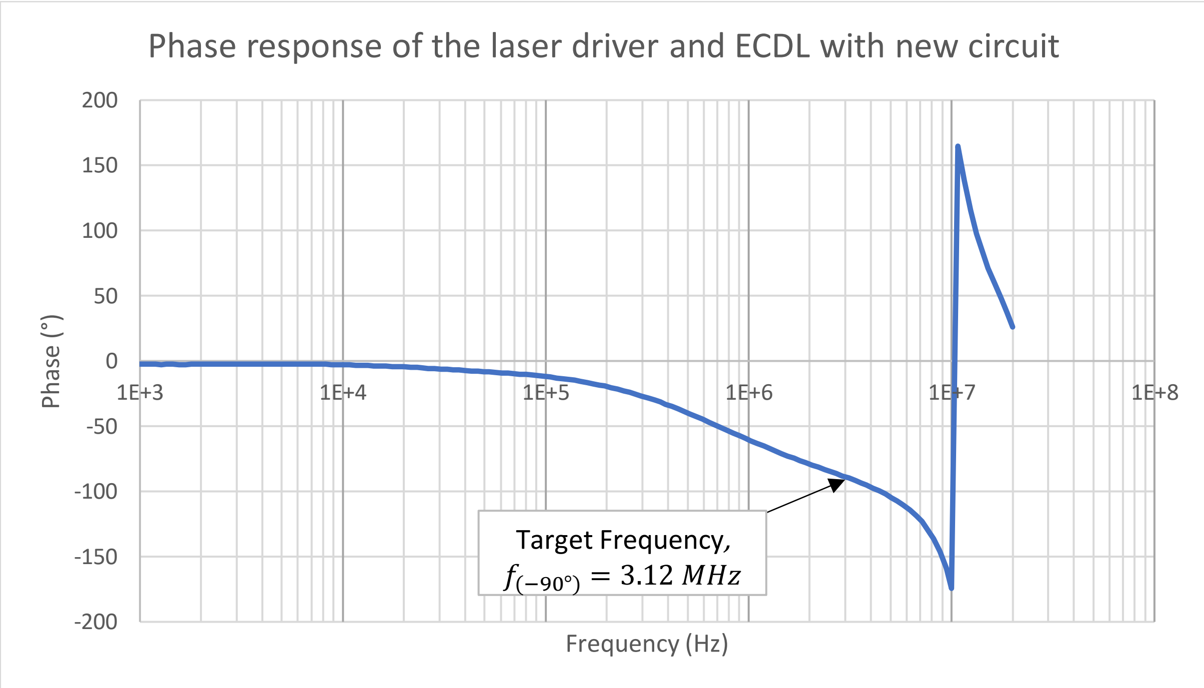


Figure 7. Phase response of the laser driver and ECDL with new circuit in ECDL. Target Frequency (at 90° phase lag), $f_{(-90°)} = 3.12MHz$

As we can see from Figure 7, there is a huge improvement in the target frequency of the laser driver and ECDL, from $f_{(-90°)} = 15.0kHz$ (in Figure 5) to $f_{(-90°)} = 3.12MHz$ (in Figure 7). Now, the target frequency of the laser driver and ECDL starts to be limited by the photodiode bandwidth and the signal propagation time (i.e., length of the cable). However, that is not a big concern as the regime of 3.12MHz target frequency is sufficient for many purposes that we are trying to fulfil by increasing the target frequency of the FM spectroscopy.

4.2.3) Measuring the Bandwidth of FM Spectroscopy with New Circuit in the ECDL

Now, we use the network analyser to measure the target frequency of the entire FM spectroscopy to determine if the target frequency has increased compared to the initial version with just the laser driver (which was $f_{(-90°)} = 1.95kHz$ shown in Figure 4). Figure 8 shows the phase response of the entire FM spectroscopy with the new circuit implemented in the ECDL.

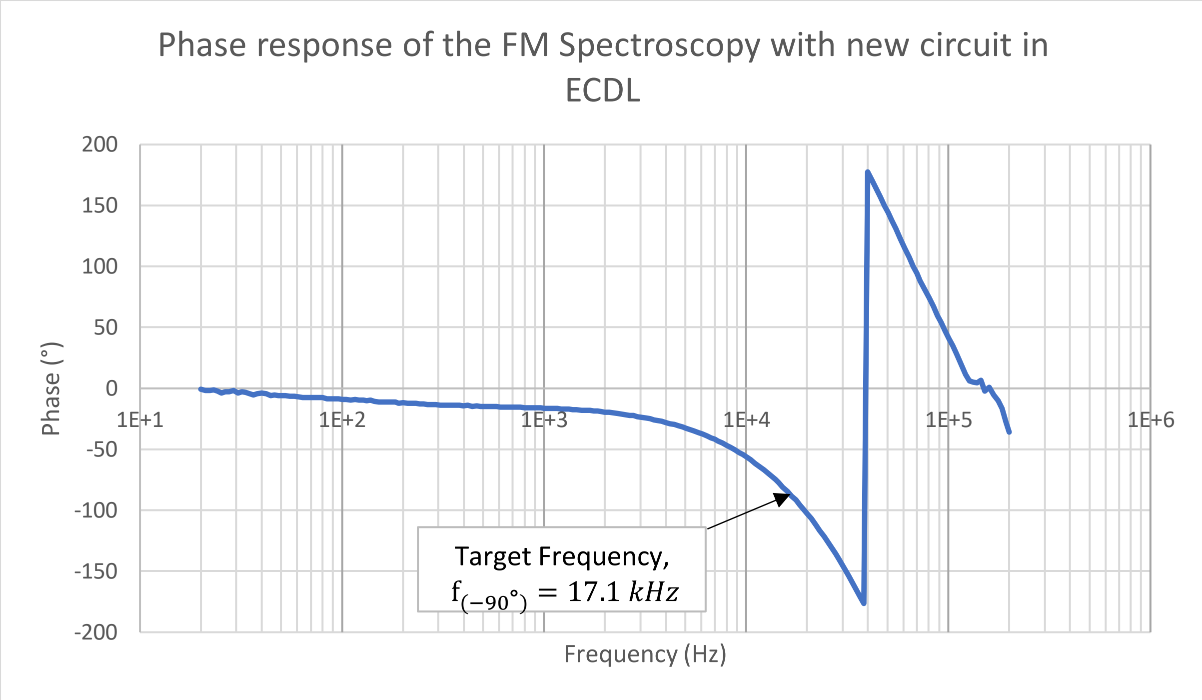


Figure 8. Phase response of the FM spectroscopy with new circuit in ECDL. Target Frequency (at 90° phase lag), $f_{(-90°)} = 17.1kHz$

As we can see from Figure 8, the target frequency of the FM spectroscopy with the new circuit implemented in the ECDL has improved from $f_{(-90°)} = 1.95kHz$ (in Figure 4) to $f_{(-90°)} = 17.1kHz$ (in Figure 8). But the new target frequency of 17.1kHz is still not satisfactory, especially since we are able to improve the target frequency of the laser driver and ECDL in the previous section to the magnitude of megahertz. Hence, there are other components of the FM spectroscopy that are still limiting its target frequency. We will investigate the bandwidth of the FM board in the next section.

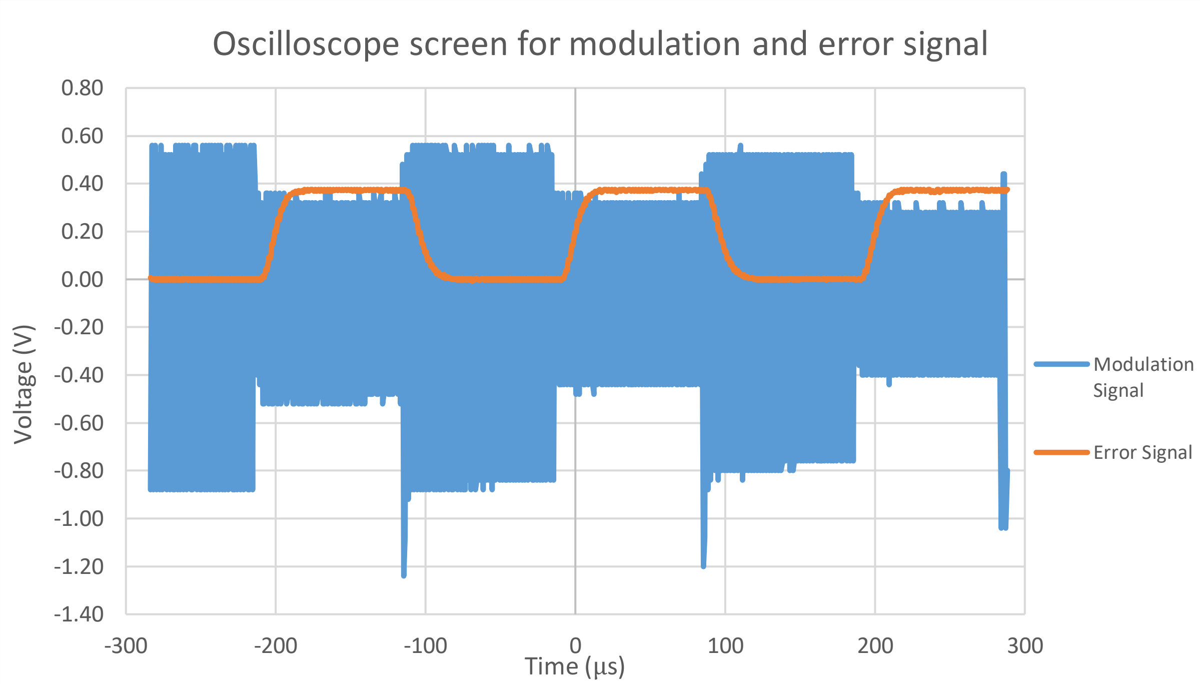
Chapter 5

Bandwidth of FM Board

5.1) Initial Measurement of Bandwidth of the FM Board

Now, with the implementation of the new circuit in place in the ECDL, we will measure the target frequency of our initial FM board. The FM board is our FM signal processing tool which we use to process the deviation of our signal from the atomic response in the form of an error signal, and to pass on the error signal to the control loop feedback system for the locking of our laser.

Since it will be a complicated task to measure the exact target frequency of the FM board, we will only get a rough estimation of this value. For our purpose, this rough estimation will be sufficient as we only need to know the order of magnitude of the target frequency of the FM board to know if it is limiting the target frequency of our FM spectroscopy. We will now explain the idea behind this measurement. Firstly, we note that the modulation level is switched between 2 binary levels in the error signal. We will record both the modulated signals and error signals using an oscilloscope. Figure 9 (top) shows the oscilloscope time trace, while Figure 9 (bottom) shows a zoomed in version of the time trace to measure the time delay and time decay.



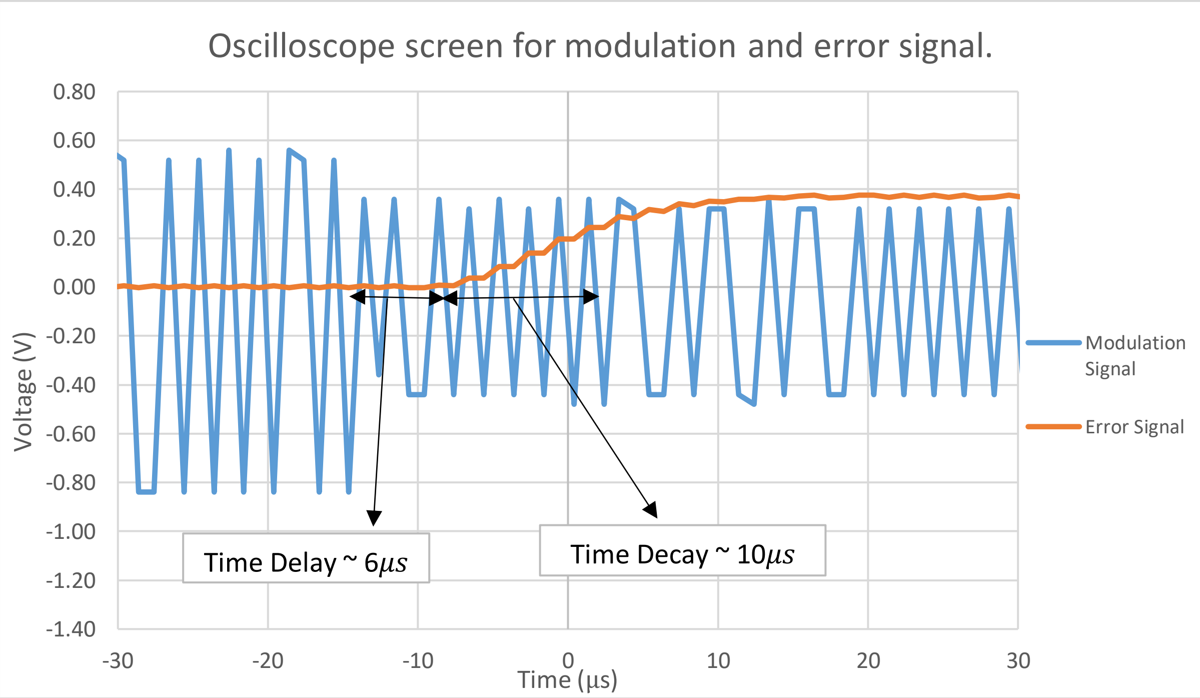


Figure 9. Oscilloscope screen for modulation and error signal (top) Original version. (bottom) Zoomed in version from -30μs to 30μs.

Referring to Figure 9, we can estimate the target frequency of our FM board with the phase delay equation, where we account for the time delay due to signal propagation delay as well as the time decay due to the resistor-capacitor (RC) constant. We have our phase delay equation:

$$Total\ Delay = -\frac{\pi}{2} = -\arctan\left(2\pi f_{(-90^{\circ})}\tau_{decay}\right) - 2\pi f_{(-90^{\circ})}\tau_{delay}, \quad (1)$$

Where $\tau_{decay} \approx 6\mu s$, $\tau_{delay} \approx 10\mu s$, and $f_{(-90^{\circ})}$ is the target frequency of our FM board. Solving the equation numerically, we have the estimated value of our target frequency to be

$f_{(-90°)} \approx 16kHz$. Hence, we can see from this low target frequency that the FM board is limiting the target frequency of our FM spectroscopy.

5.2) Improving the Bandwidth of the FM Board

Now that we know the FM board is limiting the target frequency of the FM spectroscopy, we will first study the circuit diagram of the current FM board (Figure 10) to find out what could possibly be limiting our FM board and how can we improve its target frequency.

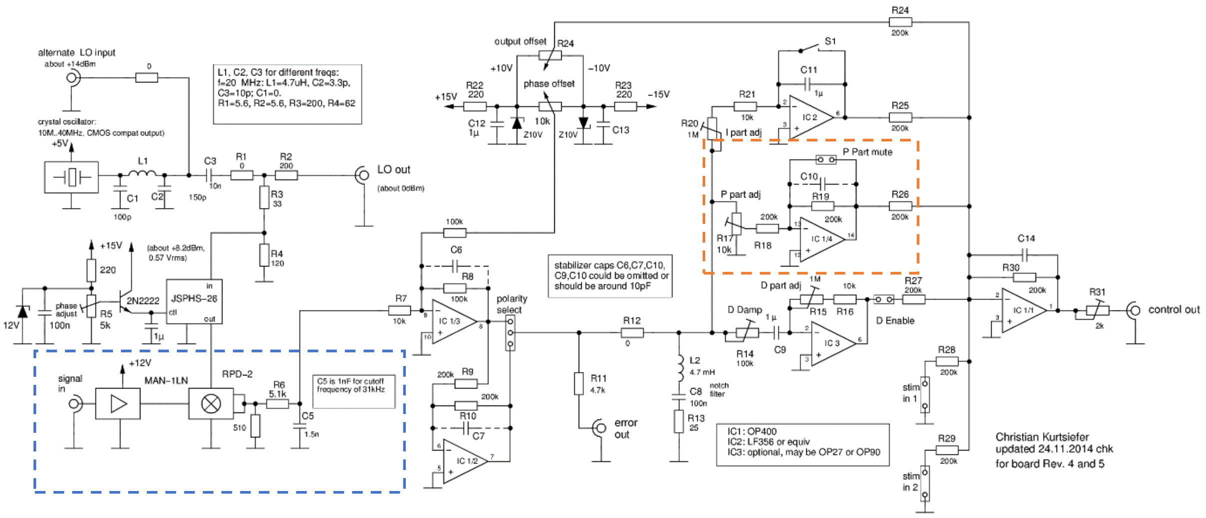


Figure 10. Circuit diagram of FM board used [9]. Box in blue will be referenced for building our own makeshift FM board. Box in orange will be referenced for building our own P control loop in Chapter 6.

From Figure 10, we hypothesise that the amplifier, mixer, and low pass filter are limiting the FM board's target frequency.

The long time decay in the old FM board is usually due to the RC constant [10], which we can improve by just replacing the resistor and capacitor to higher bandwidth ones. The long time delay on the other hand is usually due to long length of cable and the limit of the speed of light [11]. However, in the current FM board setup, the cable lengths are only of the order of $\sim 1m$, which put the time delay to be on the order of $\sim 5ns$, enough for up to $\sim MHz$ bandwidth. However, we observed a time delay on the order of $\sim 6\mu s$ which are $\sim 10^3$ orders higher than expected.

To investigate the long time delay in the FM board, we generate a 20MHz sinusoidal signal with 2 different amplitudes using the amplitude modulation (AM) function on the function generator. The FM board would perceive this as 2 different modulation levels, and the mixer would output 2 different values. Figure 11 shows the signal output of the function generator.

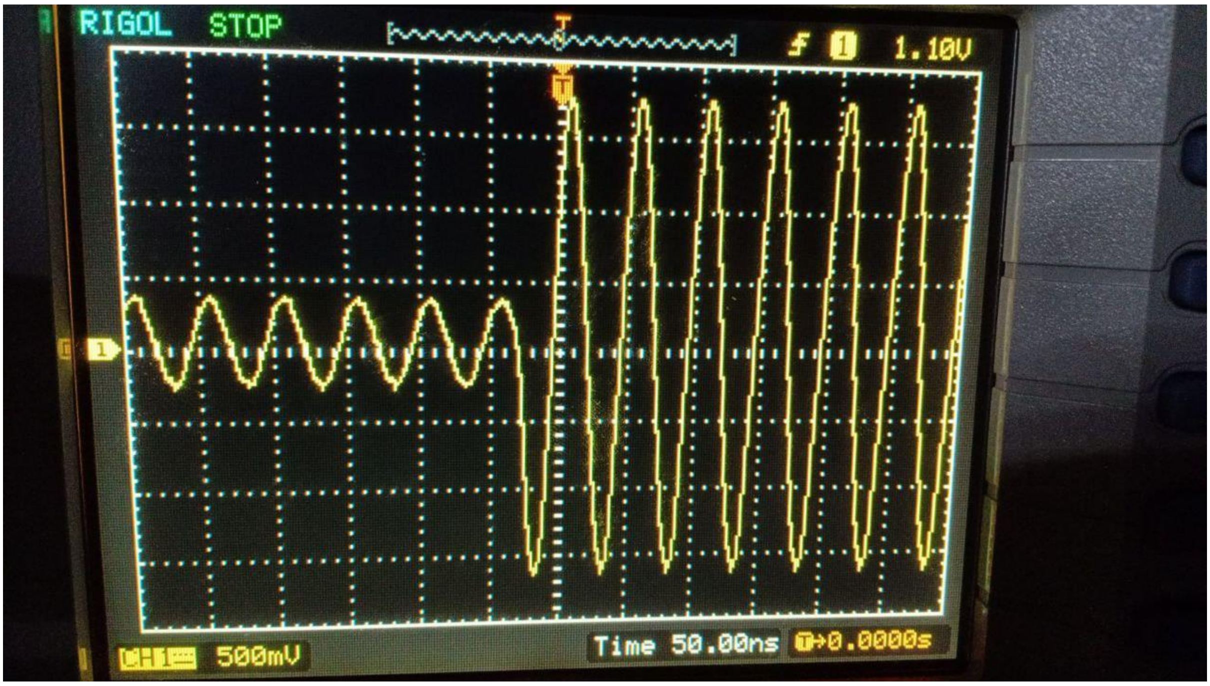


Figure 11. Sinusoidal signal output of the function generator.

Now, when we plug the sinusoidal signal in onto the FM board, we observe that the sinusoidal signal seems to be distorted as shown in Figure 12. This shows that there are multiple reflections (i.e., distortion in our sinusoidal signal) in our electronics (the amplifier and mixer).

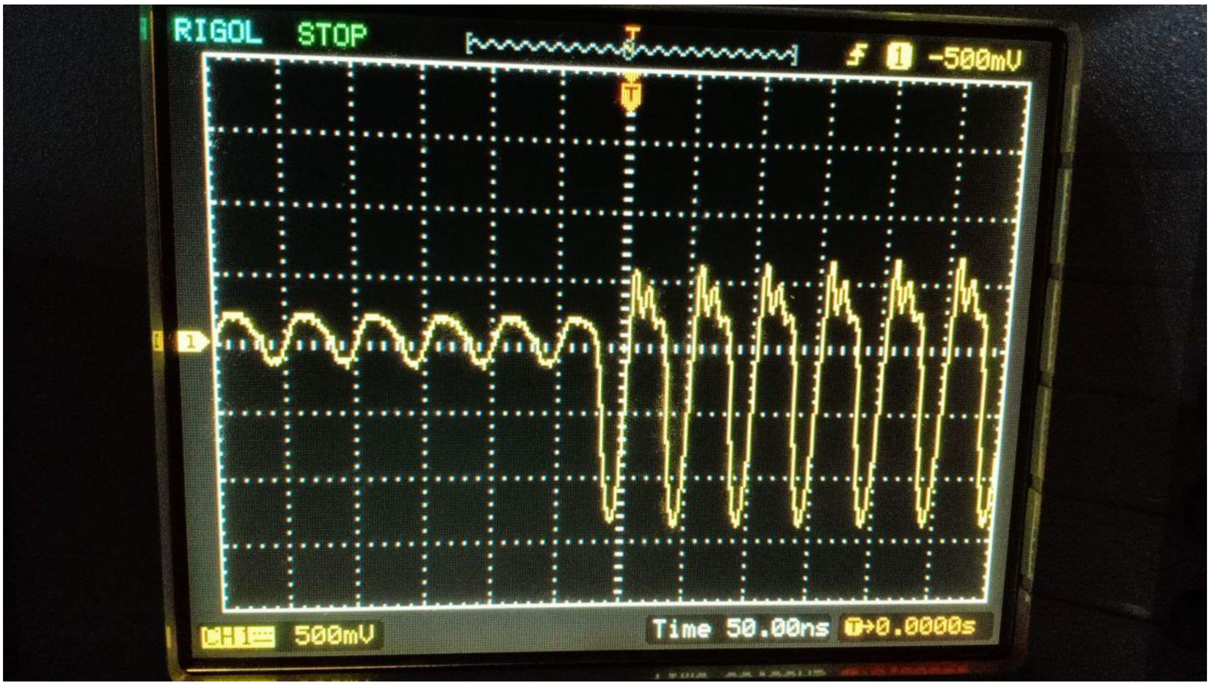


Figure 12. Signal output after passing through the FM board. Sinusoidal signal seems to be distorted.

Additionally, we also observe low pass behaviour in our signal as we zoom out on the time trace as shown in Figure 13. As we can see in Figure 13, there is a gradual move towards symmetry for a few microseconds.

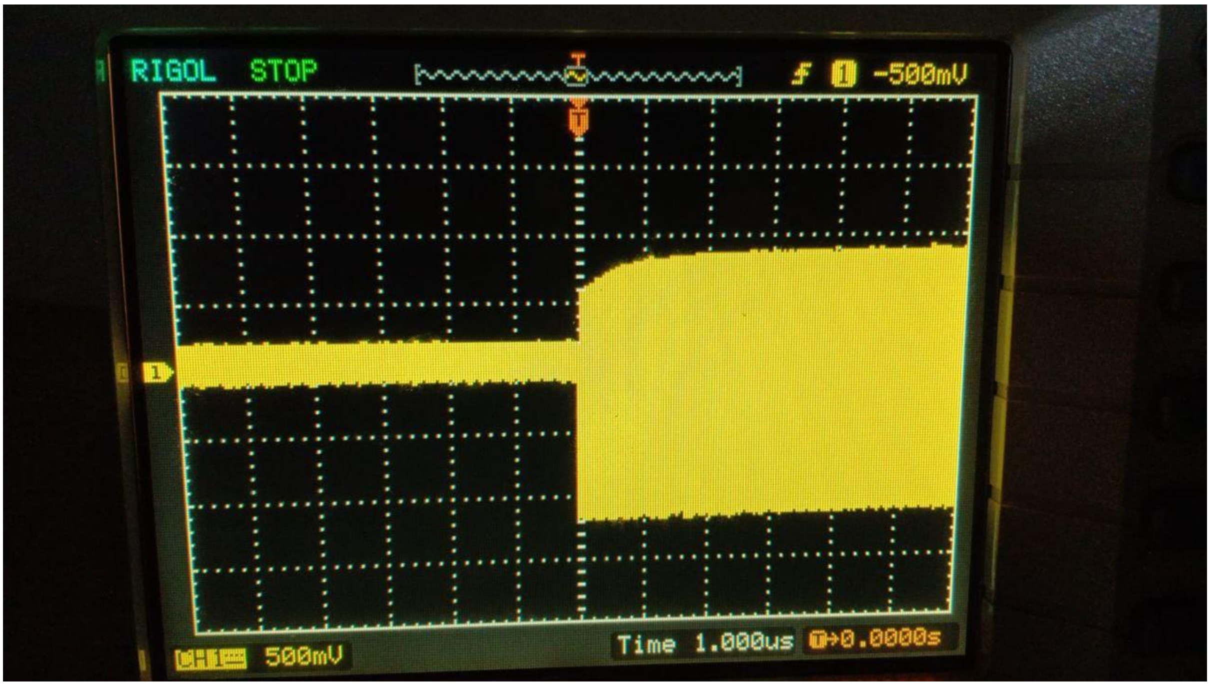


Figure 13. Zoomed-out version of signal output after passing through FM board. Low pass behaviour can be observed in the gradual move towards symmetry after a few microseconds.

From our experiment, we discovered that the long time delay is likely due to the settling time of the signal due to multiple reflections and low pass behaviour in the FM board. By design, the amplifier and the mixer should not have any multiple reflection as they are close to 50Ω terminated and the cables also have 50Ω termination. However, we observed this multiple reflection likely due to aging components of the FM board. Additionally, we also observe the low pass behaviour, likely due to the amplifier low-frequency response.

To avoid these issues, we attempt to make our own makeshift FM board with the given electronics.

For the mixer and amplifier, we have chosen the ZFM-2+ mixer and ZFL-1200G+ amplifier (as they do not exhibit multiple reflections and low pass behaviour) and connected them as shown in Figure C6 in the Appendix.

The next component is a low pass filter that passes signals with a frequency below its cut-off frequency and attenuates signals with a frequency above the cut-off frequency. The cut-off frequency for a low pass filter is the frequency at which the output voltage equals 70.7% of the input voltage [12]. For this experiment, we use the low pass filter to attenuate twice the modulation frequency of the 20MHz sideband after it passes through the mixer. We have decided to build a low pass filter of our own. We built a lowpass filter modelled after a 5th Order Chebyshev Lowpass with a cut-off frequency of 6.66MHz. Figure 14 shows the circuit diagram of the 5th Order Chebyshev Lowpass and Figure C7 in the Appendix shows the image of the lowpass filter we have built.

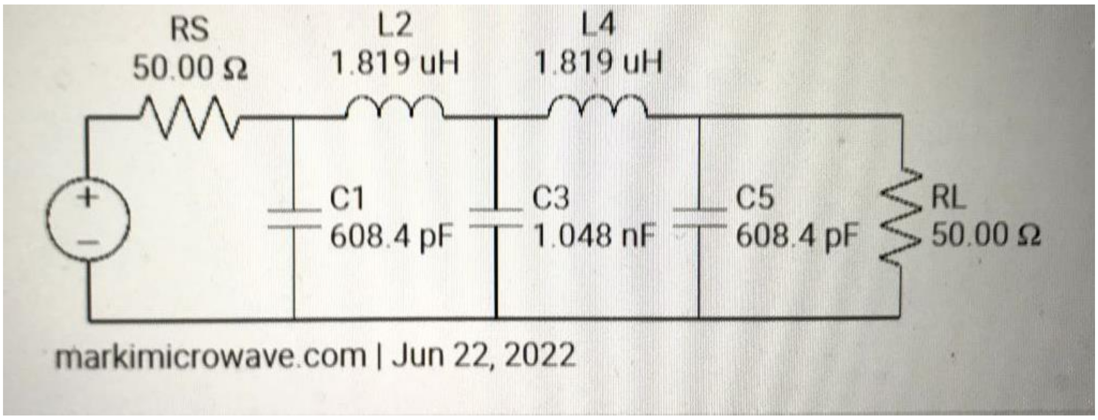


Figure 14. Circuit diagram of 5th Order Chebyshev Lowpass

Now, with the new electronics in our new makeshift FM board, we no longer observe the multiple reflections and low pass behaviour in our FM board as shown in Figure 15.

Figure 15. Sinusoidal signal output after passing through new makeshift FM board. No multiple reflection or low pass behaviour observed. (top) Zoomed-in version. (bottom) Zoomed-out version.

Now, using the network analyser, we do a measurement on the target frequency of the new makeshift FM board that we have built to check that it is sufficiently high and will not limit the target frequency of our FM spectroscopy. Figure 16 shows the phase response of the new FM board.

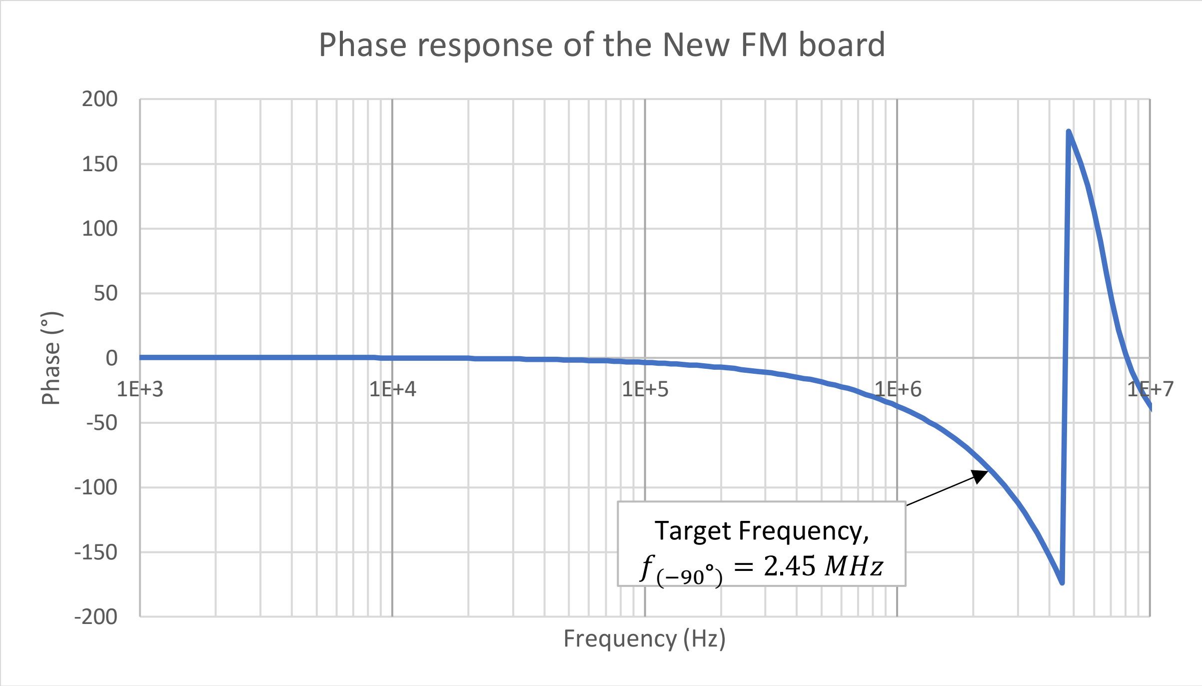


Figure 16. Phase response of the new FM board. Target Frequency (at 90° phase lag), $f_{(-90°)} = 2.45MHz$

As we can see from Figure 16, the target frequency of our new FM board is sufficiently high (at around $f_{(-90°)} = 2.45MHz$). Now, we can use the network analyser to measure the target frequency of our entire FM spectroscopy again, but this time with the implementation of the new makeshift FM board. Figure 17 shows the phase response of the FM spectroscopy with the implementation of the new makeshift FM board.

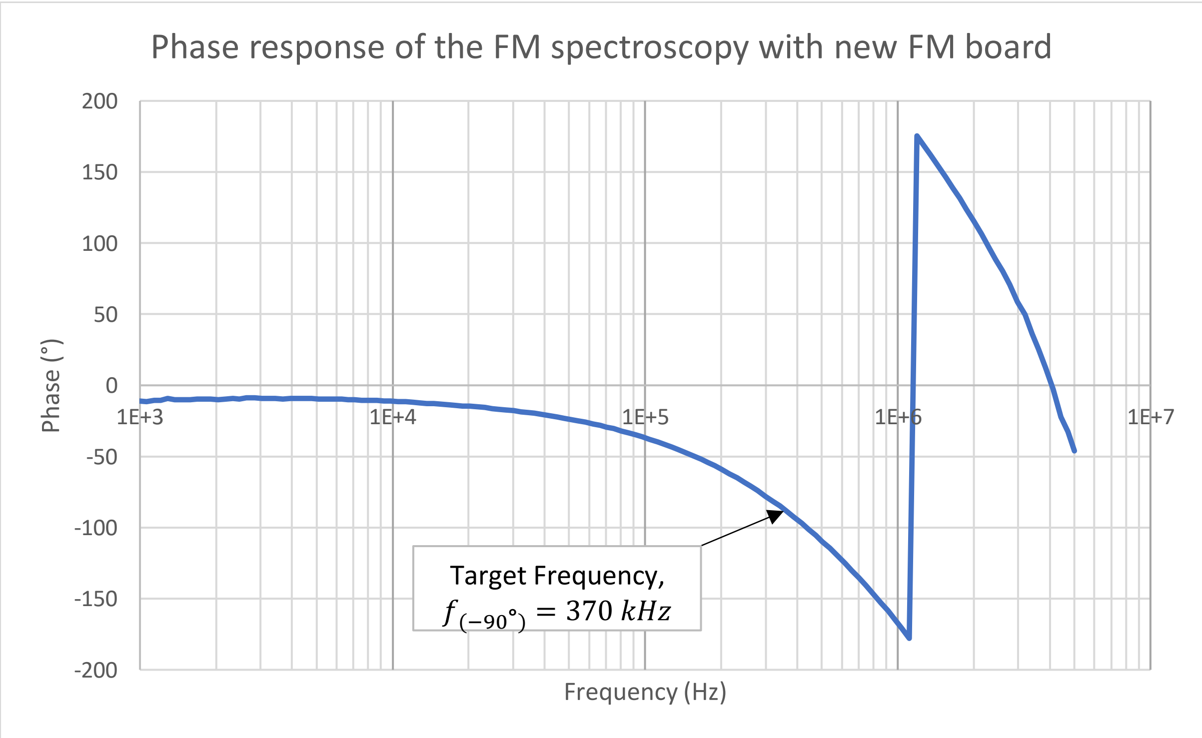


Figure 17. Phase response of the FM spectroscopy with new FM board. Target Frequency (at 90° phase lag), $f_{(-90°)} = 370kHz$.

With the implementation of the new FM board that we made, the target frequency of our FM spectroscopy improved significantly from $f_{(-90°)} = 17.1kHz$ in Figure 8 to $f_{(-90°)} = 370kHz$ in Figure 17.

Overall, with our new implementation of the new circuit for wideband current injection into the laser diode in section 4, and our new implementation of the FM board in section 5, we achieved a huge improvement (~ 200 times improvement) in the target frequency of our FM spectroscopy, from $f_{(-90°)} = 1.95kHz$ in Figure 4 to $f_{(-90°)} = 370kHz$ in Figure 17.

Chapter 6

Bandwidth of P Control Loop

Now that we have improved the target frequency of our FM spectroscopy, in this section we aim to investigate the target frequency of the control loop for our laser locking. To do so, we opt to build our own Proportional (P) control loop. We do note that the original FM spectroscopy involves a Proportional-Integral (PI) control loop system, but for our purpose we deem that the P control loop is sufficient to provide feedback and control to our laser locking. Taking reference from the FM board circuit diagram in Figure 10, we will build our P control loop according to the circuit diagram shown in Figure 18. Figure C8 in the Appendix will show the image of our implementation of the P control loop on a breadboard.

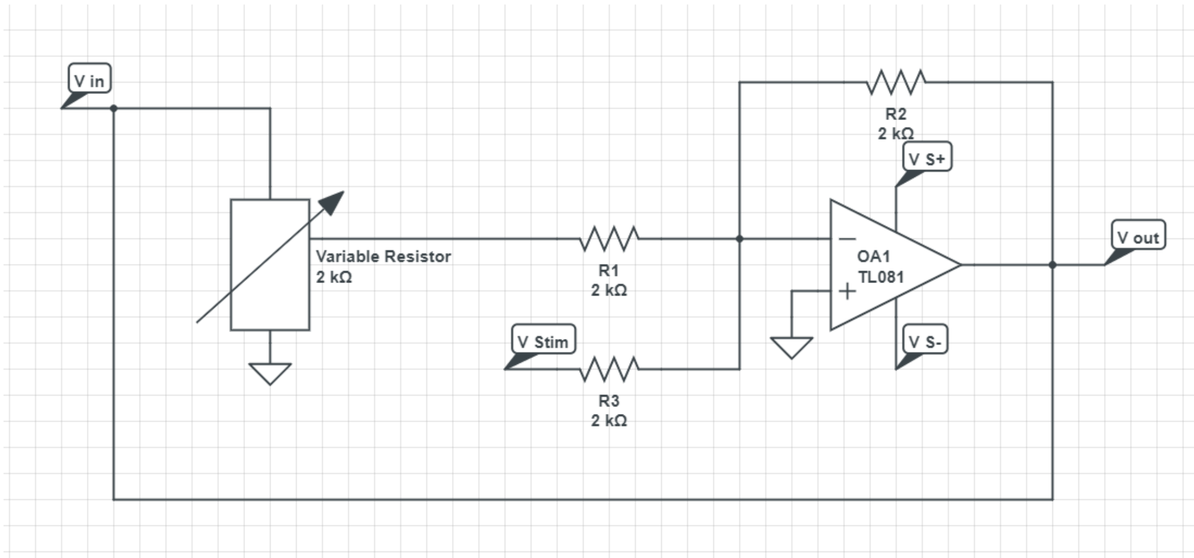


Figure 18. Circuit Diagram of P Control Loop

After we have implemented the P control loop on the breadboard, we supply +-5V to V S+ and V S- respectively and measure the target frequency of the P control loop using the network analyser. Figure 19 shows the phase response of the P control loop.

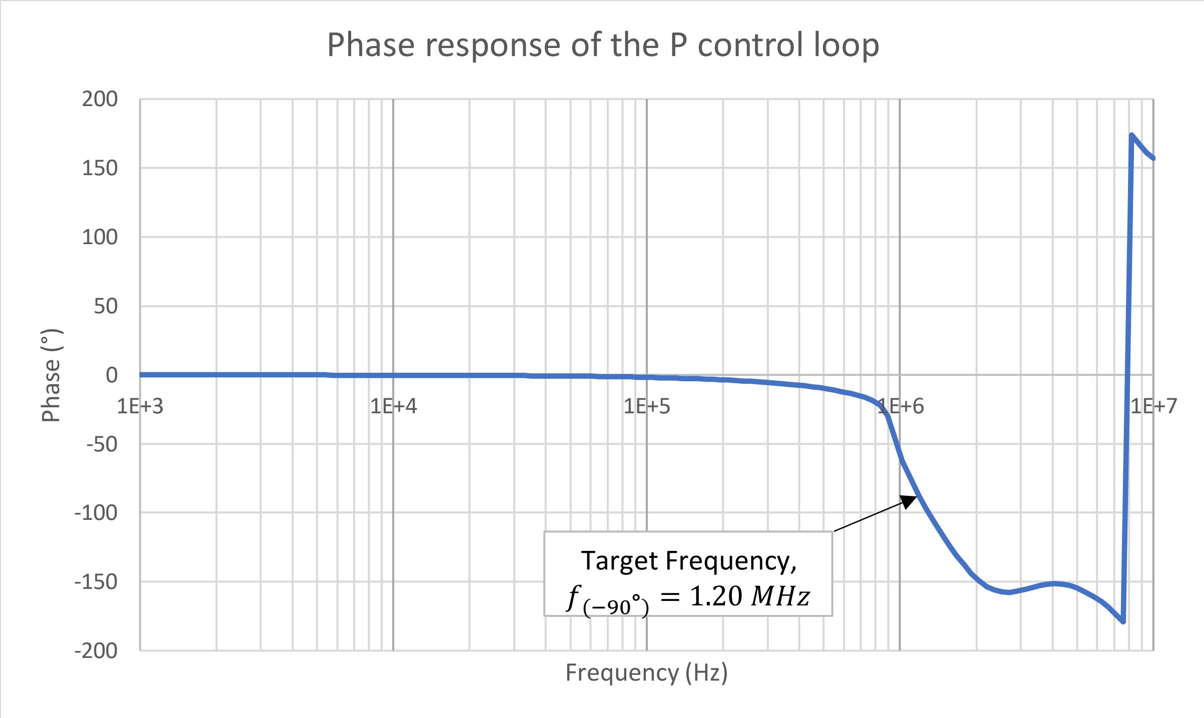


Figure 19. Phase response of the P control loop. Target Frequency (at 90° phase lag), $f_{(-90°)} = 1.20MHz$.

As we can see from Figure 19, after optimizing the resistance on the variable resistor, the target frequency of the P control loop is $f_{(-90°)} = 1.20MHz$. As the target frequency of the P control loop is sufficiently high compared to the that of the FM spectroscopy of $f_{(-90°)} = 370kHz$ (in Figure 17), we expect that it will not severely limit the target frequency of our entire FM spectroscopy.

Chapter 7

Conclusion

In this experiment, we focused on measuring and improving the bandwidth (which we chose to use the target frequency to determine) of our FM spectroscopy. We began by measuring the target frequency of our initial FM spectroscopy setup and determined it to be $f_{(-90^{\circ})} = 780Hz$ and $f_{(-90^{\circ})} = 1.95kHz$ respectively for the piezo and current modulation. From there we strive to improve those values.

We first improved the laser driver and ECDL of our FM spectroscopy and concluded that creating a circuit for a wideband current injection into the laser diode improves the target frequency by a satisfactory amount. We obtained a target frequency of $f_{(-90^{\circ})} = 3.12MHz$ for our laser driver and ECDL with the new circuit and $f_{(-90^{\circ})} = 17.1kHz$ for our entire FM spectroscopy which is an improvement from the initial values (albeit still low).

We then improved our FM board by building a makeshift FM board with a mixer, amplifier, and lowpass filter. With those components, we obtained a target frequency of $f_{(-90^{\circ})} = 2.45MHz$ for our makeshift FM board and $f_{(-90^{\circ})} = 370kHz$ for our entire FM spectroscopy which is a significant improvement from the previous values.

To end off the experiment, we set out to determine whether the control loop will limit the target frequency of our FM spectroscopy. To do so, we built a P control loop and measured its target frequency to be $f_{(-90^{\circ})} = 1.20MHz$. We were unable to determine the target frequency of our FM spectroscopy with the P control loop due to time constraint, but we will not expect the P control loop to limit the target frequency (and hence the bandwidth) of our FM spectroscopy by a significant amount.

This improvement in bandwidth of our FM spectroscopy now allows us to eliminate the ongoing technical noise, which is currently dominating at the kilohertz region.

Appendix A – Reason for choosing Frequency at 90° Phase Lag as a proxy to the Bandwidth

As mentioned in Chapter 1 – Introduction, we have chosen a standardised measure of the frequency at 90° phase lag as a proxy to the bandwidth. To understand why we make that choice, we first refer to the block diagram below (Figure A1) which illustrates our control loop for the FM spectroscopy.

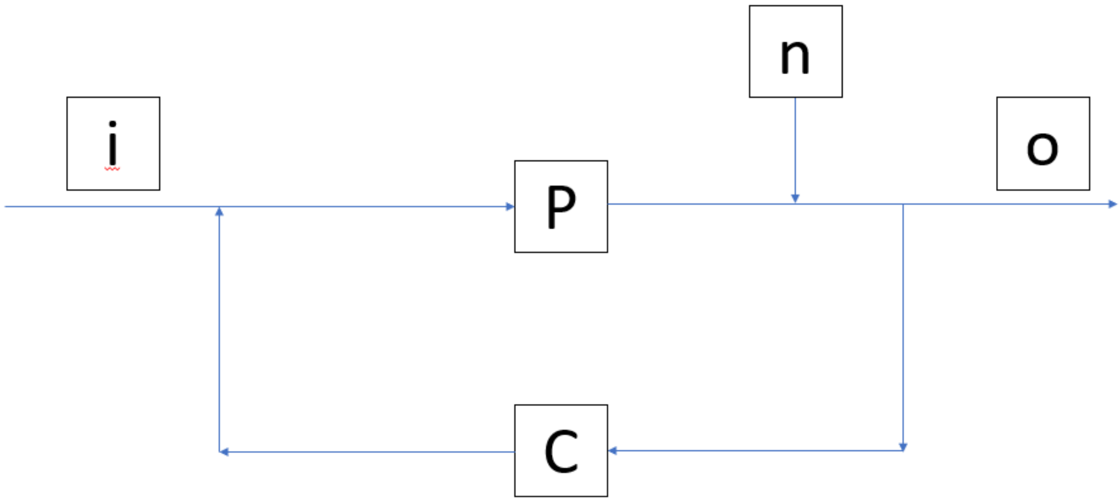


Figure A1. Block diagram for illustration of the control loop of our FM spectroscopy

Referring to Figure A1, i and o are the input and output signal of our control loop respectively. n is the noise to our system that we are aiming to reduce. P is the plant, which is our FM spectroscopy, and C is our controller. In this case, our control loop follows the equation:

$$o = \frac{1}{1+P\cdot C}n, \tag{2}$$

Hence, we can see from equation (2) that if we aim to reduce the noise, n, then we must fulfil the condition: $|1 + P \cdot C| > 1$, for all C and for simplicity, we assume the phase lag of C to be $\phi = 0$. Expanding our expression for the condition, we have:

$$\left|1 + C \cdot P_0 e^{i\phi}\right| > 1, \tag{3}$$

Or,

$$|1 + C \cdot P_0 cos\phi + iC \cdot P_0 sin\phi| > 1, \tag{4}$$

Where P_0 is the magnitude of the signal of our plant and ϕ is the phase lag of the signal of our plant. In this case from equation (4), we can see that to satisfy the inequality, we must fulfil $-90° < \phi < 90°$. In other words, the phase lag of the signal of our plant must less than $90°$ for us to reduce the noise to our system in that region. Hence, this explains our choice to measure the frequency at $90°$ as a proxy to the bandwidth.

Appendix B – Atomic D2 Transitions in ^{85}Rb and ^{87}Rb

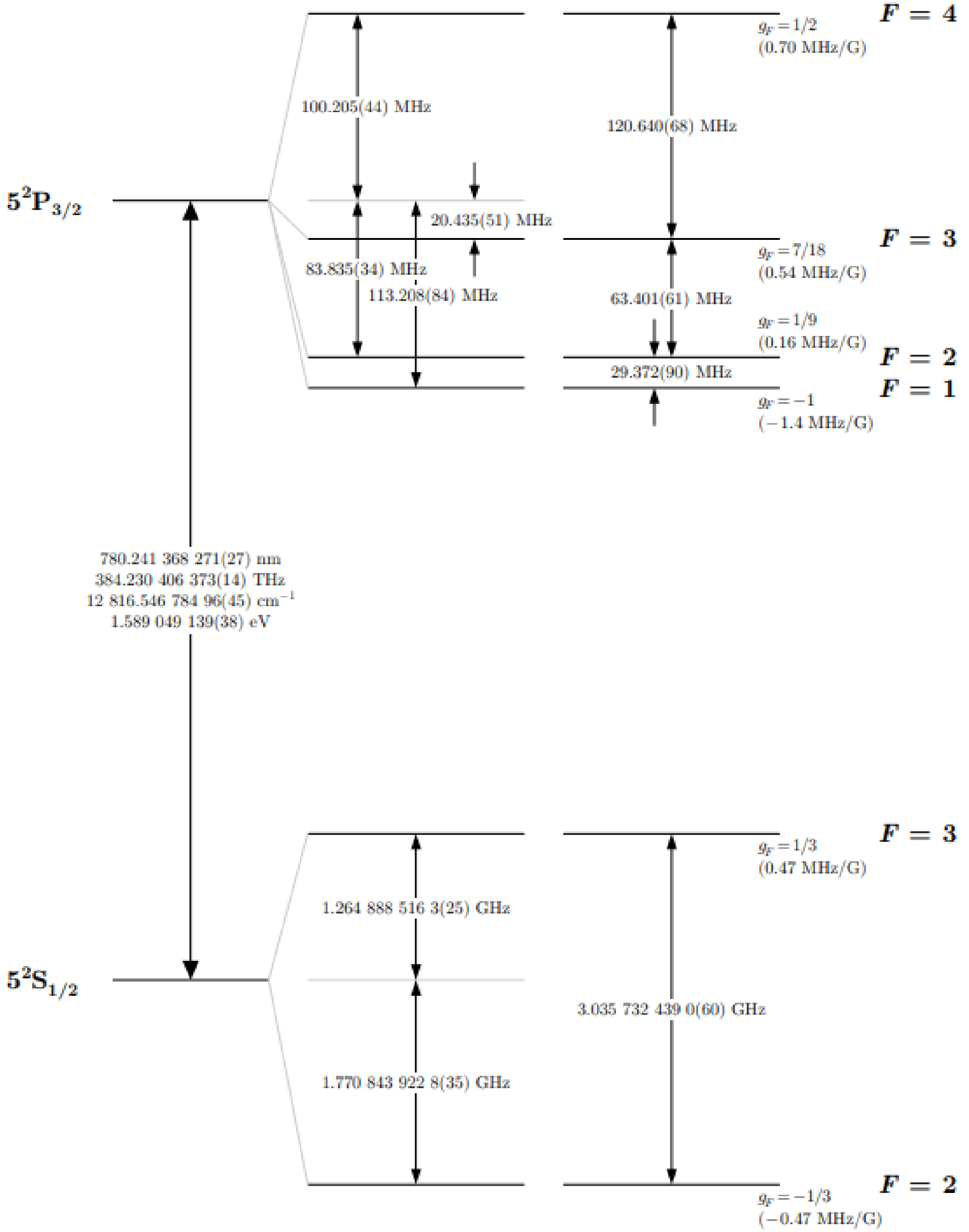


Figure B1. D2 transition in Rubidium 85 hyperfine structure, with splitting between the hyperfine energy levels. [3]

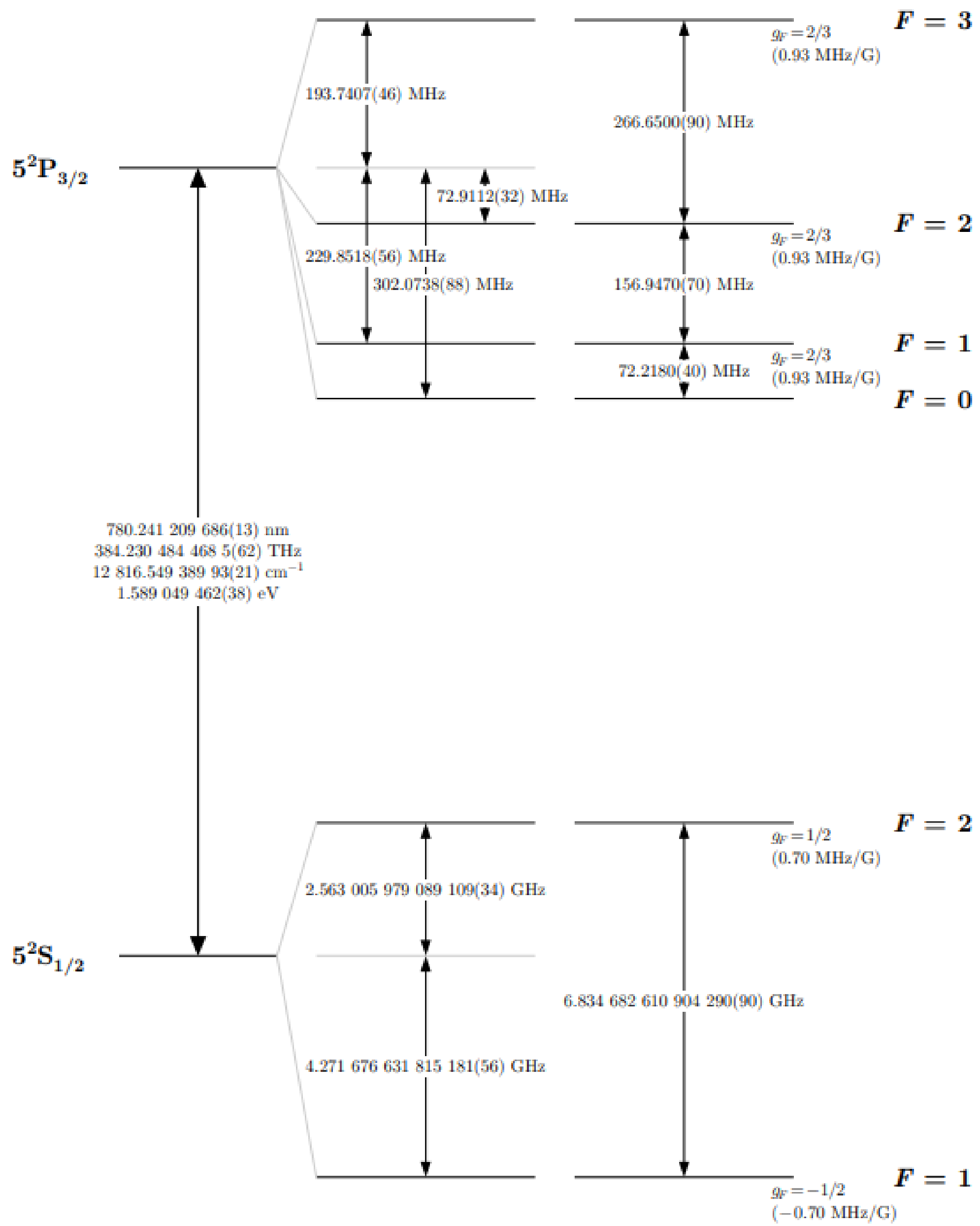


Figure B2. D2 transition in Rubidium 87 hyperfine structure, with splitting between the hyperfine energy levels. [4]

Appendix C – Images of Apparatus used during Experiment

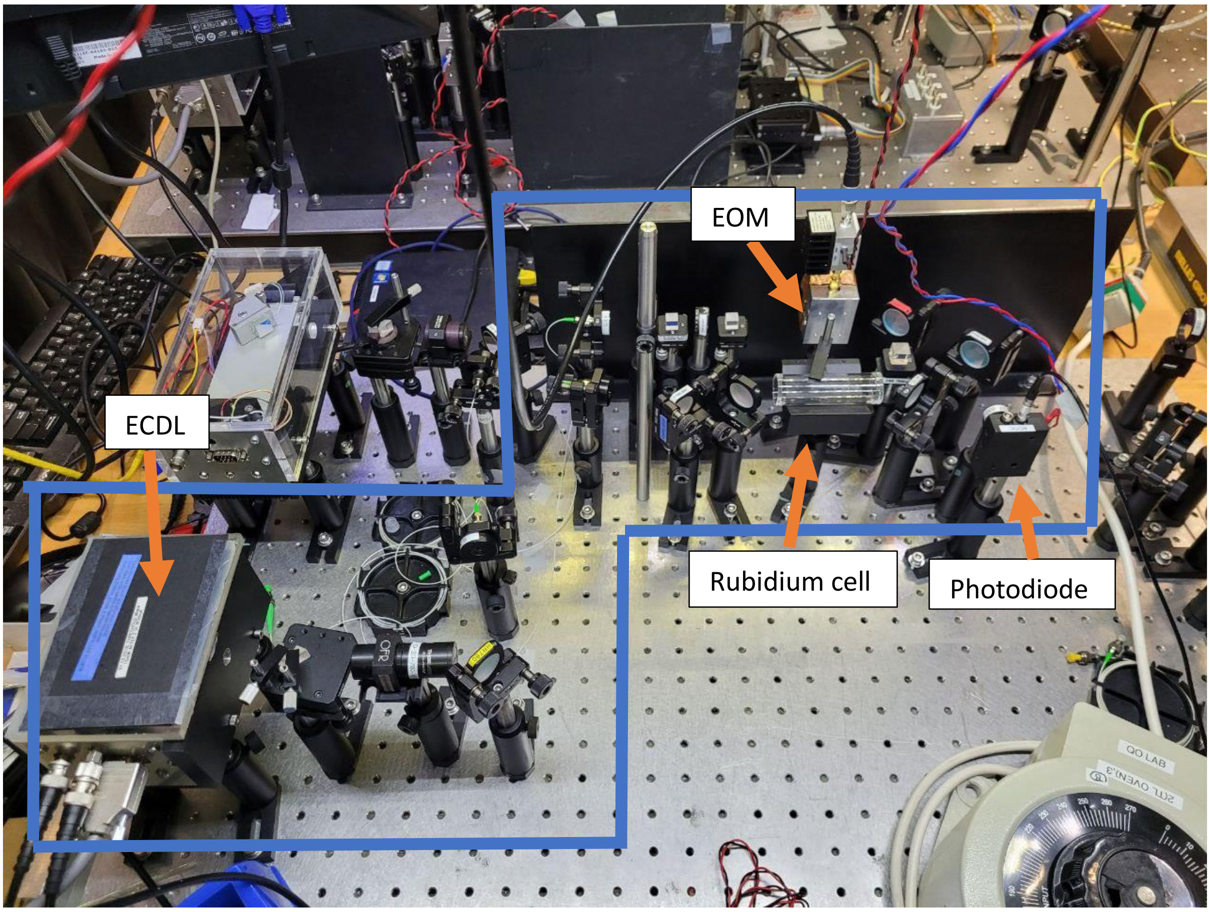


Figure C1. Image of FM Spectroscopy used in this experiment. ECDL, EOM, Rubidium vapour cell, and photodiode are labelled.

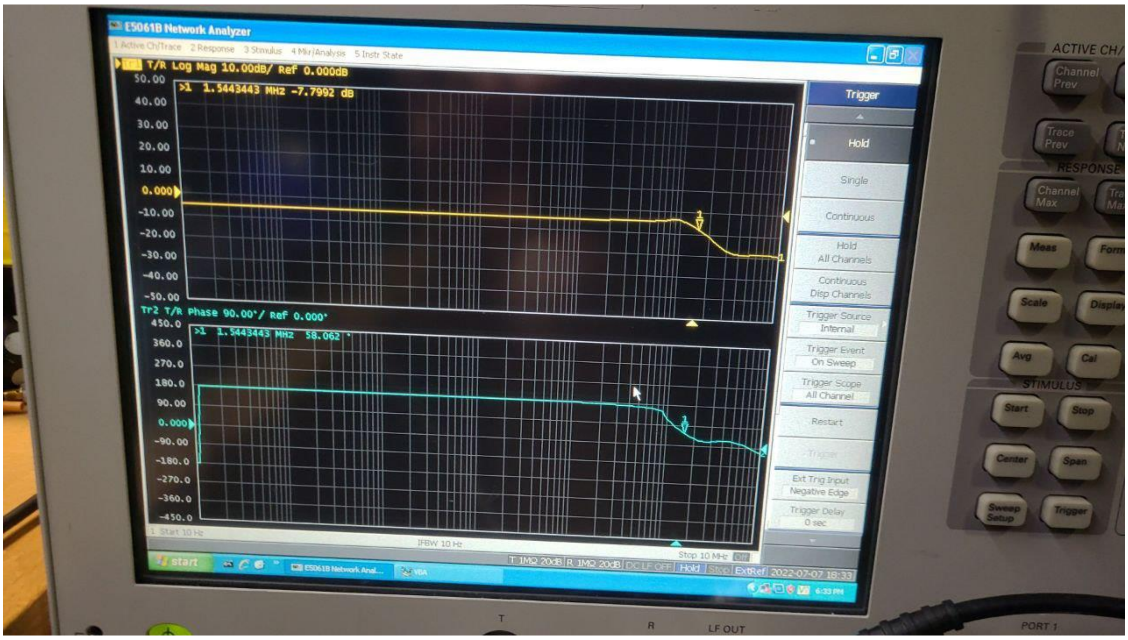


Figure C2. Image of network analyser used to measure the target frequency of our systems.

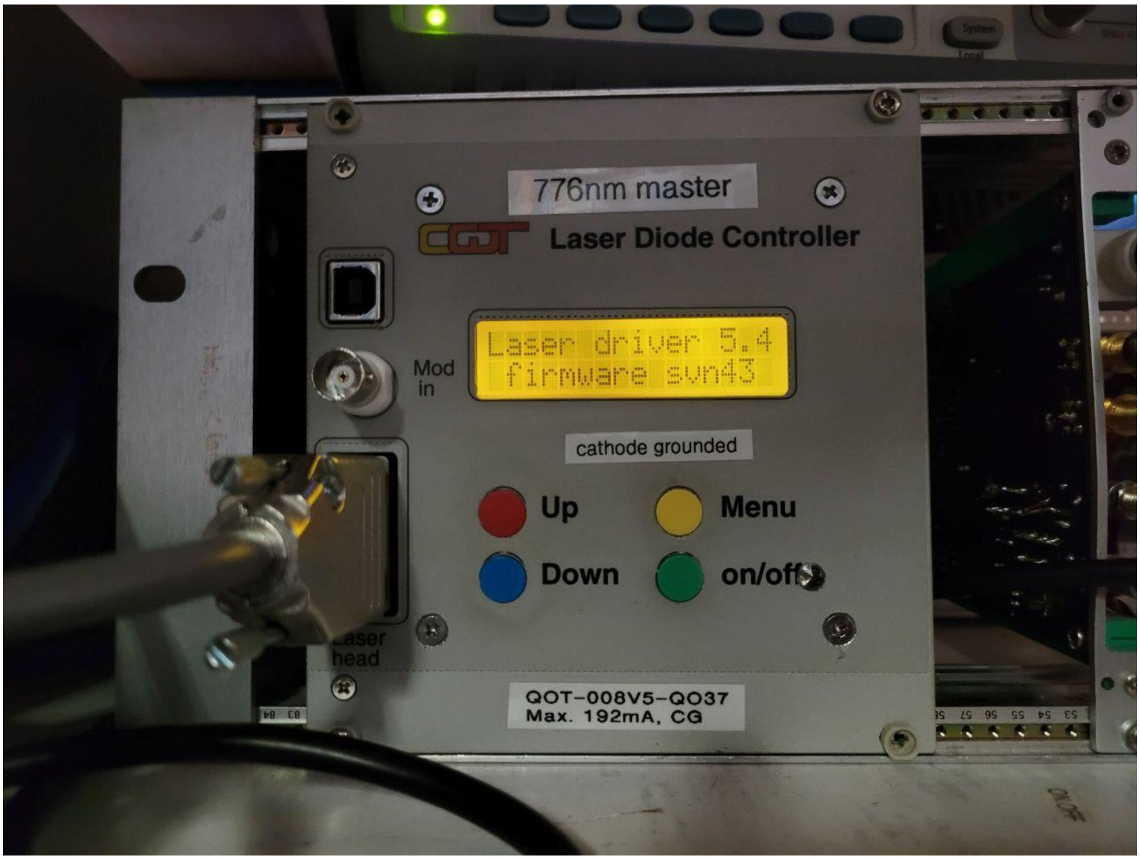


Figure C3. Image of the laser driver

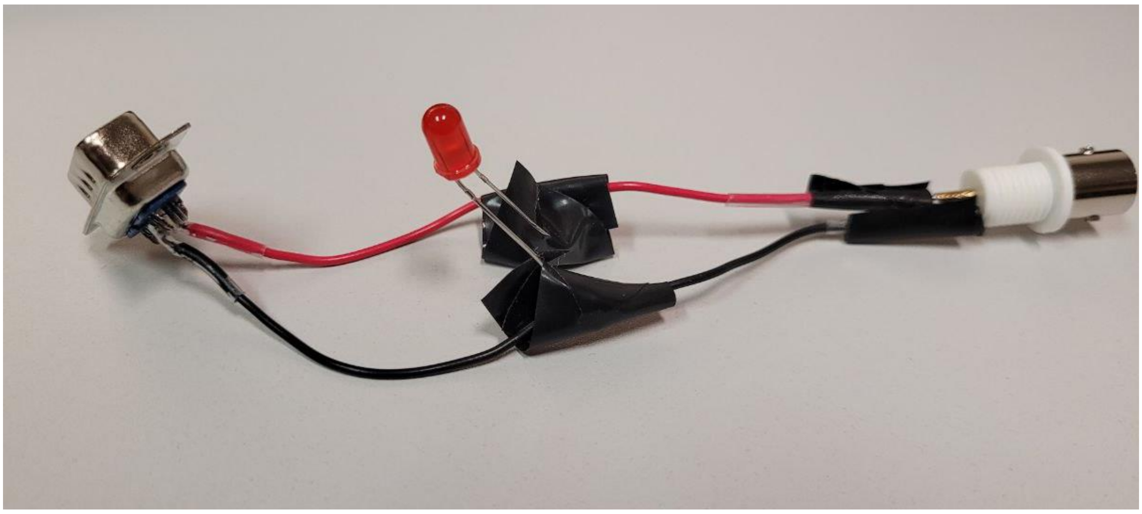


Figure C4. Image of device soldered which consists of a LED connected in a parallel circuit with the BNC connector and 9-pin connector.

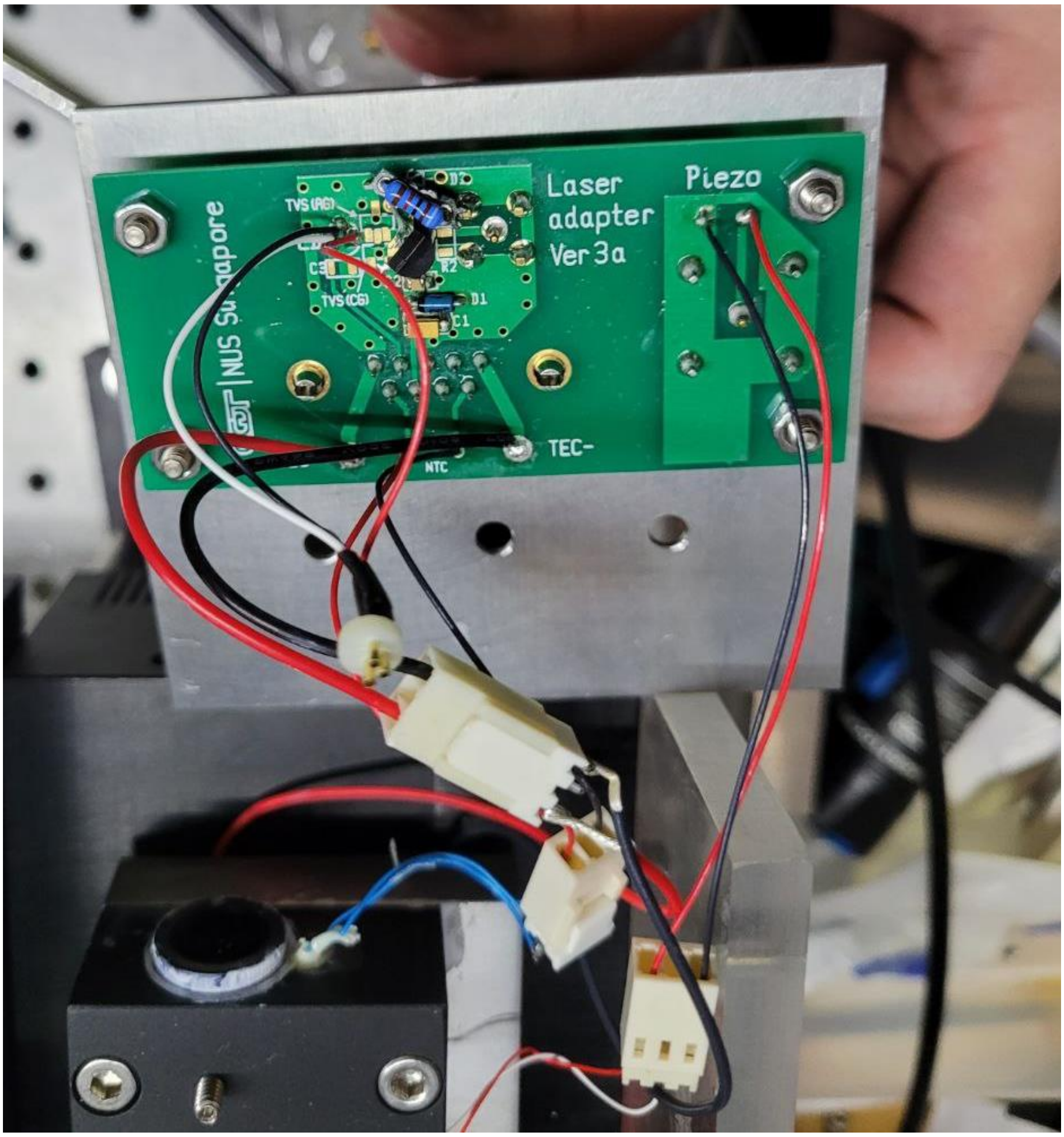


Figure C5. Image of implementation of circuit for wideband current injection into the laser diode

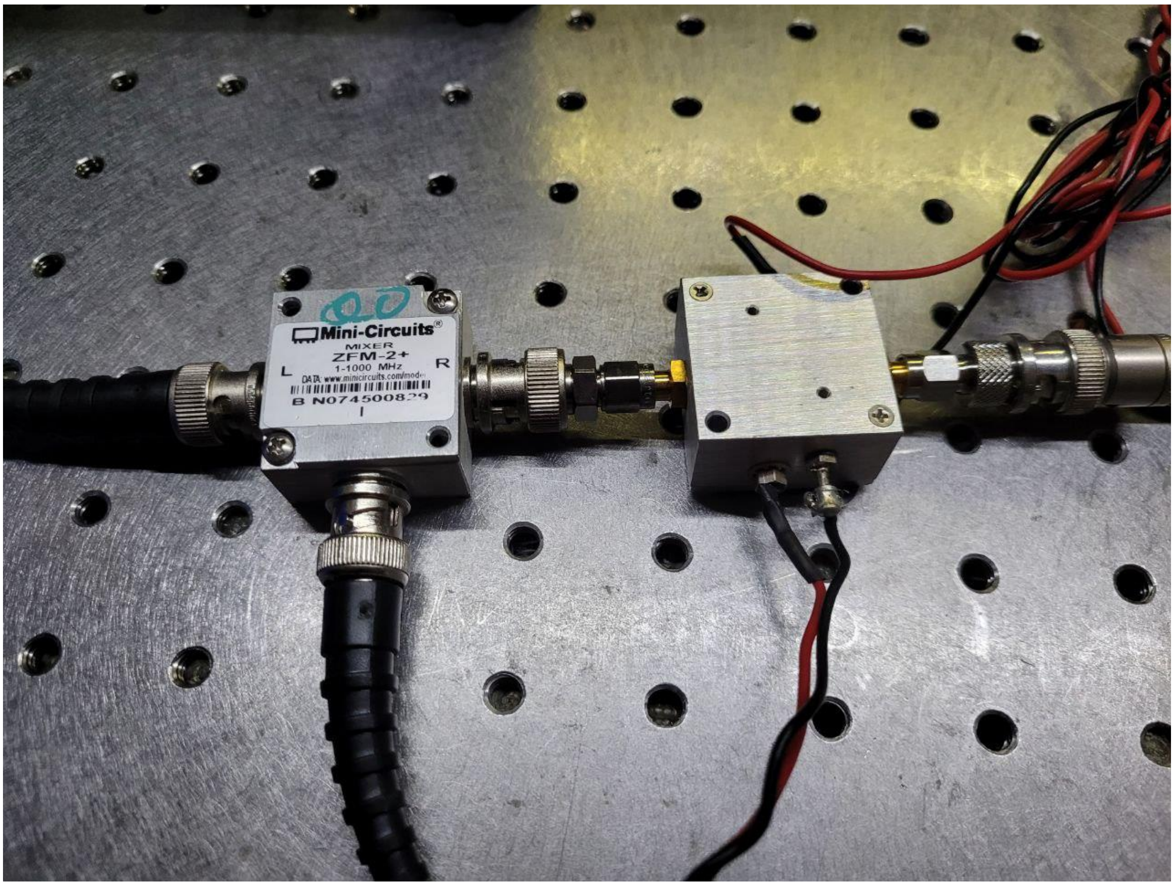


Figure C6. Image of ZFM-2+ mixer (left) and ZFL-1200G+ amplifier (right) used

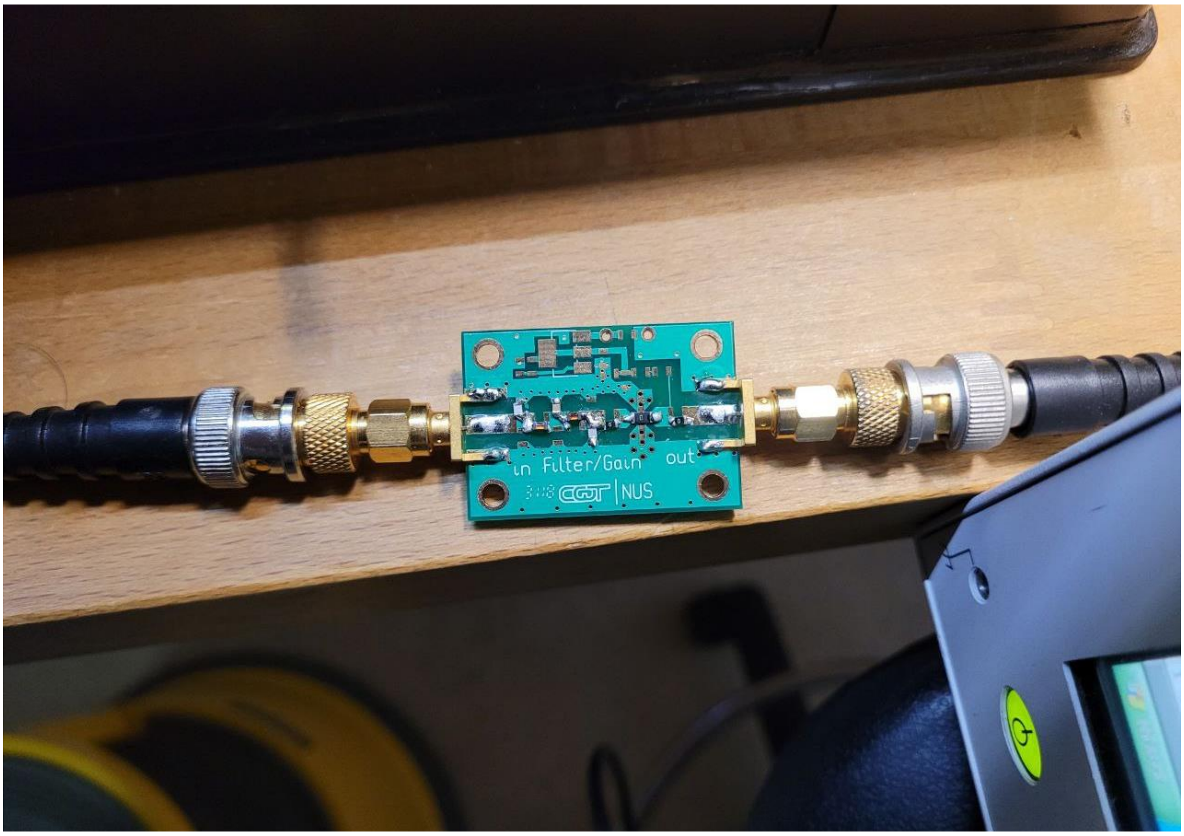


Figure C7. Image of lowpass filter we had built

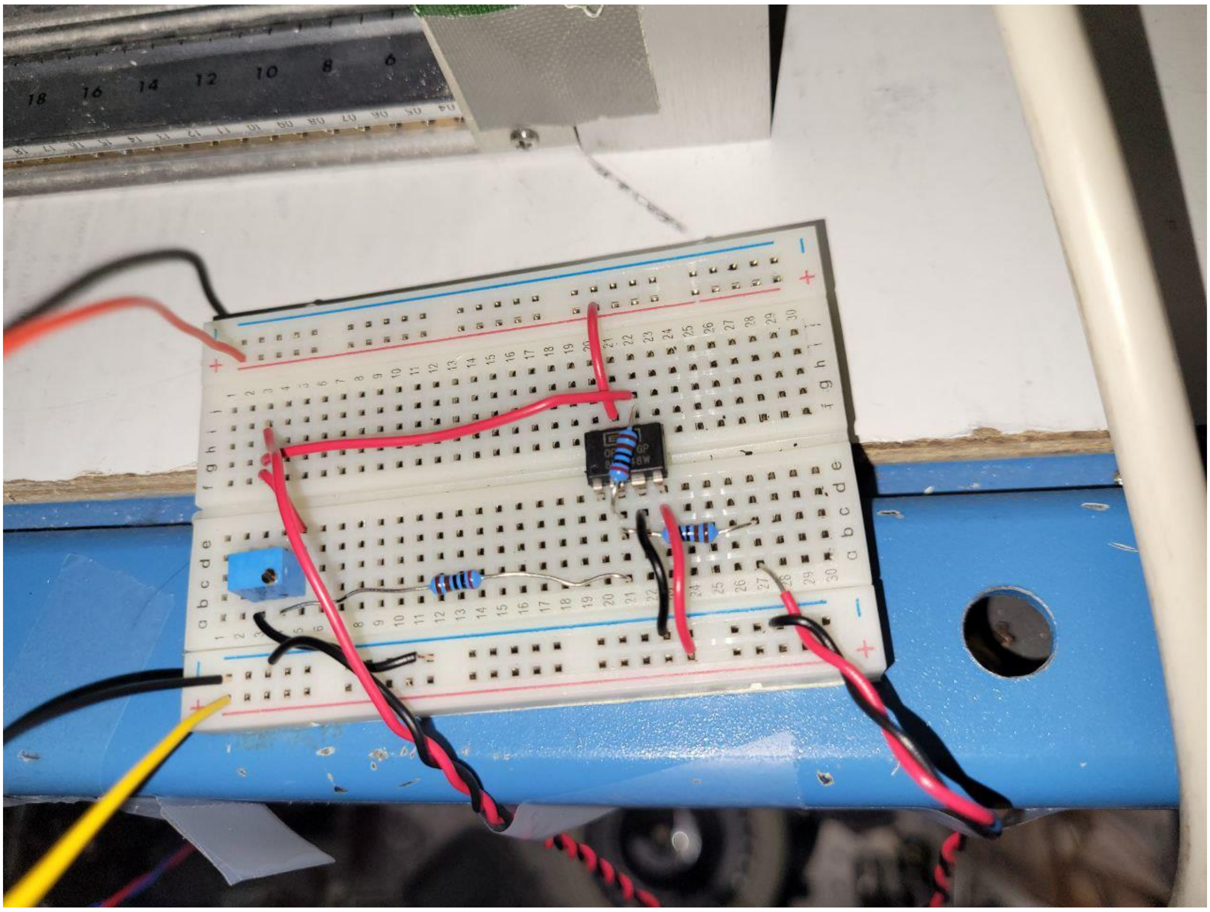


Figure C8. Image of implementation of P control loop on a breadboard

Bibliography

[1] Black, E. D. (2001). An introduction to Pound–Drever–Hall laser frequency stabilization. American Journal of Physics, 69(1), 79–87. https://doi.org/10.1119/1.1286663

[2] Andrews, J., & Dallin, P. (n.d.). Frequency modulation spectroscopy. Spectroscopy Europe, 24–26. https://www.spectroscopyeurope.com/content/frequency-modulation-spectroscopy

[3] Steck, D. A. (2021). Rubidium 85 D Line Data. http://steck.us/alkalidata

[4] Steck, D. A. (2021). Rubidium 87 D Line Data. http://steck.us/alkalidata

[5] McCarron, D. J., King, S. A., & Cornish, S. L. (2008). Modulation transfer spectroscopy in atomic rubidium. Measurement Science and Technology, 19(10), 105601. https://doi.org/10.1088/0957-0233/19/10/105601

[6] Harvey, K. C., & Myatt, C. J. (1991). External-cavity diode laser using a grazing-incidence diffraction grating. Opt. Lett., 16(12), 910–912. https://doi.org/10.1364/OL.16.000910

[7] Bjorklund, G. C. (1980). Frequency-modulation spectroscopy: a new method for measuring weak absorptions and dispersions. Opt. Lett., 5(1), 15–17. https://doi.org/10.1364/OL.5.000015

[8] IVANESCU, M. (2001). 9 - Control. In D. B. Marghitu (Ed.), Mechanical Engineer's Handbook (pp. 611–714). Academic Press. https://doi.org/https://doi.org/10.1016/B978-012471370-3/50010-3

[9] Kurtsiefer, C. (2014). Board revision 4 and 5 [Photograph]. Quantum Optics Wiki. https://qoptics.quantumlah.org/wiki/index.php/FM_spectroscopy_board

[10] Electronics Tutorials. (n.d.). Tau – The Time Constant. https://www.electronics-tutorials.ws/rc/time-constant.html

[11] Network Cable Propagation Delay. (n.d.). Fluke Corporation. https://www.flukenetworks.com/knowledge-base/dtx-cableanalyzer/propagation-delay#:~:text=Propagationdelay%2Cordelay%2Cis,is5.7nS%2Fm

[12] All About Circuits. (n.d.). Low-pass Filters. https://www.allaboutcircuits.com/textbook/alternating-current/chpt-8/low-pass-filters/#loginModal