Absorption of a single photon

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Outline

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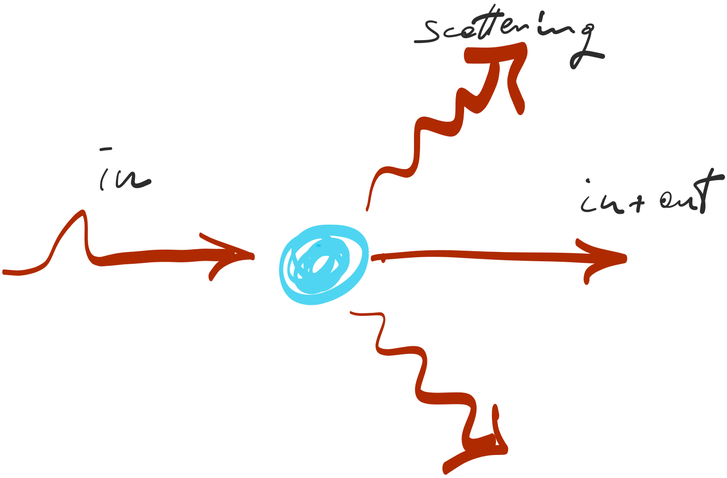
The geometry



$$|\psi(t)\rangle = C_0^e(t)e^{-i\omega_0 t}\,|e,\{0\}\rangle + \sum_{\vec{\mathbf{k}}} C^g_{1\vec{\mathbf{k}}}(t)e^{-i\omega_k t}\,|g, 1_{\vec{\mathbf{k}}}\rangle$$

General solution of the interaction Hamiltonian

Interaction Hamiltonian

$$\sum_{\vec{k}} \hbar g_{\vec{k}} \left(|e\rangle\langle g| \, \hat{a}_{\vec{k}} + |g\rangle\langle e| \, \hat{a}^{\dagger}_{\vec{k}} \right)$$

Solution for all times

$$C_0^e = e^{-\frac{\Gamma}{2}|t|}$$

$$C_{1\vec{k}}^g = \frac{i g^*_{\vec{k}}}{(\omega_k - \omega_0) - i\frac{\Gamma}{2}\mathrm{sign}(t)} \left(e^{-\frac{\Gamma}{2}|t|} e^{-i(\omega_k - \omega_0)t} - 1 \right)$$

M. Stobińska, *et al.*, Epl **86**, 14007 (2009)

Field and atom are separable only at infinite times

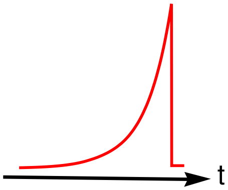
$t \to +\infty$ spontaneous decay

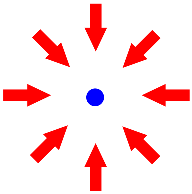
$$|\chi\rangle_+ = \lim_{t\to+\infty} \sum_{\vec{\mathbf{k}}} \frac{i g^*_{\vec{\mathbf{k}}}}{(\omega_k - \omega_0) - i\frac{\Gamma}{2}} e^{-i(\omega_k-\omega_0)t} \hat{\mathfrak{a}}_{\vec{\mathbf{k}}} |0\rangle$$

$t \to -\infty$ optimal excitation

$$|\chi\rangle_- = \lim_{t\to-\infty} \sum_{\vec{\mathbf{k}}} \frac{i g^*_{\vec{\mathbf{k}}}}{(\omega_k - \omega_0) + i\frac{\Gamma}{2}} e^{-i(\omega_k-\omega_0)t} \hat{\mathfrak{a}}_{\vec{\mathbf{k}}} |0\rangle$$

From time symmetry we need an exponentially rising temporal shape for optimal absorption





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Initial state

$$|\psi\rangle = |g\rangle_a |1\rangle_p |\{0\}\rangle_e$$

- the atom starts in the ground state $|g\rangle_a$
- the environment consists of a continuum of EM modes $|\{0\}\rangle_e$
- the input is a pulse propagating toward the atom $|1\rangle_p$

The input state

$$|1\rangle_p = \int \mathrm{d}^3k \, u_{\vec{\mathbf{k}}}(\vec{\mathbf{r}}) f(\omega_k) \hat{\mathbf{a}}^\dagger_{\vec{\mathbf{k}}} |0\rangle_p$$

Y. Wang, *et al.*, Phys. Rev. A **83**, 063842 (2011)

Interaction Hamiltonian (RWA)

$$\hat{\mathrm{H}}_{\mathrm{I}} = -i\hbar \int \mathrm{d}^3k \left[g_{\vec{\mathrm{k}}} \hat{\boldsymbol{\sigma}}_{+} \hat{\mathbf{a}}_{\vec{\mathrm{k}}} e^{-i(\omega_k - \omega_0)t} - \text{H.c.} \right]$$

with:

$\hat{\boldsymbol{\sigma}}_{+} = |e\rangle\langle g|$ rising operator

$$g_{\vec{\mathrm{k}}} = \sqrt{\frac{\omega_k}{(2\pi)^3 2\hbar\varepsilon_0}} u_{\vec{\mathrm{k}}} \left(\vec{\mathbf{d}} \cdot \vec{\epsilon}_{\vec{\mathrm{k}}} \right)$$ interaction strength

$u_{\vec{\mathrm{k}}}$ are the spatial mode functions

Time evolution of the system
Heisenberg equations

$$\frac{\partial}{\partial t}\hat{\mathbf{a}}_{\vec{\mathbf{k}}} = g^{*}_{\vec{\mathbf{k}}}\hat{\boldsymbol{\sigma}}_{-}e^{i(\omega_k-\omega_0)t}$$

$$\frac{\partial}{\partial t}\hat{\boldsymbol{\sigma}}_{z} = \hat{\zeta}_{z} - \Gamma_e(\hat{\boldsymbol{\sigma}}_{z}+1) - 2\int \mathrm{d}^3k \left[g_{\vec{\mathbf{k}}}\hat{\boldsymbol{\sigma}}_{+}\hat{\mathbf{a}}_{\vec{\mathbf{k}}}e^{-i(\omega_k-\omega_0)t} + \text{H.c.}\right]$$

$$\frac{\partial}{\partial t}\hat{\boldsymbol{\sigma}}_{-} = \hat{\zeta}_{-} - \frac{\Gamma_e}{2}\hat{\boldsymbol{\sigma}}_{-} + \hat{\boldsymbol{\sigma}}_{z}\int \mathrm{d}^3k\, g_{\vec{\mathbf{k}}}\hat{\mathbf{a}}_{\vec{\mathbf{k}}}e^{-i(\omega_k-\omega_0)t}$$

Γ_e rate of decay into the environment

$\hat{\zeta}$ noise operators, interaction with the environment

Let's deal with the EM field first

from the formal integration of Eq.(1):

$$\hat{a}_{\vec{k}}(t) = \underbrace{\hat{a}_{\vec{k}}(t_0)}_{\text{free field}} + \underbrace{g^*_{\vec{k}} \int_{t_0}^{t} \mathrm{d}t' \, \hat{\sigma}_-\big(t'\big) e^{i(\omega_k - \omega_0)t'}}_{\textit{radianting}}$$

we can now substitute into (2) and (3), and we find an old friend:

$$\int \mathrm{d}^3 k \, \big|g_{\vec{k}}\big|^2 \int_{t_0}^{t} \mathrm{d}t \, e^{i(\omega_k - \omega_0)t'}$$

WW approximation

For timescales $t \gg 1/\omega_0$, then $\int_{t_0}^{t} \to \int_0^{\infty}$

$$\int_{t_0}^{t} \mathrm{d}t\, e^{i(\omega_k - \omega_0)t'} \to \pi\delta(\omega_k - \omega_0)$$

$$\Gamma_p = \frac{\left|\vec{\mathbf{d}}\right|^2 \omega_0^3}{6\pi^2\hbar\varepsilon_0 c^3} \underbrace{\int \mathrm{d}\Omega \left|u_{\vec{\mathbf{k}}}\right|^2 \left|\hat{\mathbf{d}} \cdot \vec{\epsilon}_{\vec{\mathbf{k}}}\right|^2}_{\Lambda}$$

The spatial overlap $\Lambda \in \left[0, \frac{8\pi}{3}\right]$

Bloch Equations

$$\frac{\partial}{\partial t}\hat{\sigma}_z = \hat{\zeta}_z - \Gamma(\hat{\sigma}_z + 1) - 2\int \mathrm{d}^3k \left[g_{\vec{k}}\hat{\sigma}_+\hat{a}_{\vec{k}}(t_0)e^{-i(\omega_k - \omega_0)t} + \text{H.c.}\right]$$

$$\frac{\partial}{\partial t}\hat{\sigma}_+ = \hat{\zeta}_+ - \frac{\Gamma}{2}\hat{\sigma}_+ + \hat{\sigma}_z \int \mathrm{d}^3k\, g_{\vec{k}}\hat{a}^\dagger_{\vec{k}}(t_0)e^{i(\omega_k - \omega_0)t}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_- = \hat{\zeta}_- - \frac{\Gamma}{2}\hat{\sigma}_- + \hat{\sigma}_z \int \mathrm{d}^3k\, g_{\vec{k}}\hat{a}_{\vec{k}}(t_0)e^{-i(\omega_k - \omega_0)t}$$

where

$$\Gamma = \underbrace{\Gamma_e}_{\text{environment}} + \overbrace{\Gamma_p}^{\text{back to pulse mode}}$$

Excitation probability in time

The operator σ_z tells us about the population of the excited state

$$P_e(t) = |\langle\psi_0|\hat{\sigma}_z|\psi_0\rangle|^2 = \frac{1}{2}(|\langle\psi_0|\hat{\sigma}_z|\psi_0\rangle| + 1)$$

We need to find $\langle\hat{\sigma}_z\rangle$ by solving the Bloch equations also for $\langle\hat{\sigma}_+\rangle$ and $\langle\hat{\sigma}_-\rangle$.

Let's define the state vector

$$s(t) = \begin{pmatrix} \langle g_a, 1_p, \{0\}_e | \, \hat{\boldsymbol{\sigma}}_z(t) \, | g_a, 1_p, \{0\}_e \rangle \\ \\ \langle g_a, 1_p, \{0\}_e | \, \hat{\boldsymbol{\sigma}}_+(t) \, | g_a, 0_p, \{0\}_e \rangle \\ \\ \langle g_a, 1_p, \{0\}_e | \, \hat{\boldsymbol{\sigma}}_-(t) \, | g_a, 1_p, \{0\}_e \rangle \end{pmatrix}$$

Bloch Equations

$$\frac{\partial}{\partial t}\hat{\sigma}_z = \hat{\zeta}_z - \Gamma(\hat{\sigma}_z + 1) - 2\int \mathrm{d}^3k \left[g_{\vec{k}}\hat{\sigma}_+ \hat{a}_{\vec{k}}(t_0) e^{-i(\omega_k - \omega_0)t} + \text{H.c.} \right]$$

$$\frac{\partial}{\partial t}\hat{\sigma}_+ = \hat{\zeta}_+ - \frac{\Gamma}{2}\hat{\sigma}_+ + \hat{\sigma}_z \int \mathrm{d}^3k \, g_{\vec{k}} \hat{a}^\dagger_{\vec{k}}(t_0) e^{i(\omega_k - \omega_0)t}$$

$$\frac{\partial}{\partial t}\hat{\sigma}_- = \hat{\zeta}_- - \frac{\Gamma}{2}\hat{\sigma}_- + \hat{\sigma}_z \int \mathrm{d}^3k \, g_{\vec{k}} \hat{a}_{\vec{k}}(t_0) e^{-i(\omega_k - \omega_0)t}$$

We can express the system as

$$\frac{\partial s(t)}{\partial t} = M\,s(t) + b$$

with

$$M = \begin{pmatrix} -\Gamma & -2g(t) & -2g^*(t) \\ 0 & -\Gamma/2 & 0 \\ 0 & 0 & -\Gamma/2 \end{pmatrix} \quad b = \begin{pmatrix} -\Gamma \\ -g^*(t) \\ -g(t) \end{pmatrix}$$

and initial conditions

$$s(t_0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

What's under the rug?

Reminder: the input state

$$|1\rangle_p = \int \mathrm{d}^3 k \, u_{\vec{\mathbf{k}}}(\vec{\mathbf{r}}) f(\omega_k) \hat{\mathbf{a}}^\dagger_{\vec{\mathbf{k}}} |0\rangle_p$$

the spatial integral + WW approximation return

$$\Gamma_p = \frac{\left|\vec{\mathbf{d}}\right|^2 \omega_0^3}{6\pi^2 \hbar \varepsilon_0 c^3} \int \mathrm{d}\Omega \, \left|u_{\vec{\mathbf{k}}}\right|^2 \left|\hat{\mathbf{d}} \cdot \vec{\epsilon}_{\vec{\mathbf{k}}}\right|^2$$

The temporal part instead give us the Fourier transform of the single photon wavepacket:

$$\xi(t) = \frac{1}{2\pi} \int \mathrm{d}\omega_k \, f(\omega_k) e^{-i(\omega_k - \omega_0)t}$$

Finally

$$g(t) = \Gamma \frac{\Lambda}{8\pi/3}\, \xi(t)$$

The interaction depends on the temporal and spatial shape of the input light.

What happened to the environment?

When we average out over initial states, the assume the environment is empty, effectively giving:

$$\left\langle \hat{\zeta} \right\rangle = 0$$

It's a hat trick, but it works :)

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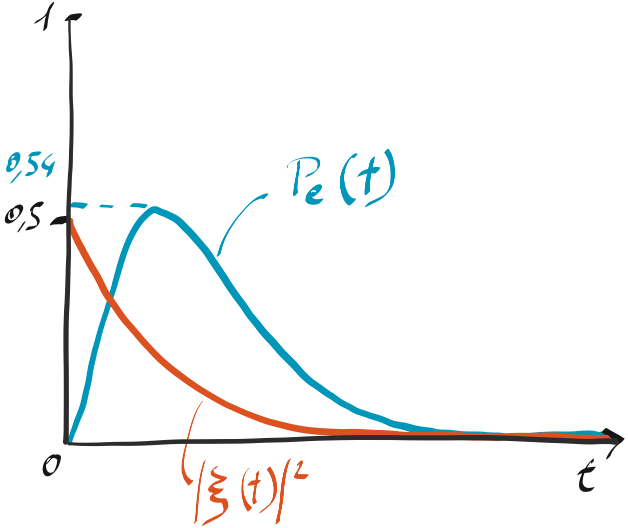
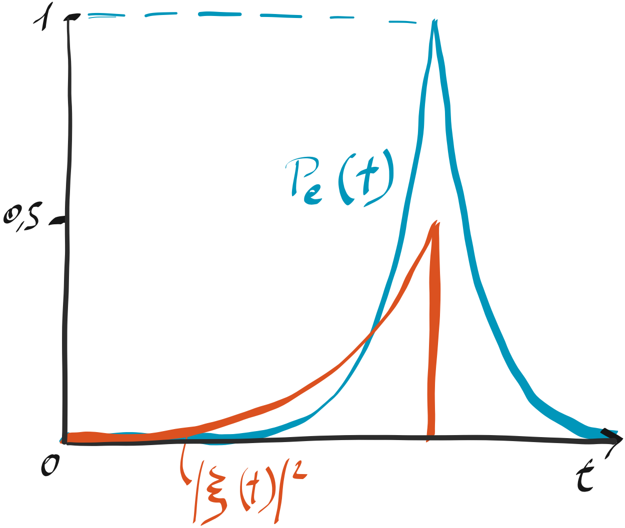
Exponentially decaying bandwidth $\Gamma_f = \Gamma$



Exponentially rising
bandwidth $\Gamma_f = \Gamma$

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