

# *Substantial scattering of a Weak Coherent Beam by a Single Atom*

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Centre for  
Quantum  
Technologies



# *Atom-Photon interface*

## **Motivation:**

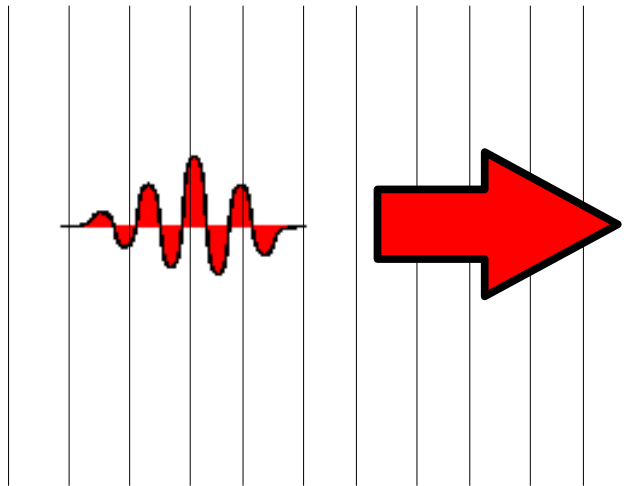
- Atoms and photons are good for different quantum information tasks – allow an exchange of quantum information between them
- Understand elementary interaction between flying qubits and single atoms
- Explore possibilities of controlled phase gates & friends for photonic qubits

## **Key idea:**

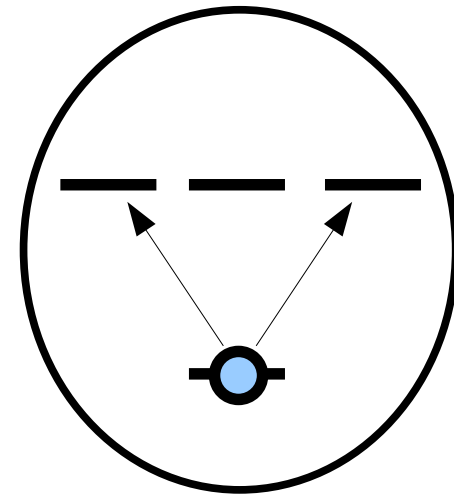
- Try to **mode-match** traveling qubit modes to field modes of spontaneous emission of a single atom

# Why is this interesting?

- e.g. transfer of information from flying qubits into a quantum memory



$$|\Psi_L\rangle = \alpha|L\rangle + \beta|R\rangle$$

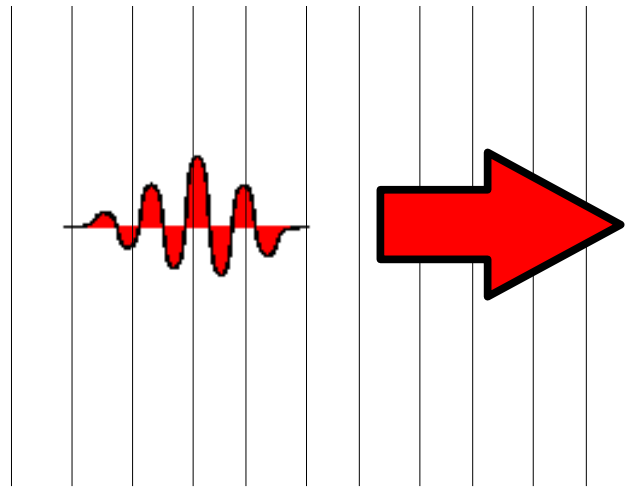


$$|\Psi_A\rangle = \alpha|m=-1\rangle + \beta|m=+1\rangle$$

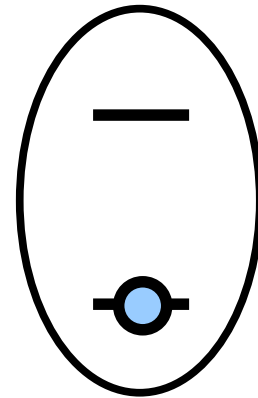
- requires internal states of atom and an **absorption process**

# *The basic problem*

- Get strong coupling between an atom and a light field on the single photon level

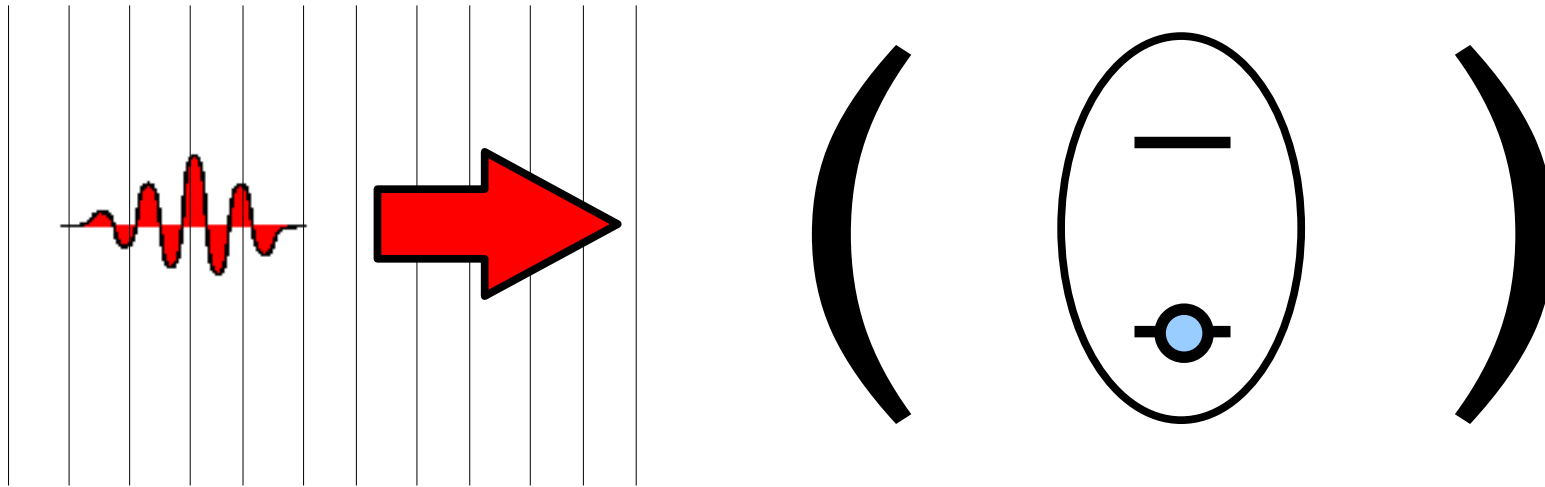


**electromagnetic field / photon**



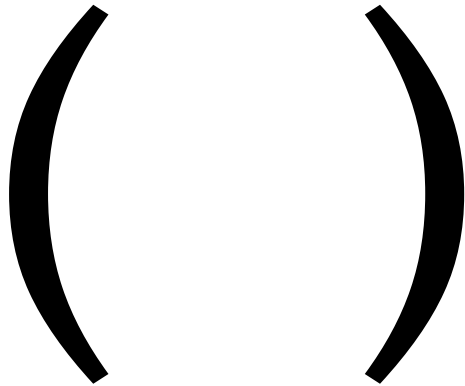
**2-level atom**

# *One solution: Use a cavity*



- High electrical field strength even for a single photon
- Preferred spontaneous emission into the cavity mode
- A cavity can enhance the interaction between a propagating external mode and an atom

# Why cavities are nice



- It's clear what photons in a cavity are  
discrete mode spectrum, 'textbook' energy eigenstates for the electromagnetic field

$$\hat{H}_{field} = \frac{\epsilon_0}{2} \int (\hat{\mathbf{E}}^2 + c^2 \hat{\mathbf{B}}^2) dV = \hbar \omega (\hat{n} + \frac{1}{2})$$

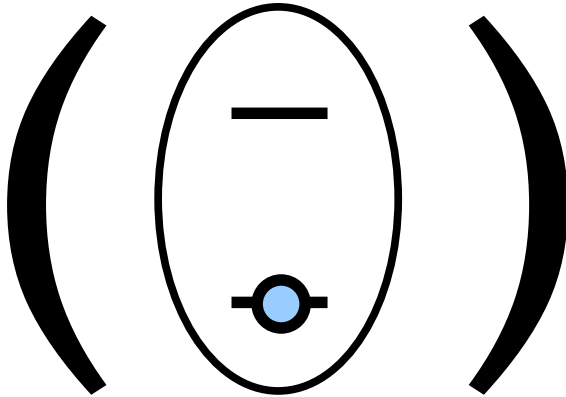
Electrical field operator (single freq):

$$\hat{\mathbf{E}}(x, y, z) = i \sqrt{\frac{\hbar \omega}{2 \pi \epsilon_0 V}} (\underset{\substack{\uparrow \\ \text{mode function, e.g.}}}{\mathbf{g}}(x, y, z) \hat{a}^+ - \mathbf{g}^*(x, y, z) \hat{a})$$

mode function, e.g.

$$\mathbf{g}(x, y, z) = \mathbf{e} \sin kz e^{-\frac{x^2 + y^2}{w^2}}$$

# Atom in a cavity



- atom Hamiltonian

$$\hat{H}_{atom} = E_g |g\rangle\langle g| + E_e |e\rangle\langle e|$$

- electric dipole interaction

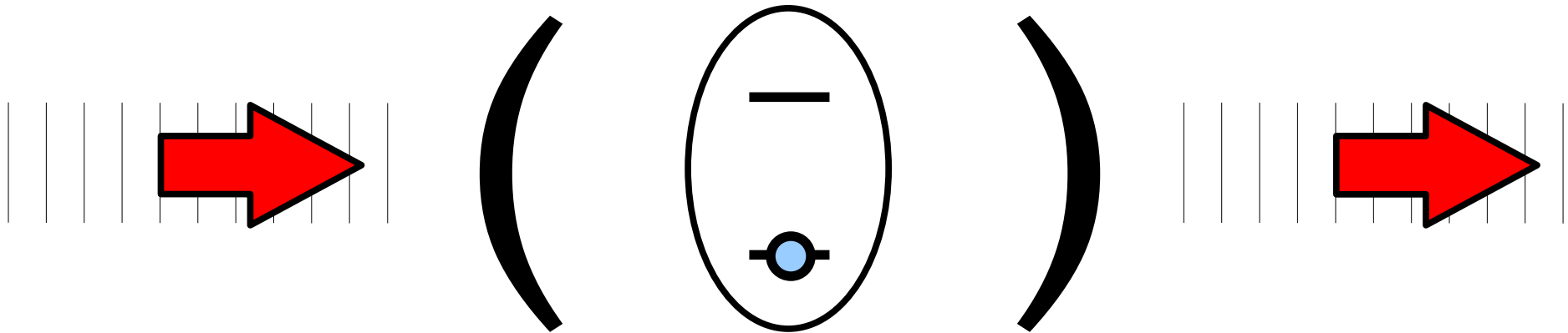
$$\hat{H}_I = \hat{\mathbf{E}} \cdot \hat{\mathbf{d}} \quad \text{with} \quad \hat{\mathbf{d}} = \mathbf{e} d_{eff} (|e\rangle\langle g| + |g\rangle\langle e|)$$

- (treat other field mode as losses)...

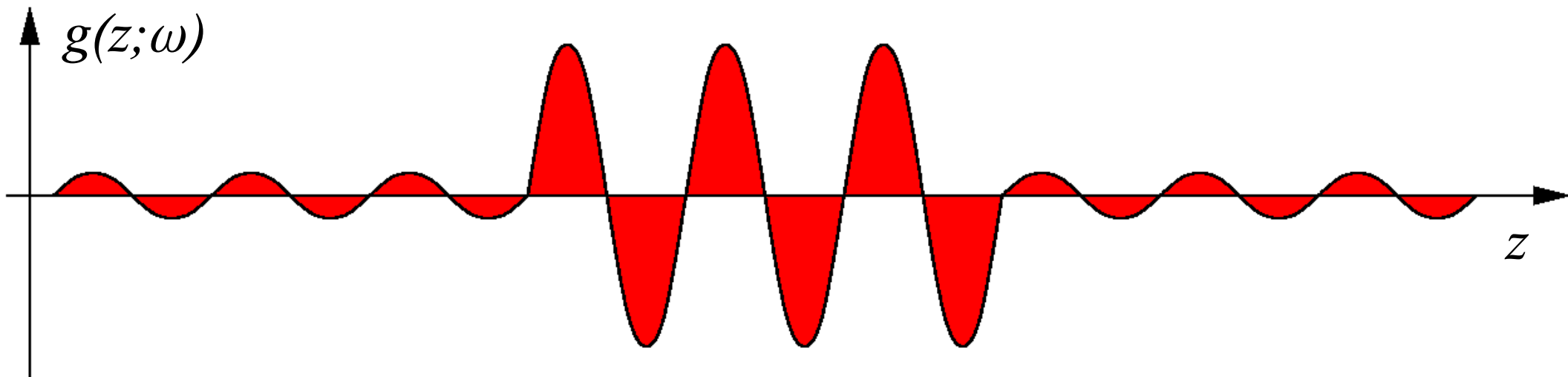
**.....Jaynes-Cummings model with all its aspects**

- treat external fields as perturbation/spectator of internal field

# *External view of cavity+atom*

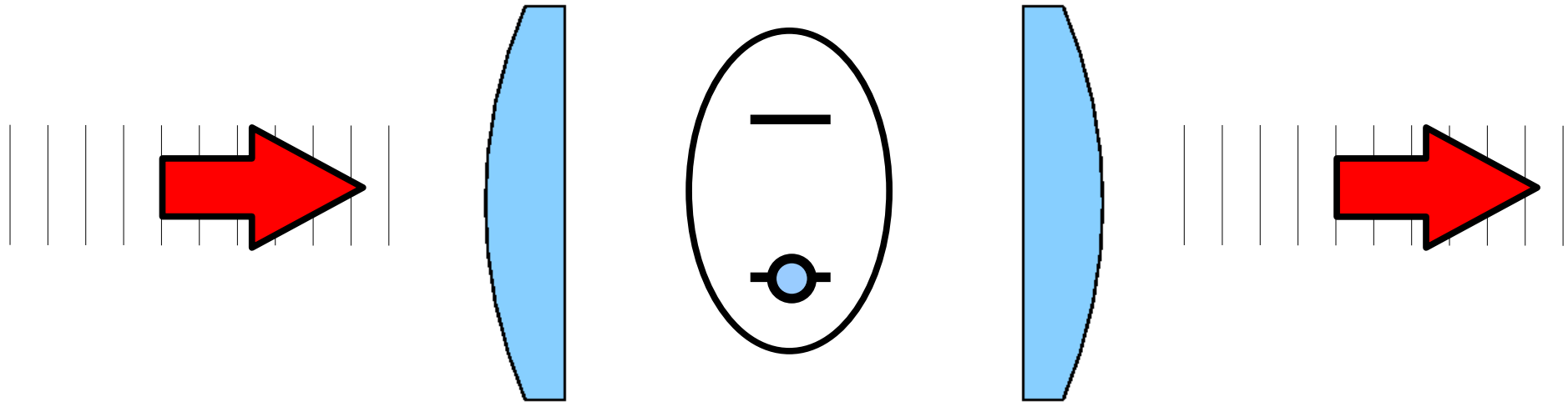


- continuous mode spectrum with enhanced/reduced field mode function:

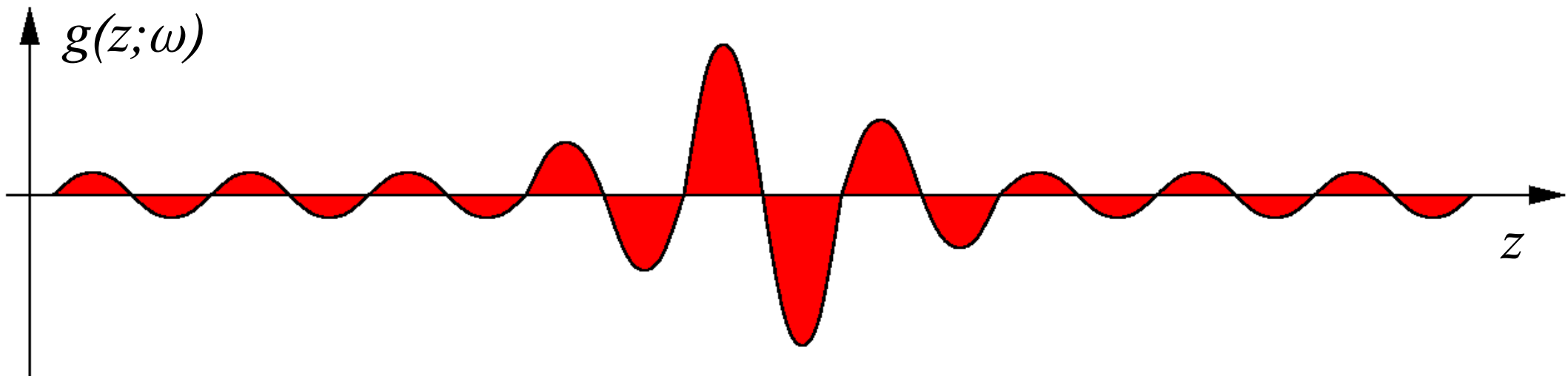




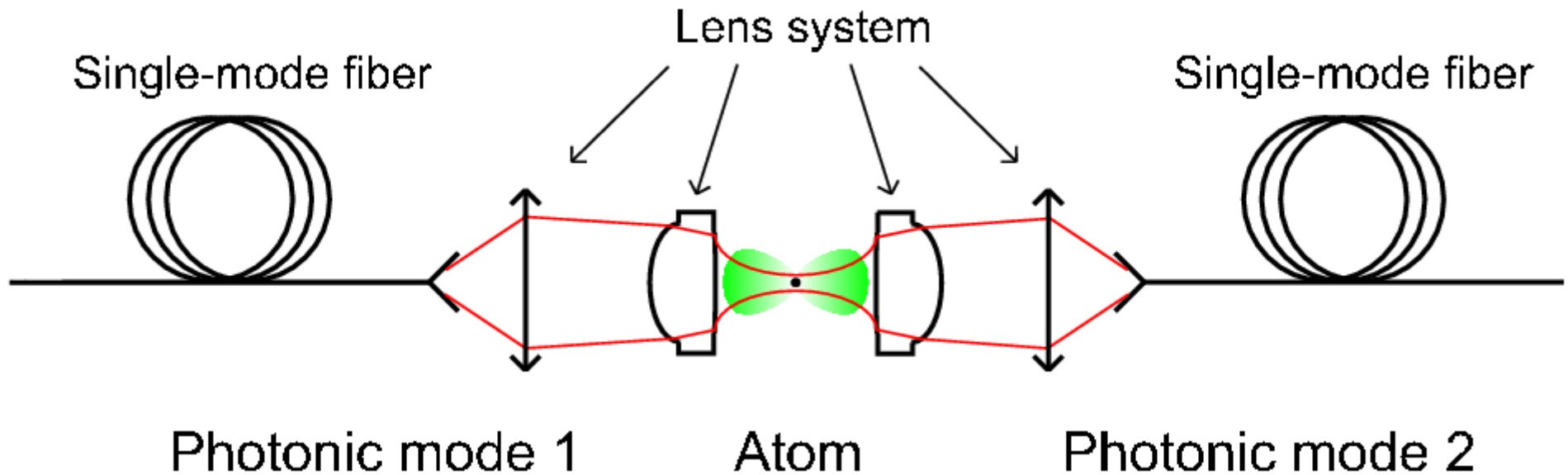
# *An alternative approach*



- use a **focusing lens pair** to enhance center mode function:



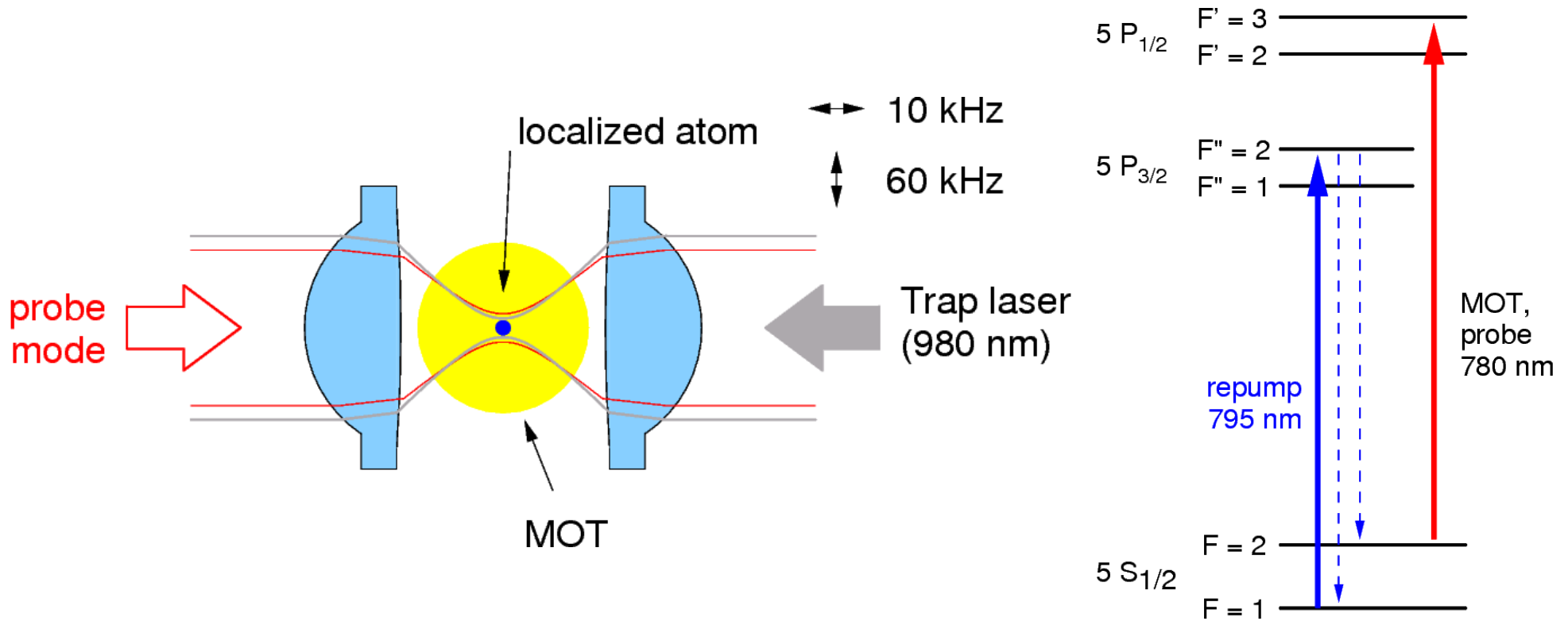
# Concept of an experiment



- achieve a small focal spot
- = high central field amplitude
- = good mode match between atomic emission mode and propagating light field

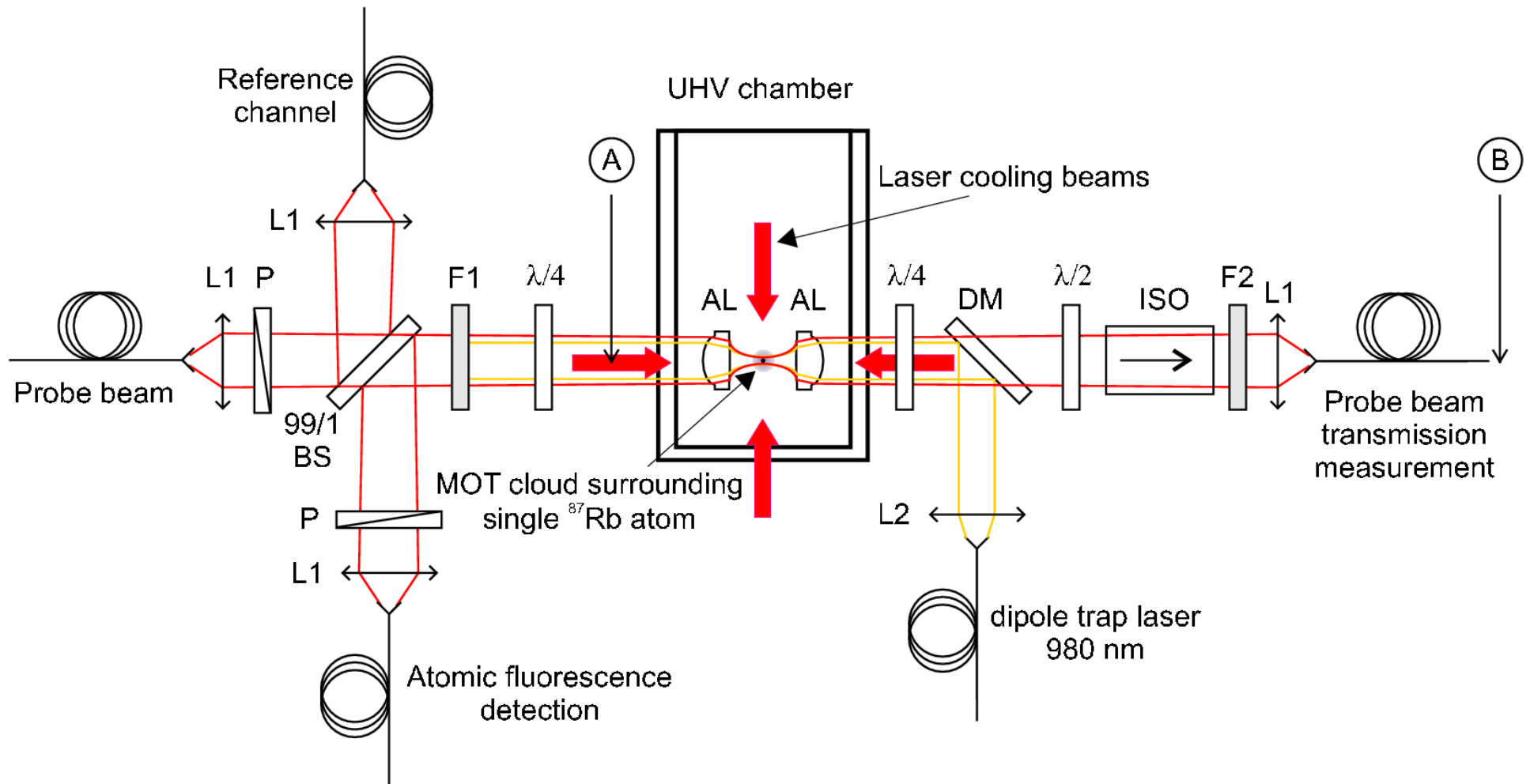
# More technical details

## One atom in an optical dipole trap, loaded from a MOT



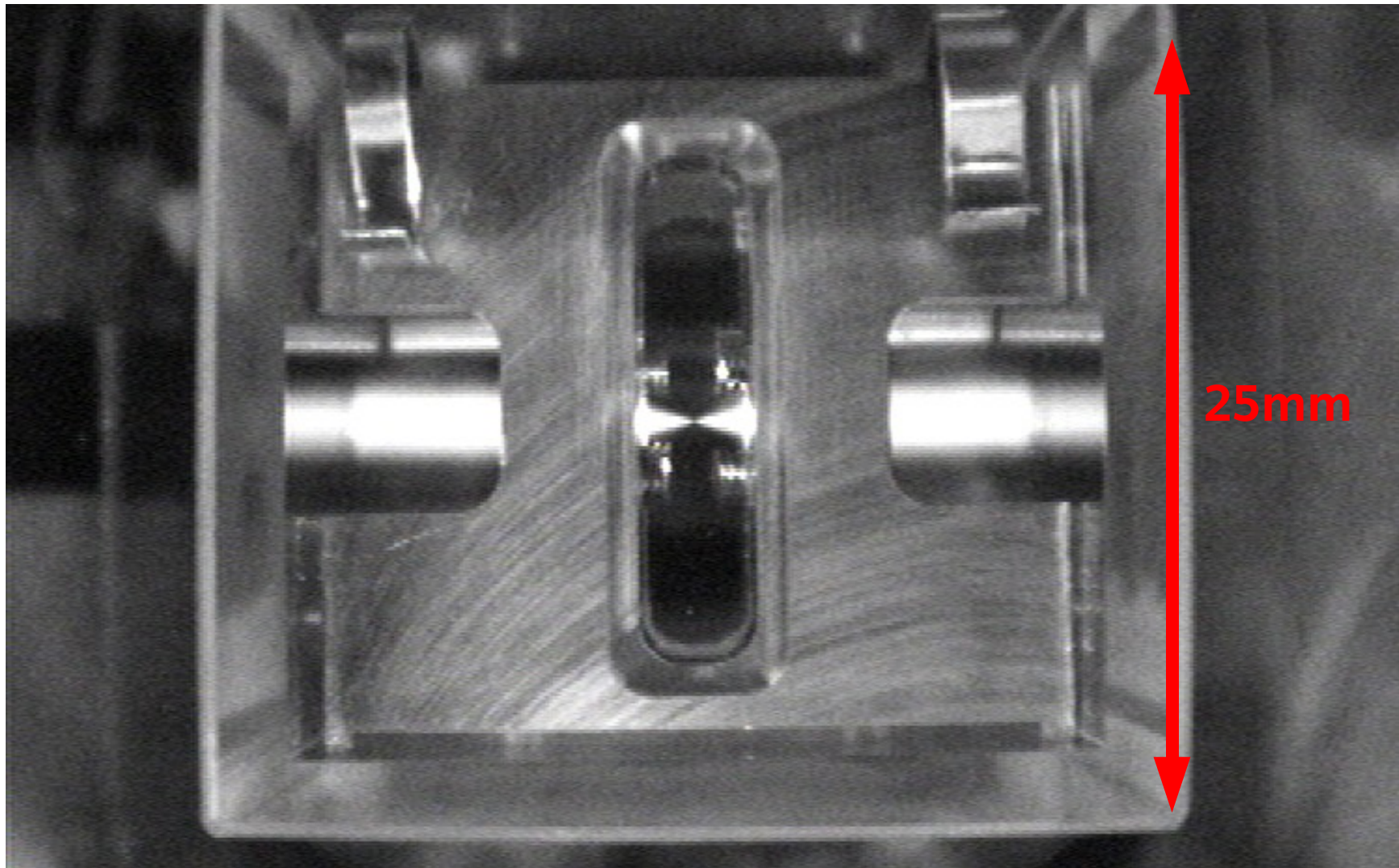
- use Rubidium-87 atom because it is convenient

# *Almost the real exp setup*



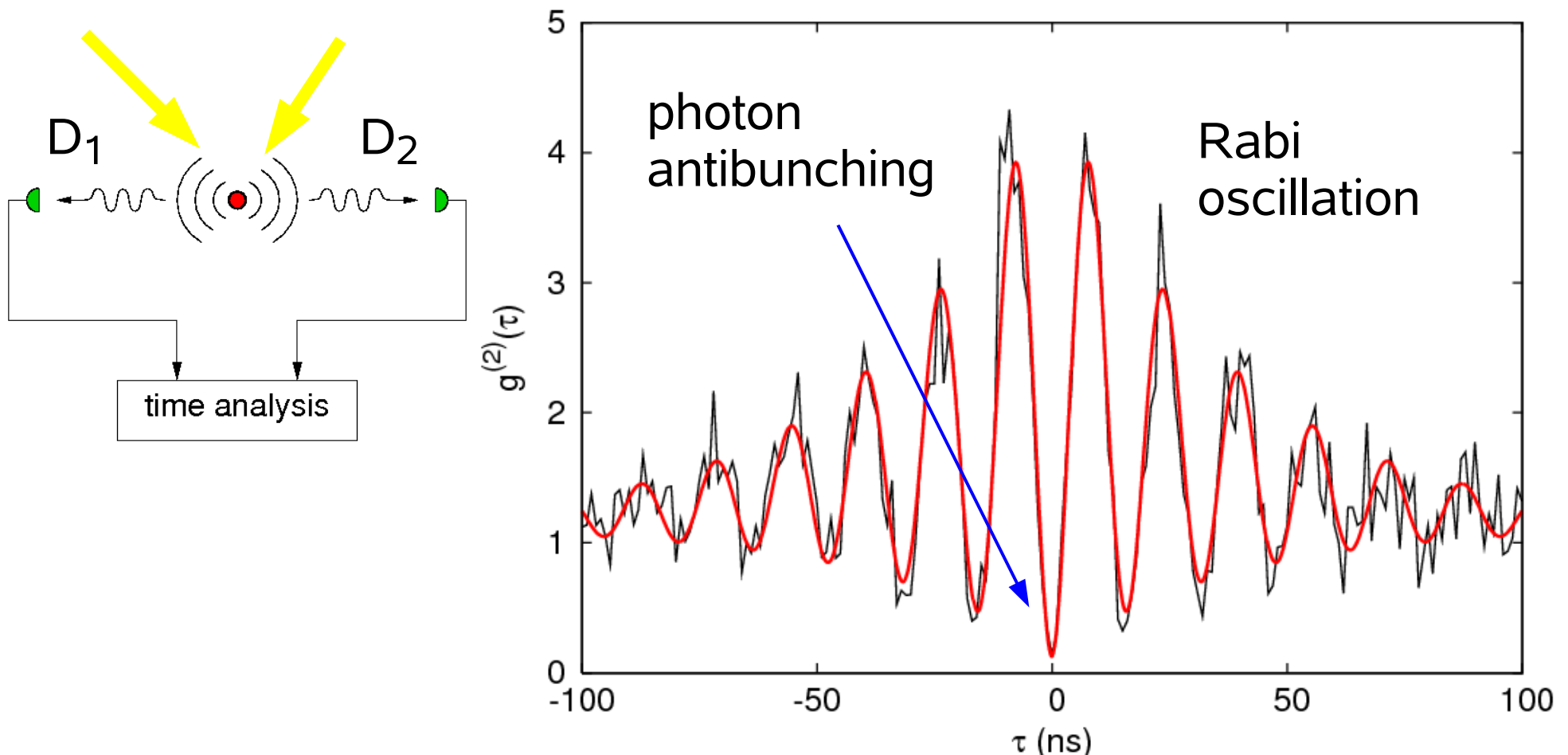
# *Focusing geometry...*

**...as seen by a CCTV camera at high Rb pressure**

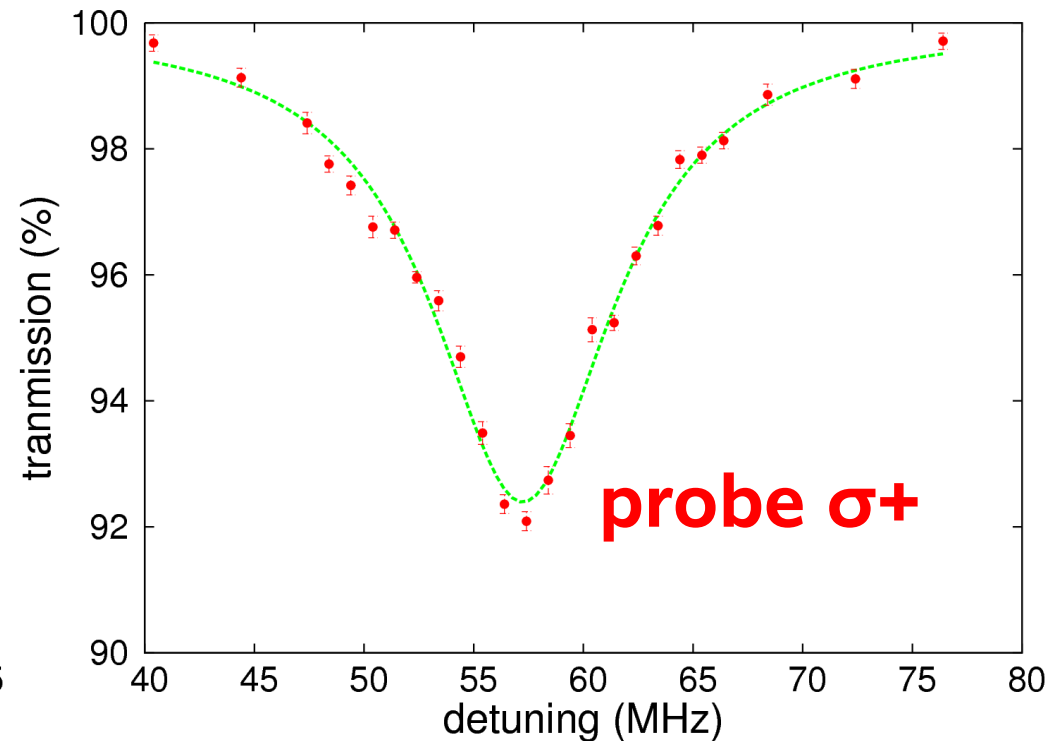
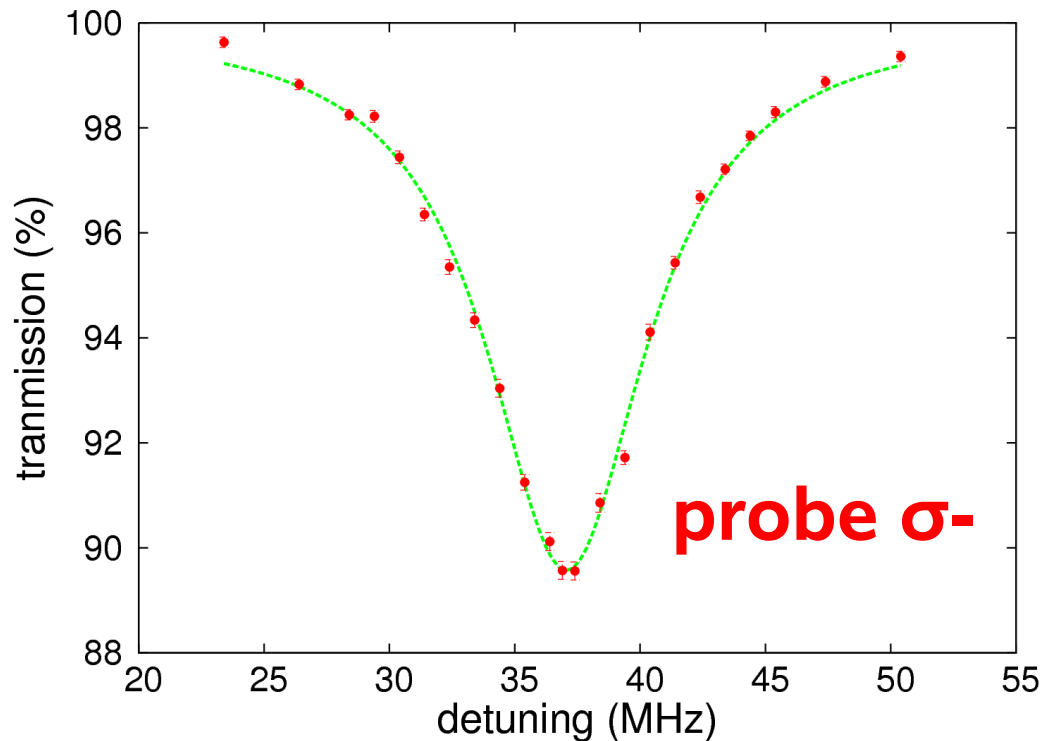


# Single atom evidence

## (almost) Hanbury-Brown—Twiss experiment on atomic fluorescence during cooling

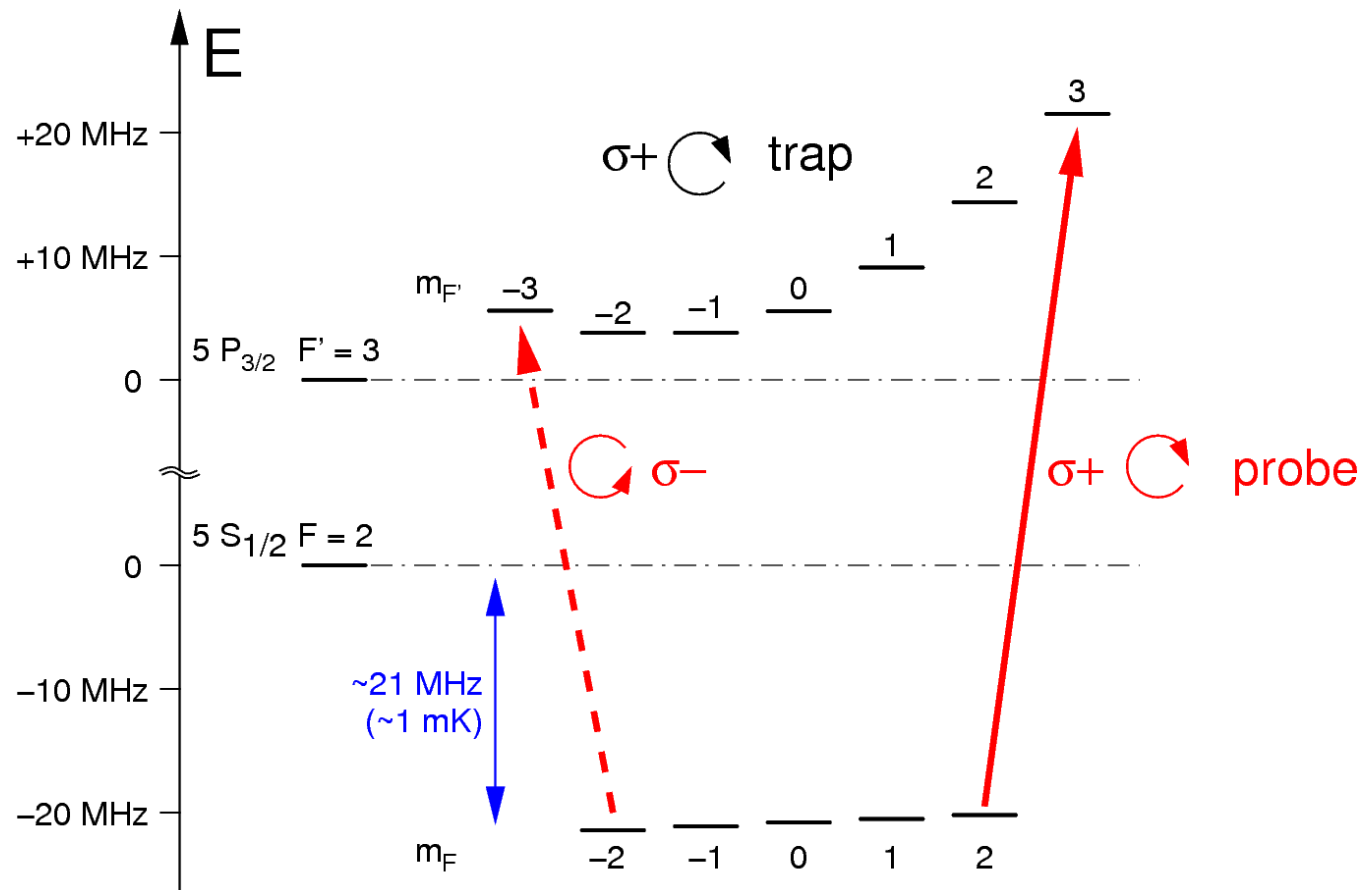


# Transmission results



- almost natural line width of atomic transition
- different resonances for different probe polarizations

# Atomic levels in a dipole trap

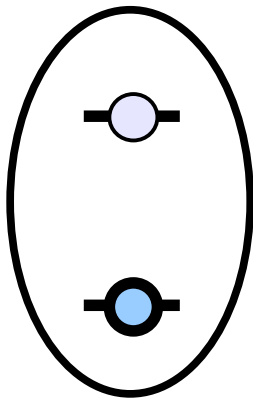


- optically pump with the probe beam into 2-level system



# Step 1: Scattering from an atom

## two - level atom in external driving field (quick & dirty)



- stationary excited state population:

$$\rho_{ee} = \frac{\Omega^2/4}{\delta^2 + \Omega^2/2 + \Gamma^2/4}$$

$$\Omega = E_A |d_{12}| / \hbar \quad \text{Rabi frequency}$$

$$\Gamma = \frac{\omega_{12}^3 d_{12}^2}{3\pi \epsilon_0 \hbar c^3} \quad \text{excited state decay rate}$$

- photon emission rate  $\rho_e \Gamma$
- use this to obtain atomic susceptibility

# *A simple scattering model*

- Electrical field in laser beam before lens

$$\mathbf{E} = E_L \frac{1}{\sqrt{2}} e^{-\frac{\rho^2}{w_L^2}} (\hat{x} \cos \omega t + \hat{y} \sin \omega t)$$

- Total excitation power

$$P_{in} = 1/4 \epsilon \pi c w_l^2 E_L^2$$

- Total power scattered by the atom

$$P_{sc} = 3 \epsilon_0 c \lambda^2 E_A^2 / 4 \pi$$

# Simple model II

- “Scattering ratio”

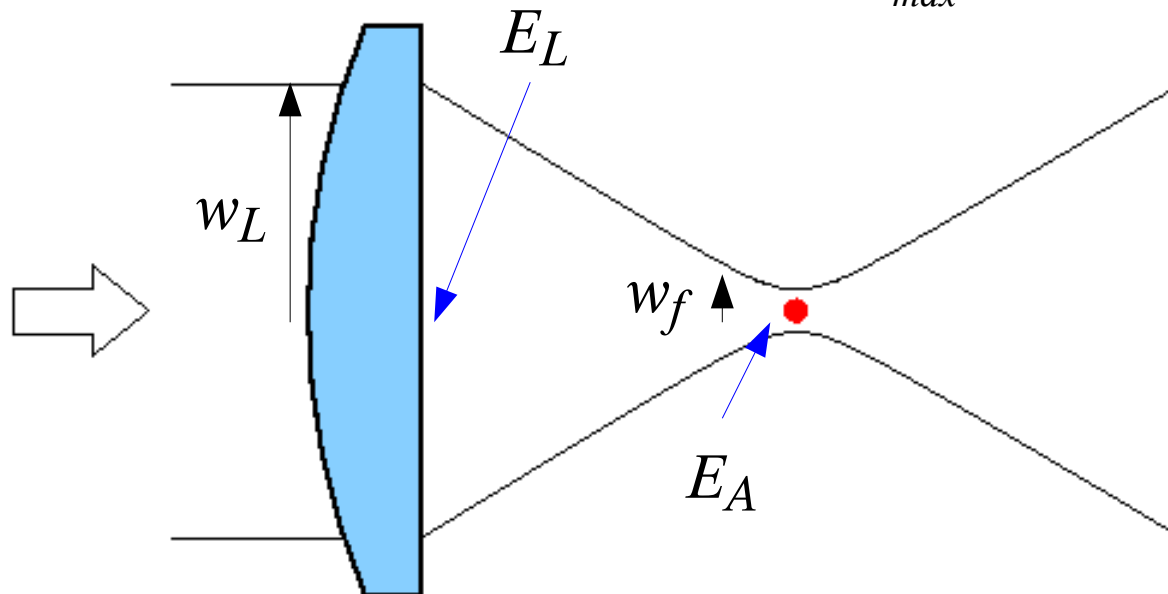
paraxial approximation

$$R_{sc} = \frac{P_{sc}}{P_{in}} = \frac{3\lambda^2}{\pi w_L^2} \left( \frac{E_A}{E_L} \right)^2 \approx \frac{3\lambda^2}{\pi w_f^2} \approx \sigma_{max} / A$$

focal area  
 $A \approx \pi w_f^2 / 2$

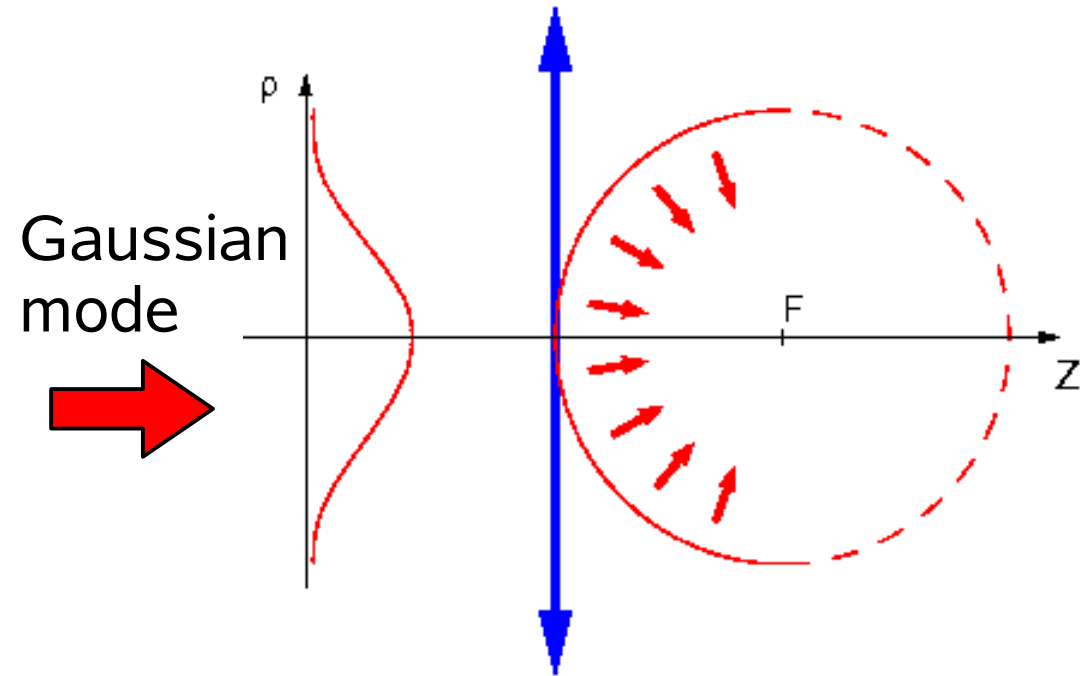
atomic scattering  
cross section

$$\sigma_{max} = 3\lambda^2 / 2\pi$$



# *A more careful model*

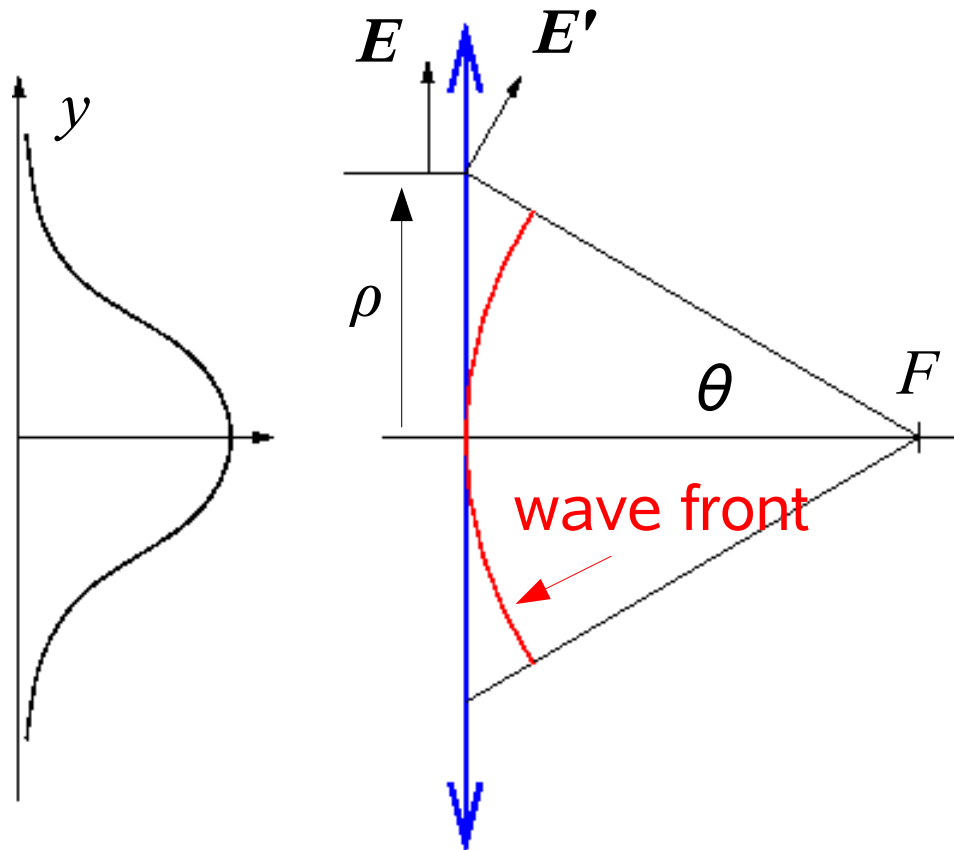
- assume spherical wave front and field compatible with Maxwell equations to get field at atom location
- determine atom response from semiclassical excitation probability for a given field
- combine atom response and original field



## Step 2: Get field in focus

### Action of an ideal lens on a collimated, circularly polarized Gaussian beam

$$E = E_L \hat{e}_+ e^{-\frac{\rho^2}{w_l^2}}$$



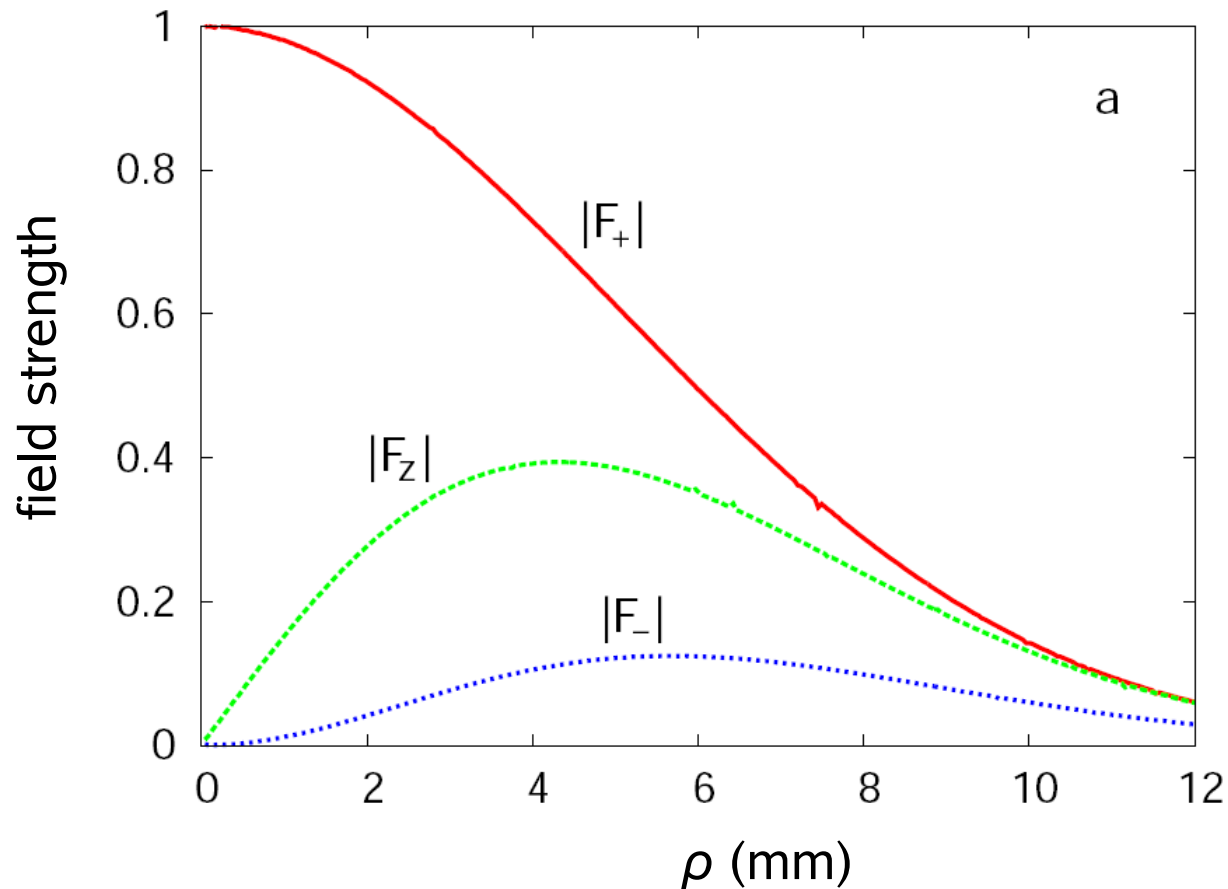
- spherical wave front
- locally transverse
- conserve power through each small area

(Richardson, Wolf, 1959)

(cyl. coordinates)

# Directly after lens:

$$\mathbf{E}' = E_L e^{-\frac{\rho^2}{w_l^2}} \frac{1}{\sqrt{\cos \theta}} \times e^{-ik\sqrt{\rho^2 + f^2}} \times \left( \frac{1 + \cos \theta}{2} \hat{\mathbf{e}}_+ + \frac{\sin \theta e^{i\phi}}{\sqrt{2}} \hat{\mathbf{z}} + \frac{\cos \theta - 1}{2} e^{2i\phi} \hat{\mathbf{e}}_- \right)$$



- different polarization components appear

beam parameter:  
 $w_l = 7$  mm

focal length:  
 $f = 4.5$  mm

# *Propagate field to focus*

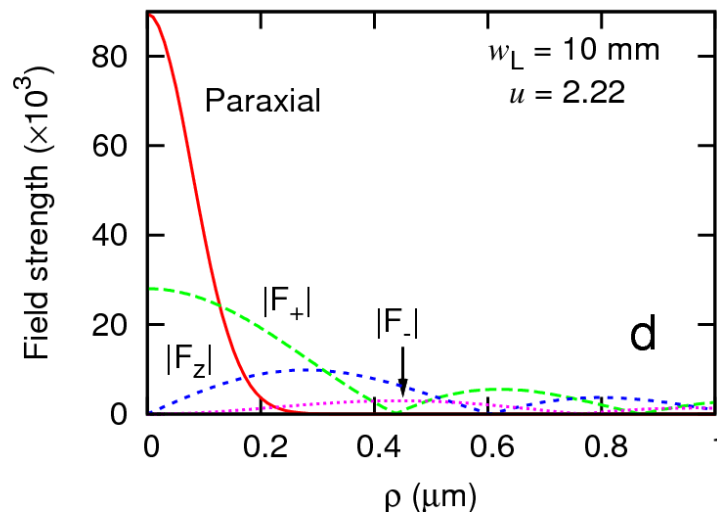
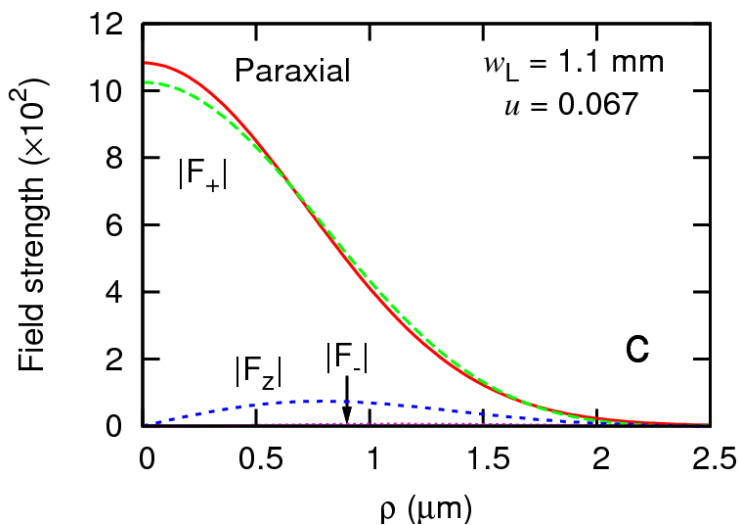
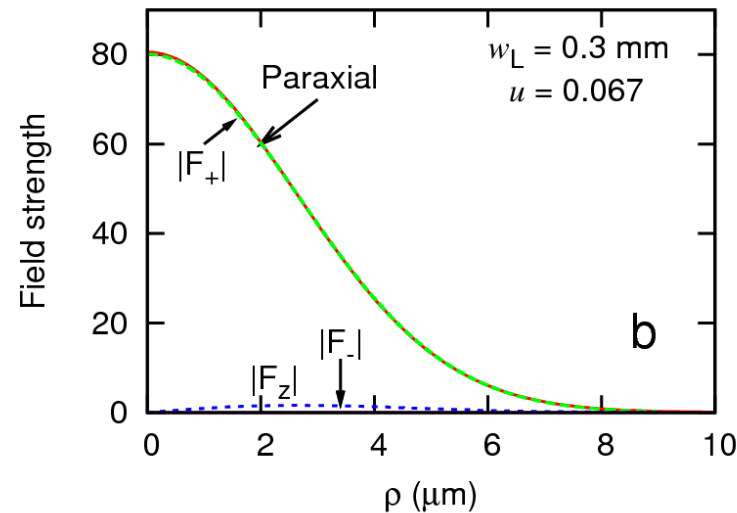
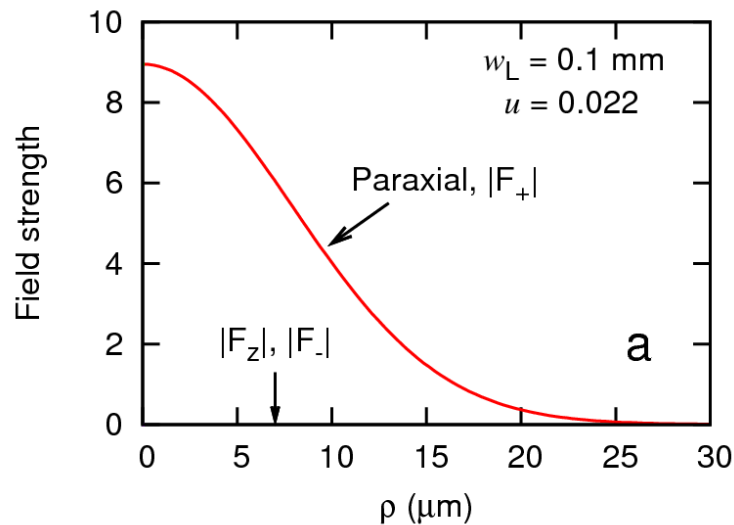
- numerical method:
  - decompose initial field into cylindrical harmonics
  - analytically propagate to focus, allows to obtain field around focal point
- closed expression for field at focus via Green theorem

$$\mathbf{E}(z=f, \rho=0) = \sqrt{\frac{\pi P_{in}}{\epsilon_0 c \lambda^2}} \cdot \frac{1}{u} e^{1/u^2} \left[ \sqrt{\frac{1}{u}} \Gamma\left(-\frac{1}{4}, \frac{1}{u^2}\right) + \sqrt{u} \Gamma\left(\frac{1}{4}, \frac{1}{u^2}\right) \right] \hat{\mathbf{e}}_+$$

with focusing strength  $u := w_L / f$

# Focal fields for different $w_L$

- paraxial approximation starts to break down late...



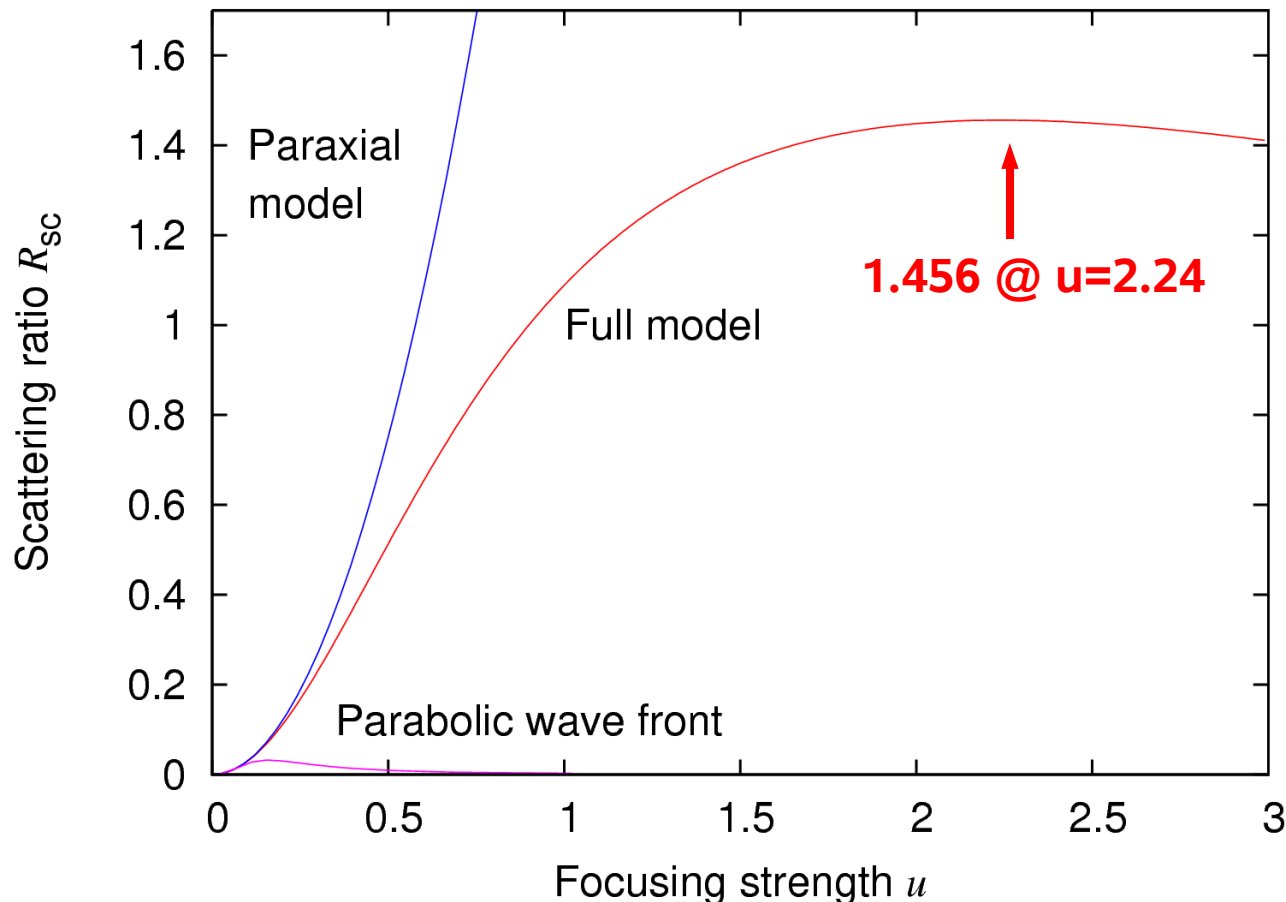
( $f = 4.5$  mm)



# Ooops...strange scattering?

- scattering ratio like in plane wave excitation mode:

$$R_{sc} := \frac{P_{sc}}{P_{in}} = \frac{3}{4u^3} e^{-2/u^2} \left[ \Gamma\left(-\frac{1}{4}, \frac{1}{u^2}\right) + u \Gamma\left(\frac{1}{4}, \frac{1}{u^2}\right) \right]^2$$



???

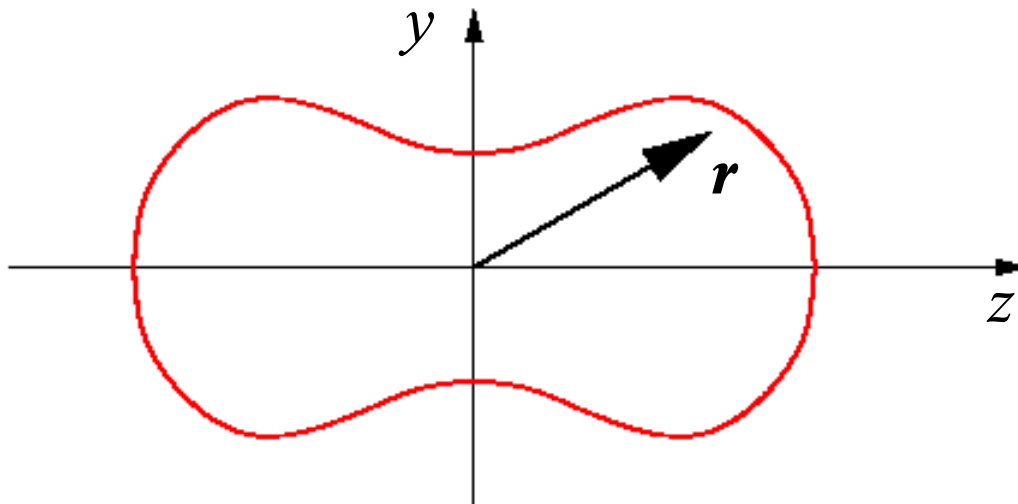
# Atomic response II

**scattered field has electric dipole characteristic  
corresponding to  $\sigma^+$  transition**

$$\mathbf{E}_{sc}(\mathbf{r}) = E_A \frac{3}{2} \frac{e^{ikr + \pi/2}}{kr} [\hat{\epsilon}_+ - (\hat{\epsilon}_+ \cdot \hat{r}) \hat{r}]$$

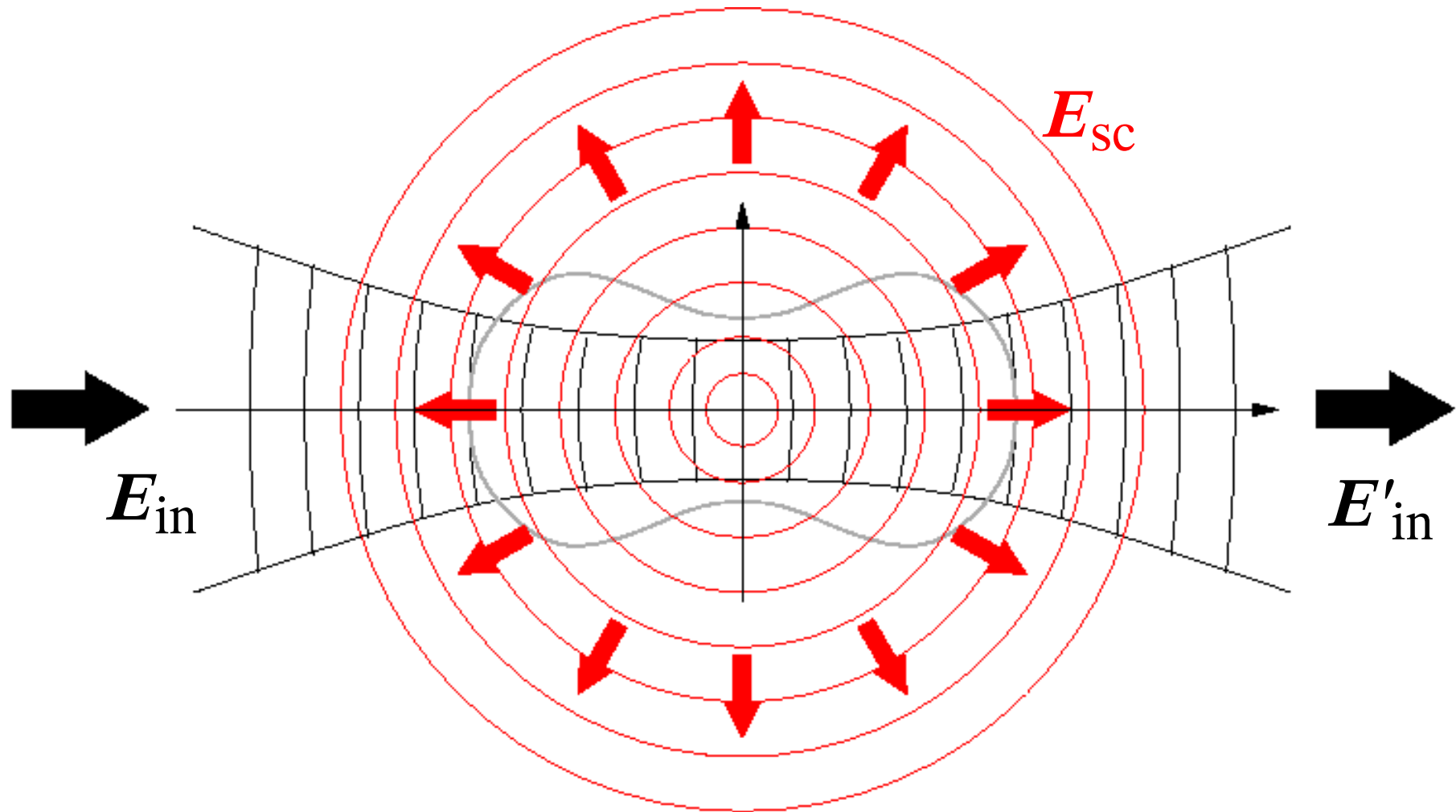
$$\hat{r} = \frac{1}{|\mathbf{r}|} \mathbf{r} \quad \text{radial unit vector}$$

$$\hat{\epsilon}_+ = \frac{\hat{x} + i\hat{y}}{\sqrt{2}} \quad \text{circular unit vector}$$



# Step 3: Combine with probe

$$E_{\text{Tot}} = E_{\text{in}} + E_{\text{sc}}$$



# Collection into Gaussian mode

- Project total field onto Gaussian mode of collection fiber

$$P_{out} = \left| \langle \vec{g}, \vec{E}_{Tot} \rangle \right|^2 \quad \langle \vec{g}, \vec{E} \rangle := \int_{\vec{x} \in S} \vec{E}_{Tot}(\vec{x}) \cdot \vec{g}(\vec{x}) (\vec{k}_g \cdot \vec{n}) dA$$

- Forward transmission: cross section fiber mode

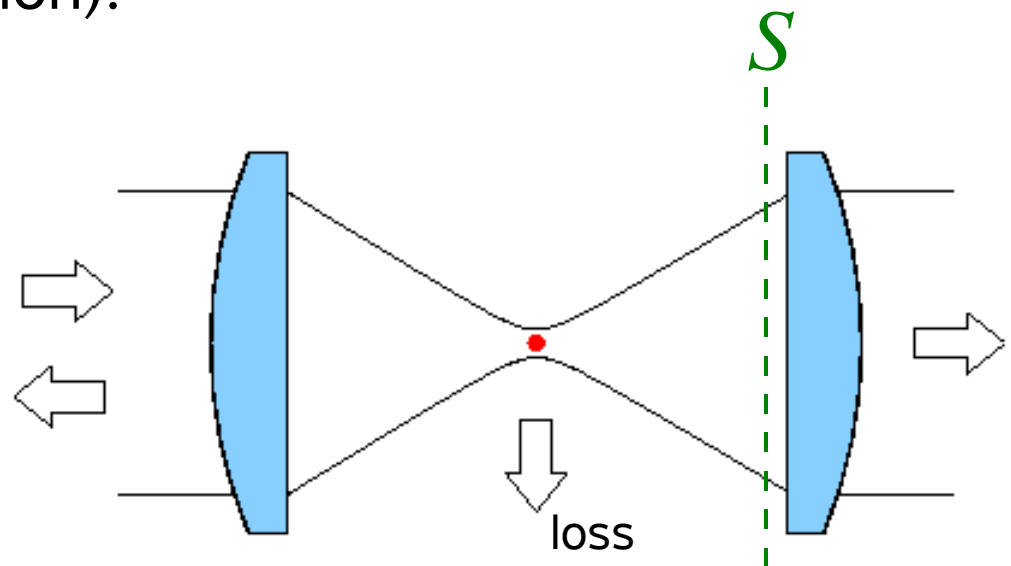
$$T = 1 - \epsilon = \frac{P_{out}}{P_{in}} = \left| 1 - \frac{R_{sc}}{2} \right|^2$$

- Reflectivity (backward direction):

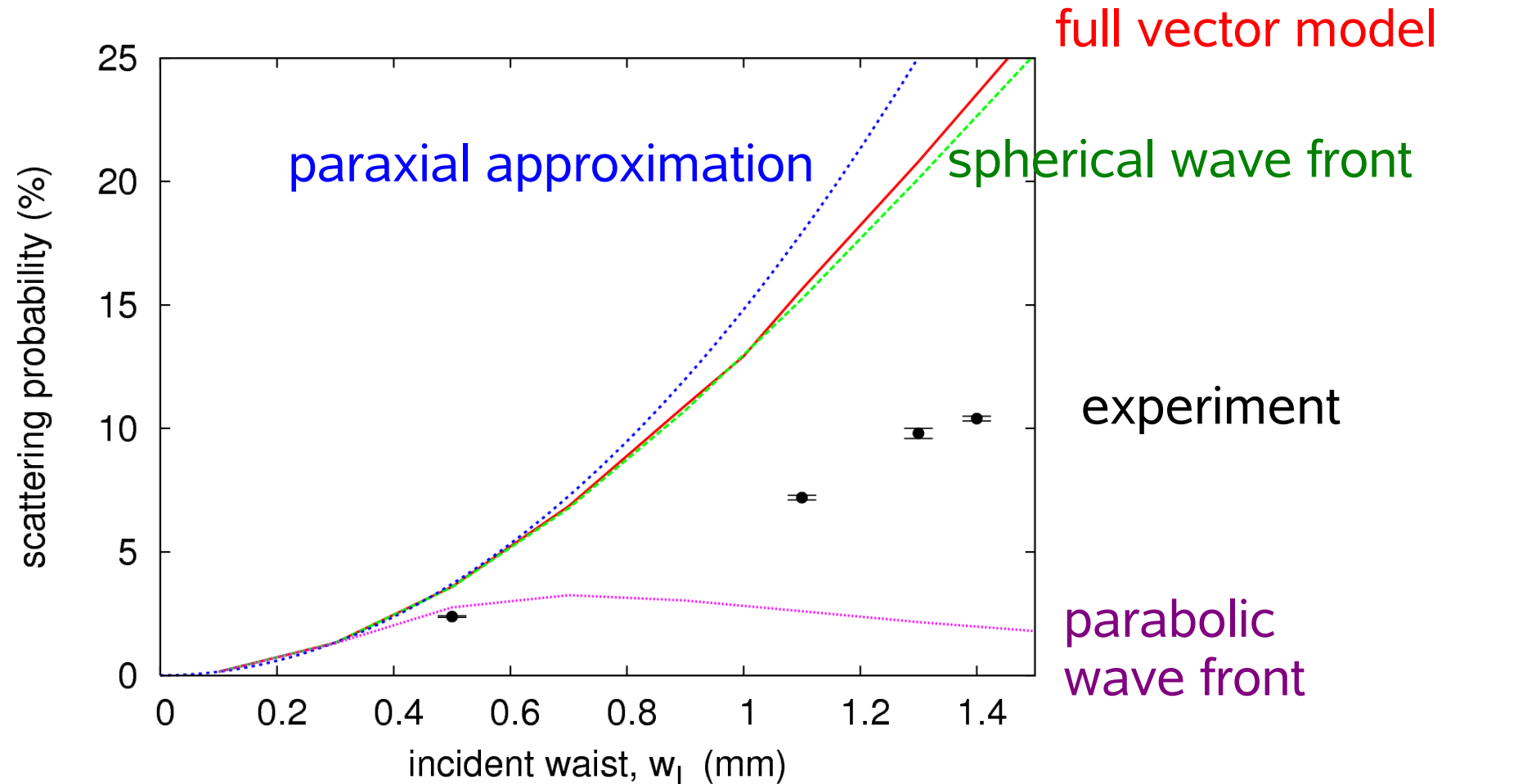
$$R = \frac{R_{sc}^2}{4}$$

- Loss:

$$L = R_{sc} - \frac{R_{sc}^2}{2}$$



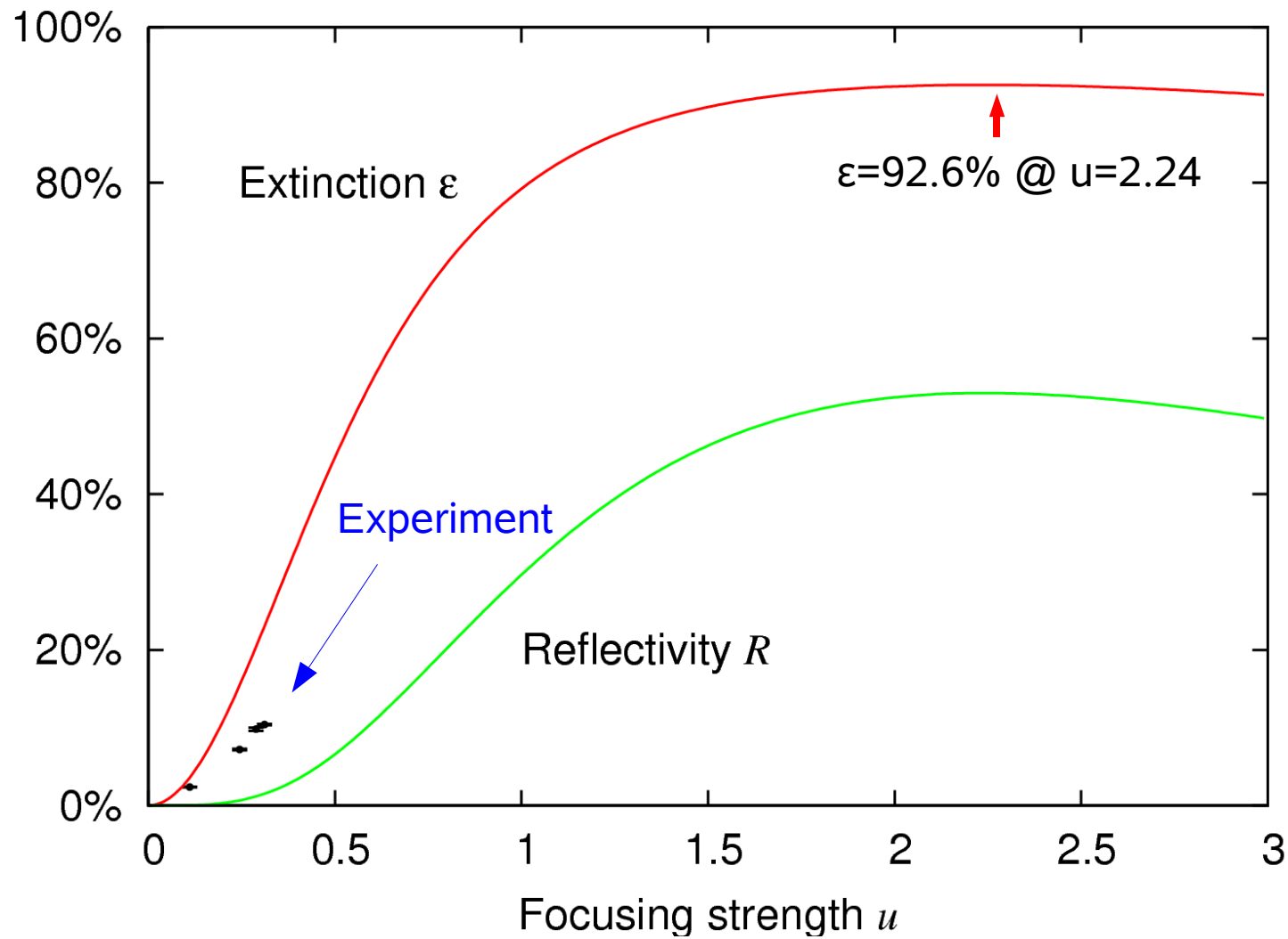
# Scattering vs. focusing



( experimental  $P_{sc}$  extracted out of  
transmission measurement )

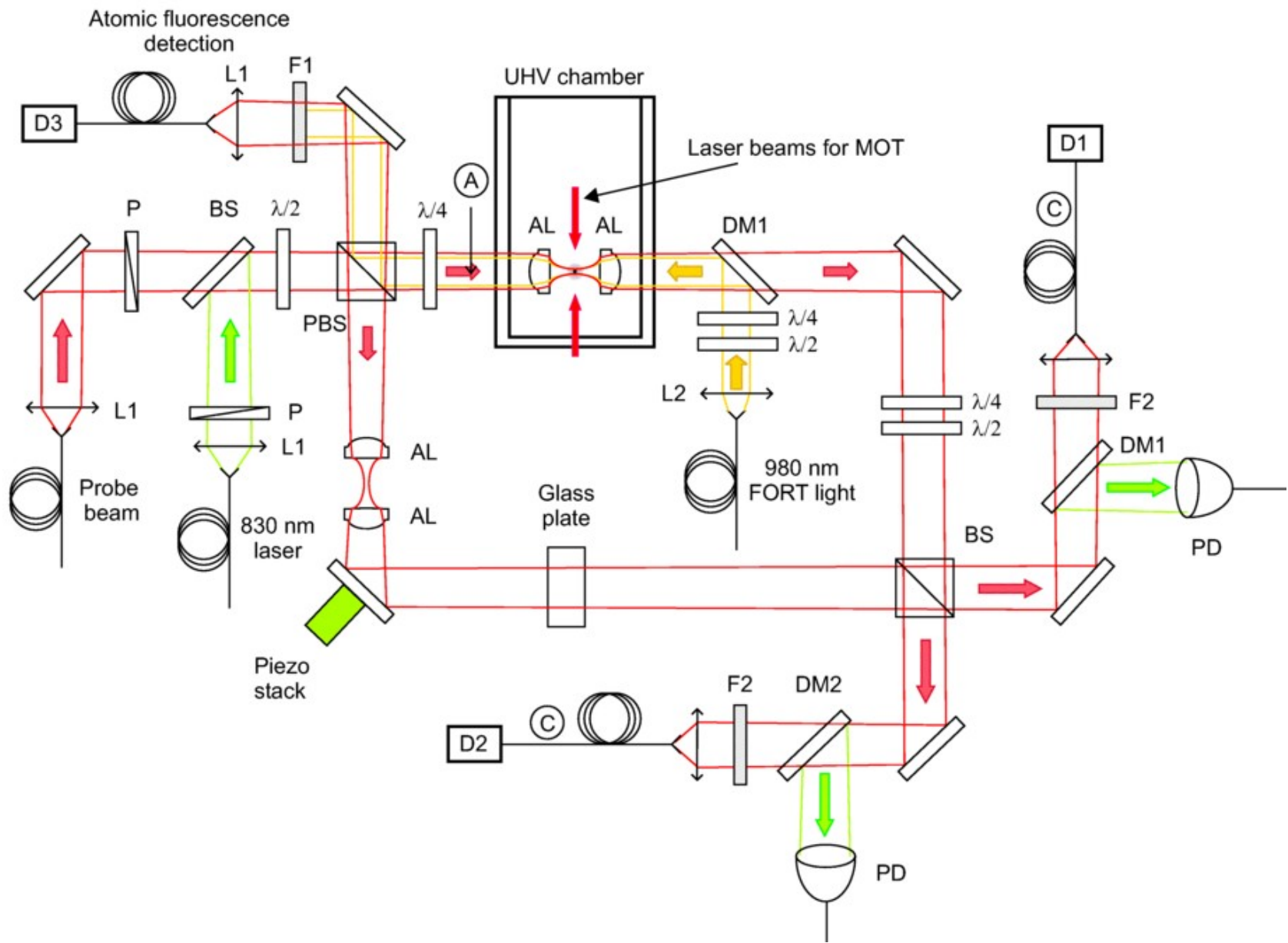
( $f = 4.5$  mm)

# *How far does this go?*

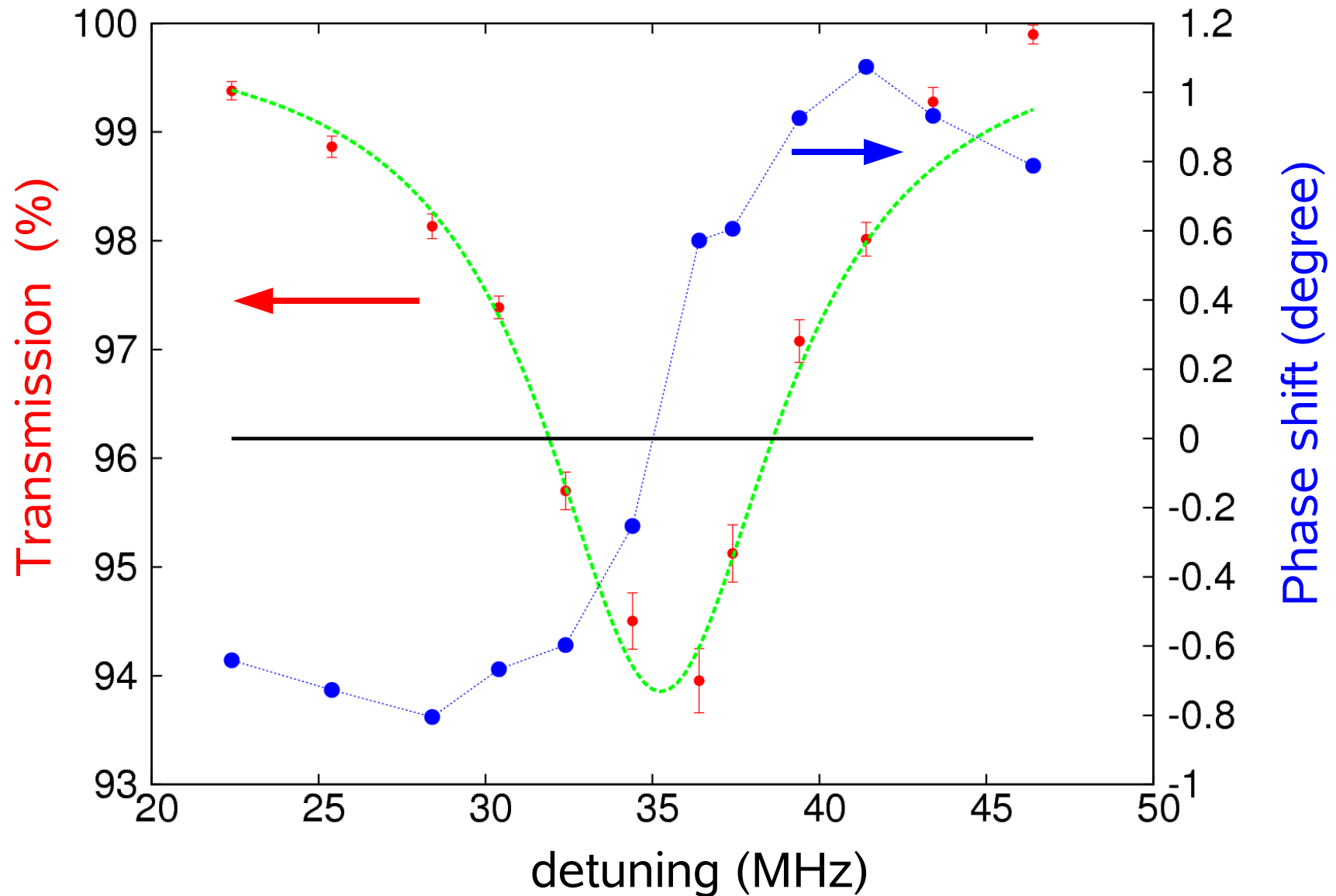


# Single atom phase shift

## Mach-Zehnder interferometer setup



# Phase shift / Transmission





# *Comparison to cavity QED*

- Could strong focusing replace cavities for strong coupling?

Probably not: imperfect mode match  
Gaussian modes --- atomic dipole modes

- Can strong focusing help in cavity QED experiments?

Probably yes: field enhancement due to focusing  
can lower cavity finesse

- What is the balance of technical problems?

high NA lenses vs. high finesse mirrors (similar effort?)

# *Next steps*

---

- Improve laser cooling
- Try larger numerical apertures
- Look for backscattered light
- Connect to nonclassical light sources....

# *Thank you!*



*Meng Khoon Tey (now UIBK)*  
*Syed Abdullah Aljunid*  
*Zilong Chen (now JILA)*  
*Florian Huber (now Harvard)*  
*Brenda Chng*  
*Jianwei Lee*

*Timothy Liew*  
*Gleb Maslennikov*

*Valerio Scarani*  
*Christian Kurtsiefer*

**<http://www.qolah.org>**  
(has also this talk)

# Results (collect full NA)

- Extinction

$$\epsilon = \frac{P_{sc}^{\rho_0}}{2 P_{in} (1 - e^{-2\rho_0^2/w_L^2})} \left[ 1 + \frac{4 f^3 + 3 f \rho_0^2}{4 (f^2 + \rho_0^2)^{3/2}} \right]$$

- Reflectivity (backward direction)

$$R = \frac{P_{sc}^{\rho_0}}{2 P_{in}} \left[ 1 - \frac{4 f^3 + 3 f \rho_0^2}{4 (f^2 + \rho_0^2)^{3/2}} \right]$$

- No energy gets lost

