How did physicists detect Gravitational Waves?

Some tools that revealed the GW150914 event

C. Kurtsiefer, Physics enrichment camp 2016 @ NUS

The Story in the News

Selected for a Viewpoint in *Physics*

PRL **116,** 061102 (2016) PHYSICAL REVIEW LETTERS week ending 12 FEBRUARY 2016

Observation of Gravitational Waves from a Binary Black Hole Merger

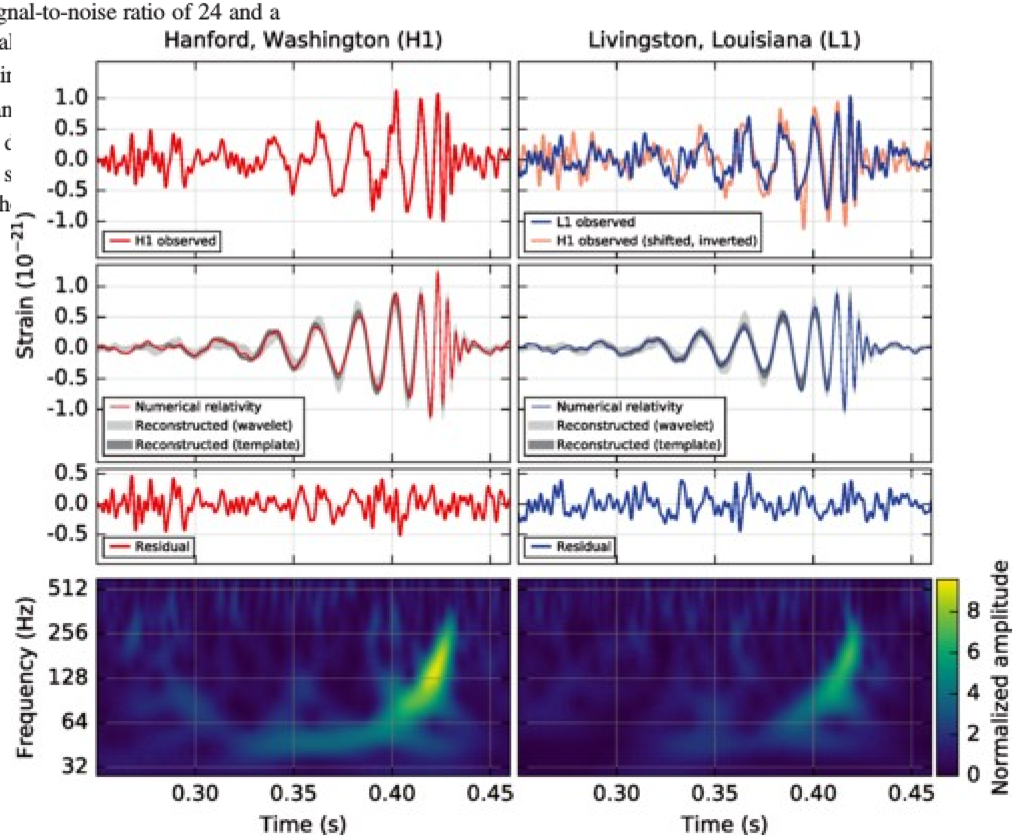
B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)

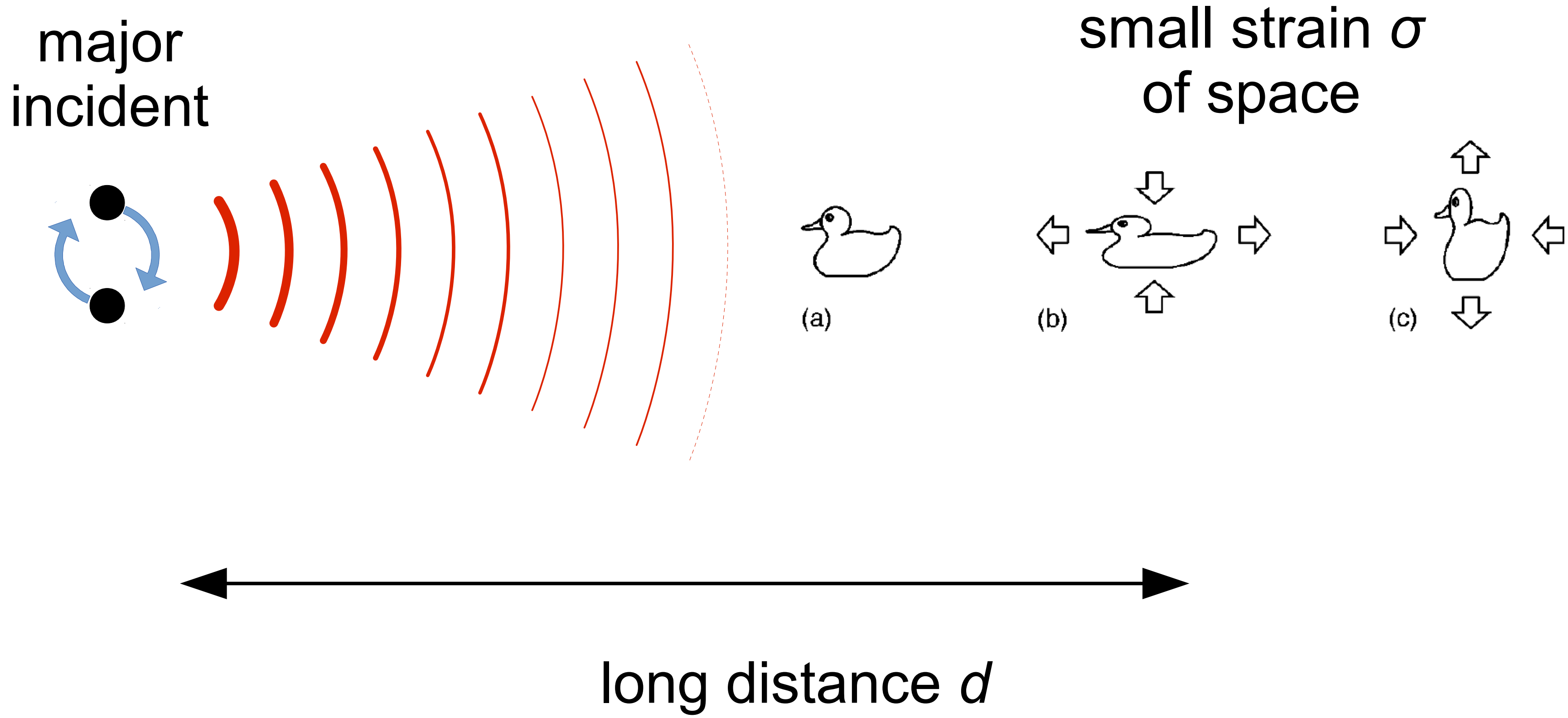
(Received 21 January 2016; published 11 February 2016)

On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory simultaneously observed a transient gravitational-wave signal. The signal sweeps upwards in frequency from 35 to 250 Hz with a peak gravitational-wave strain of 1.0×10^{-21}. It matches the waveform predicted by general relativity for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. The signal was observed with a matched-filter signal-to-noise ratio of 24 and a false alarm rate estimated to be less than 1 event per 203 000 years, equival
than 5.1σ. The source lies at a luminosity distance of 410^{+160}_{-180} Mpc correspondi
In the source frame, the initial black hole masses are $36^{+5}_{-4}M_{\odot}$ and $29^{+4}_{-4}M_{\odot}$, an
$62^{+4}_{-4}M_{\odot}$, with $3.0^{+0.5}_{-0.5}M_{\odot}c^2$ radiated in gravitational waves. All uncertainties c
These observations demonstrate the existence of binary stellar-mass black hole s
detection of gravitational waves and the first observation of a binary black h

DOI: 10.1103/PhysRevLett.116.061102



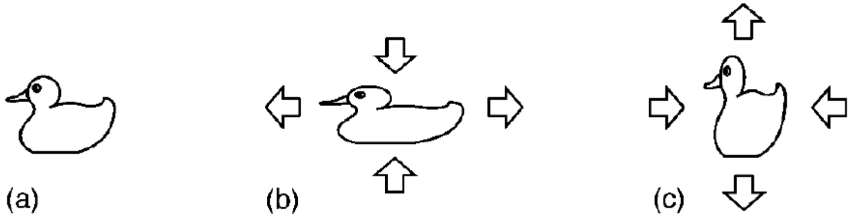
The Situation



$d \approx 410$ Mpc $\approx 1.34 \cdot 10^{9}$ ly $\approx 1.26 \cdot 10^{25}$ m

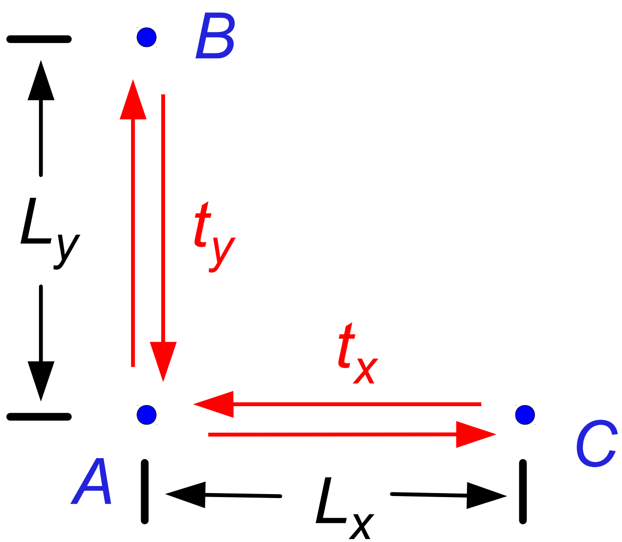
$\sigma = \Delta l \, / \, l \approx 10^{-21}$

How to test strain σ?



$$\sigma = \frac{\Delta L_x}{L_x} = -\frac{\Delta L_y}{L_y}$$

Round trip times of light between fixed points:



A, B, C: masses at rest

$L_x = L_y$ in quiet times

$t_{x,y} = 2\, L_{x,y} / c_0$

$t_x - t_y = 0$: no strain

$t_x - t_y \neq 0$: strain

Michelson and Morley 1887

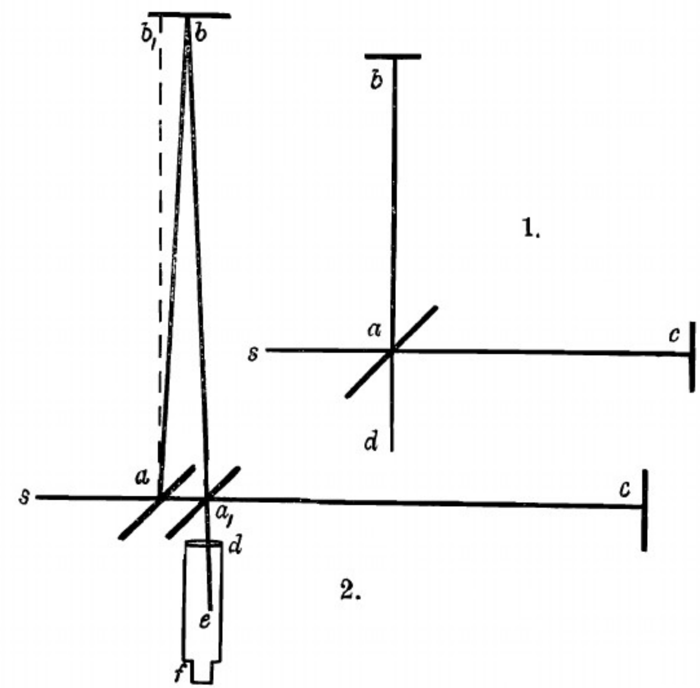
THE

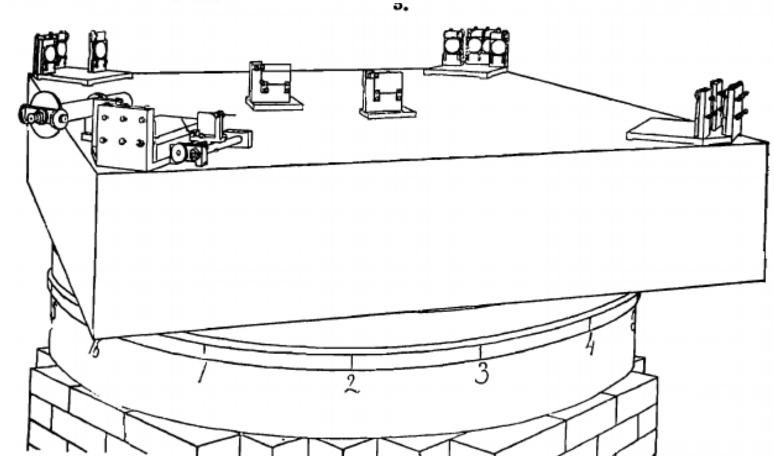
AMERICAN JOURNAL OF SCIENCE.

[THIRD SERIES.]

ART. XXXVI.—*On the Relative Motion of the Earth and t*
Luminiferous Ether; by ALBERT A. MICHELSON an
EDWARD W. MORLEY.*

THE discovery of the aberration of light was soon followed by an explanation according to the emission theory. The effect was attributed to a simple composition of the velocity of light with the velocity of the earth in its orbit. The difficulties in



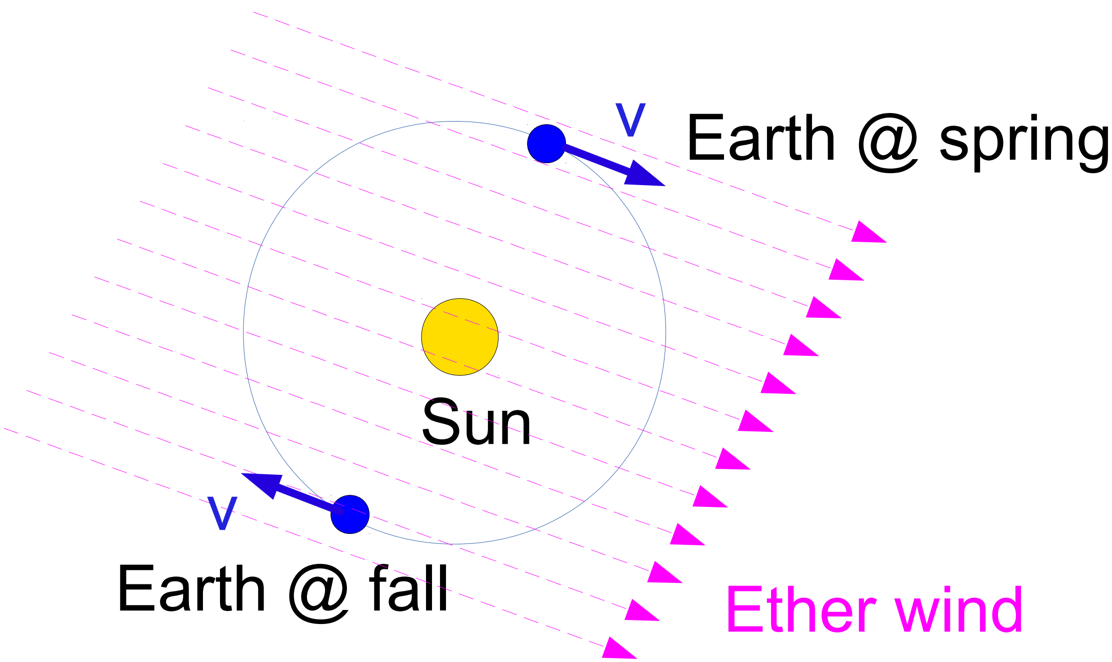


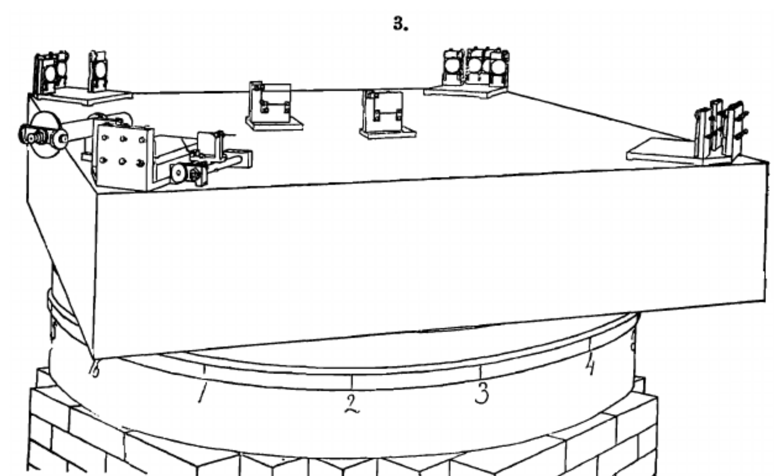
Michelson and Morley Result:

- Speed of light does not depend on direction of propagation
- Same speed of light at different times in the year

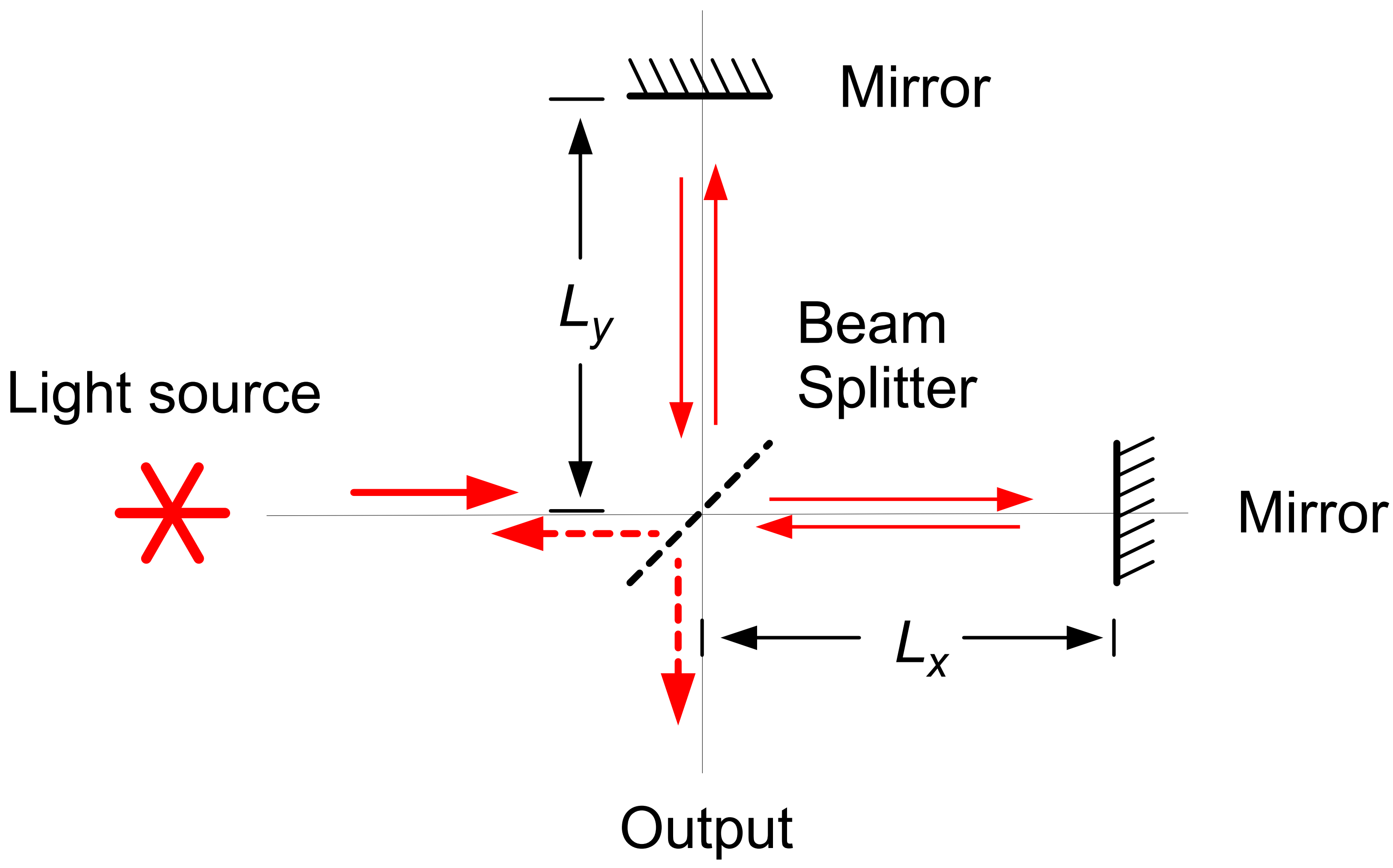
- No “ether” or reference that supports the propagation of light

- One of the starting points for theory of special relativity

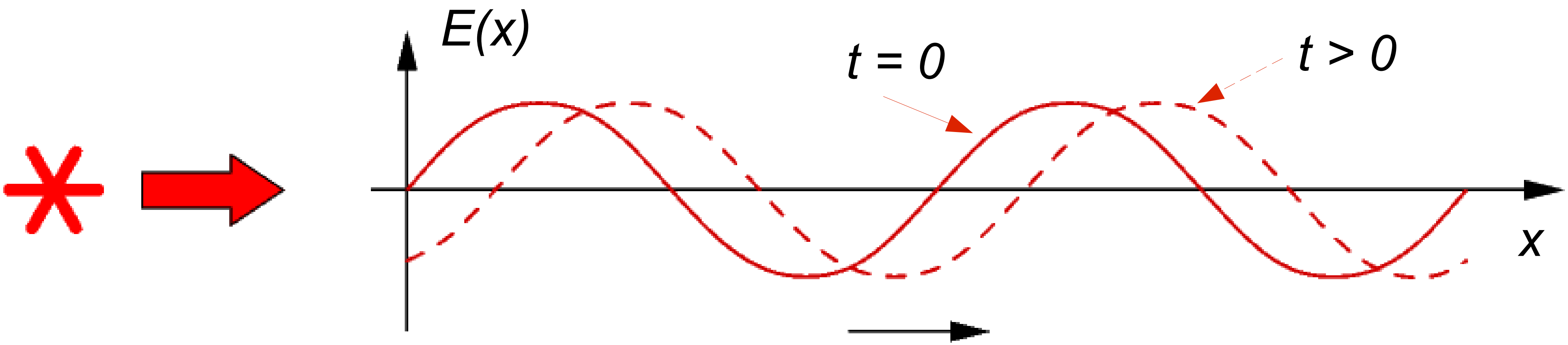




Michelson Interferometer



Light as a wave

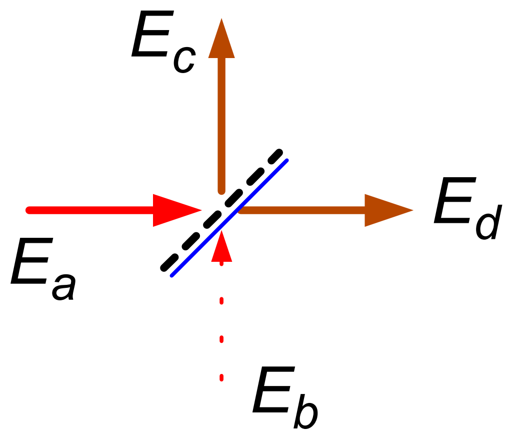


Light of fixed frequency *f*: $E(x,t) = E_0 \sin(kx - \omega t)$

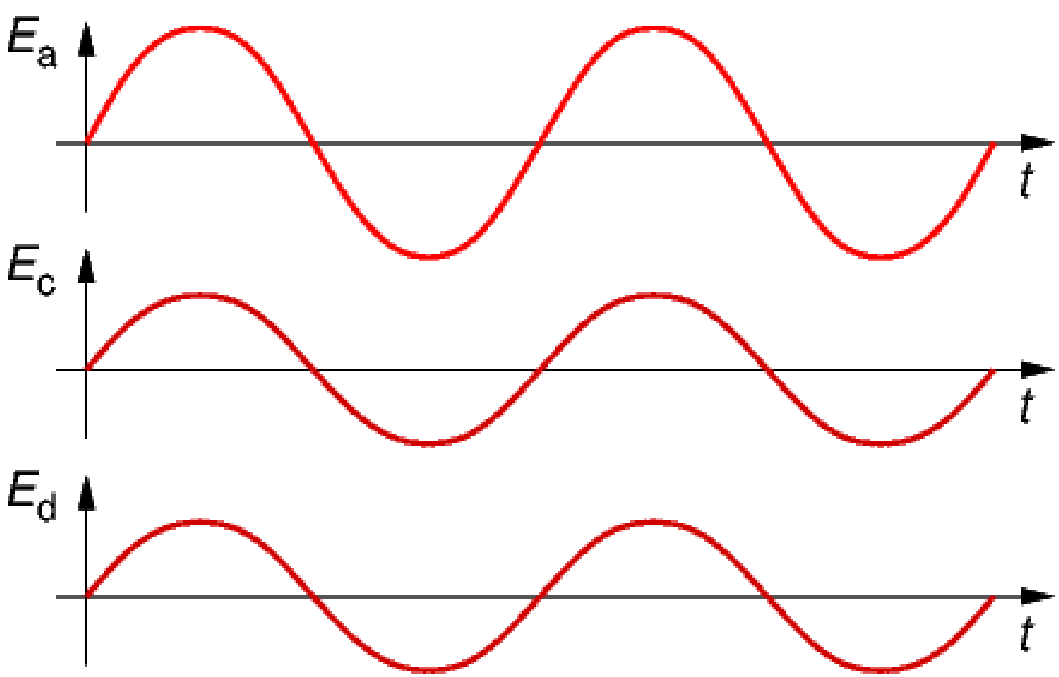
$$k = \frac{2\pi}{\lambda}, \quad \omega = 2\pi f$$

Speed of light c_0 is constant: $\omega = c_0 \cdot k, \quad c_0 = \lambda \cdot f$
(and independent of direction
and reference frame)

Symmetric Beam Splitter

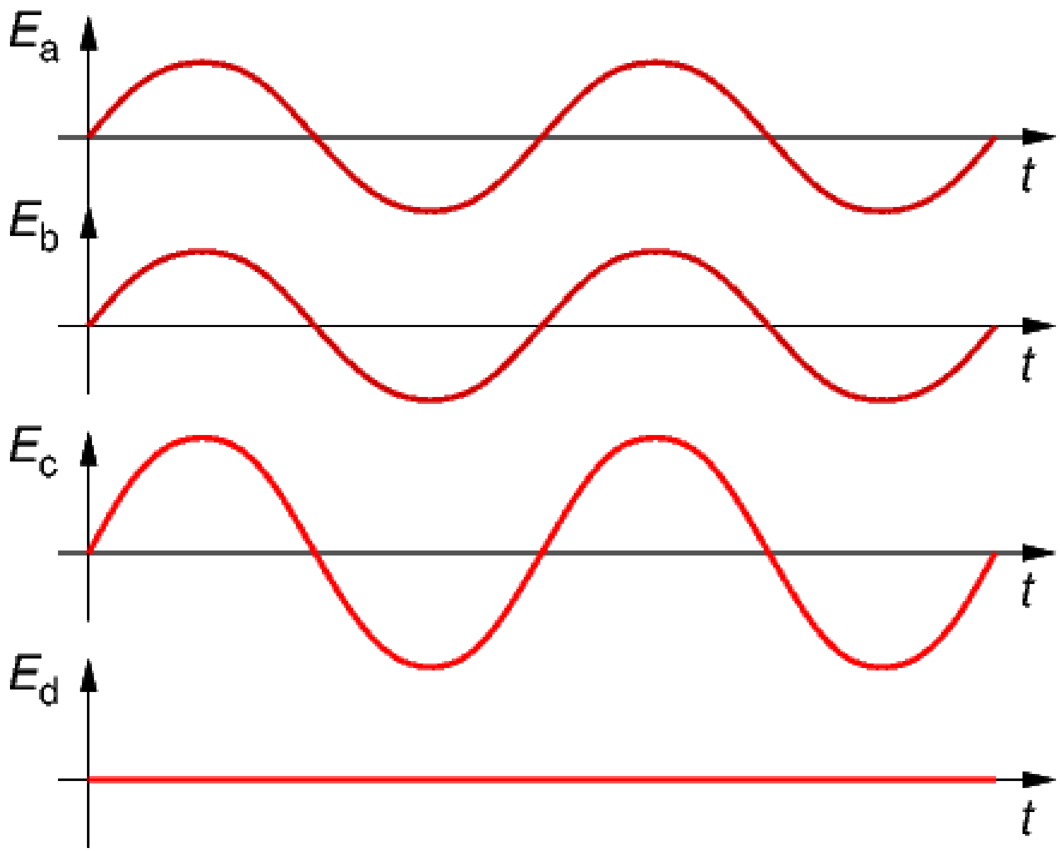


$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

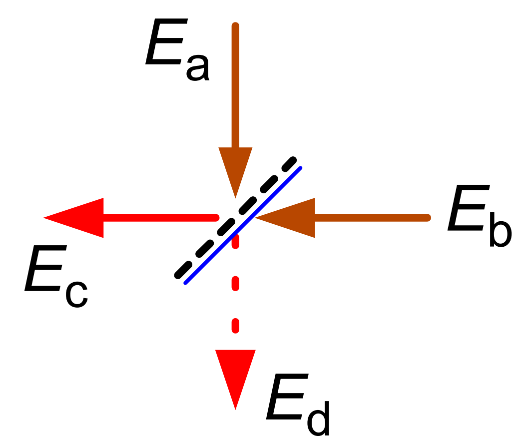


output beams
with amplitude
reduced by √2

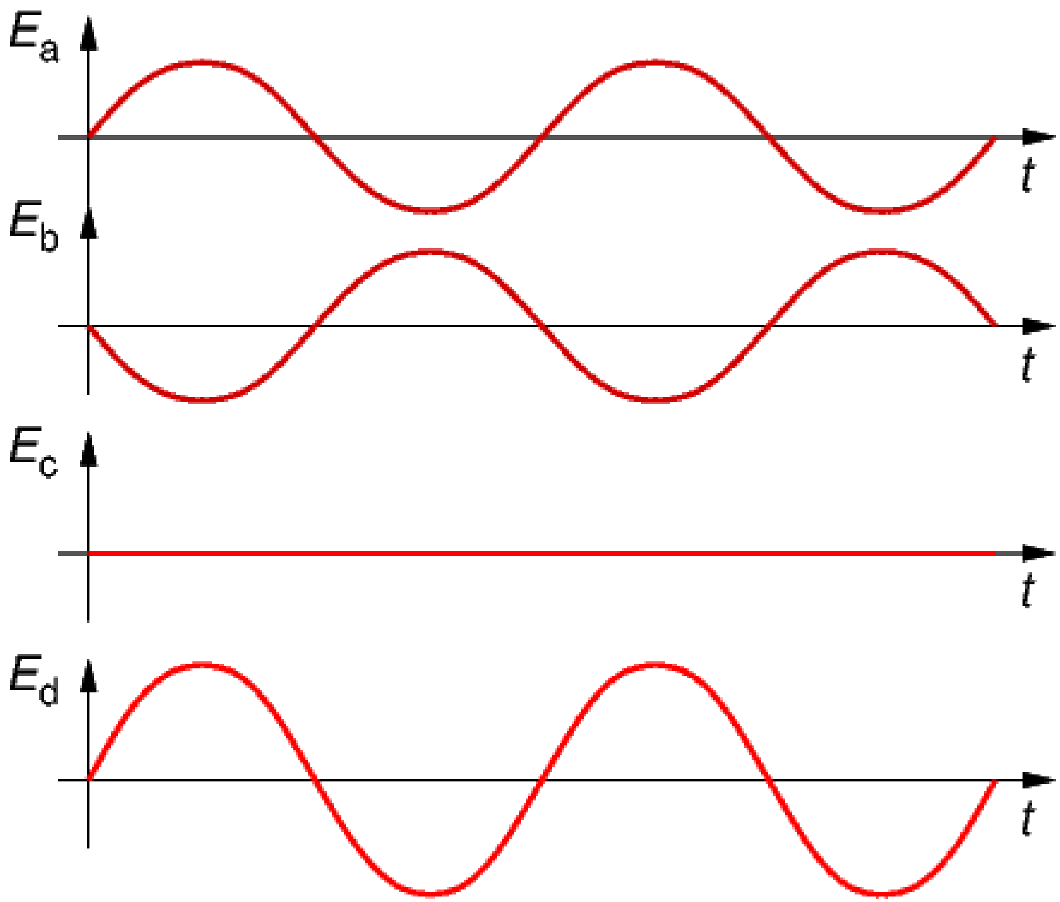
Superposition of light waves I



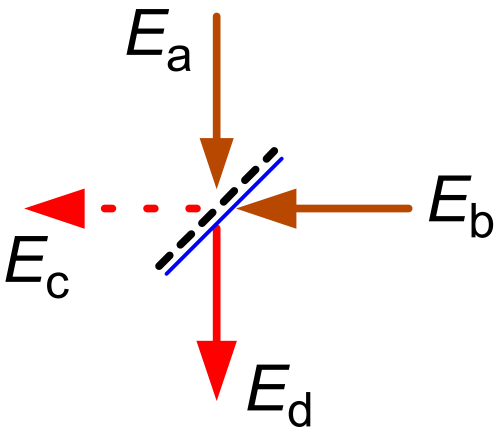
$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$



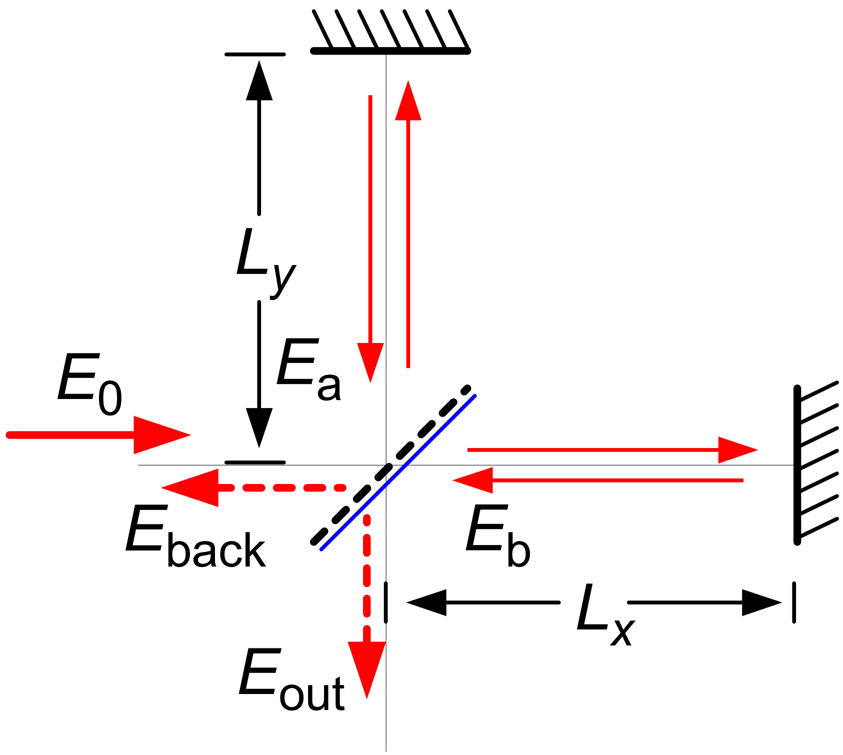
Superposition of light waves II



$$\begin{pmatrix} E_c \\ E_d \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$



Adding two amplitudes



$$\begin{pmatrix} E_{back} \\ E_{out} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} E_a \\ E_b \end{pmatrix}$$

Just before recombination:

$$E_a = \frac{E_0}{\sqrt{2}} \cos(2kL_y - \omega t)$$

$$E_b = \frac{E_0}{\sqrt{2}} \cos(2kL_x - \omega t)$$

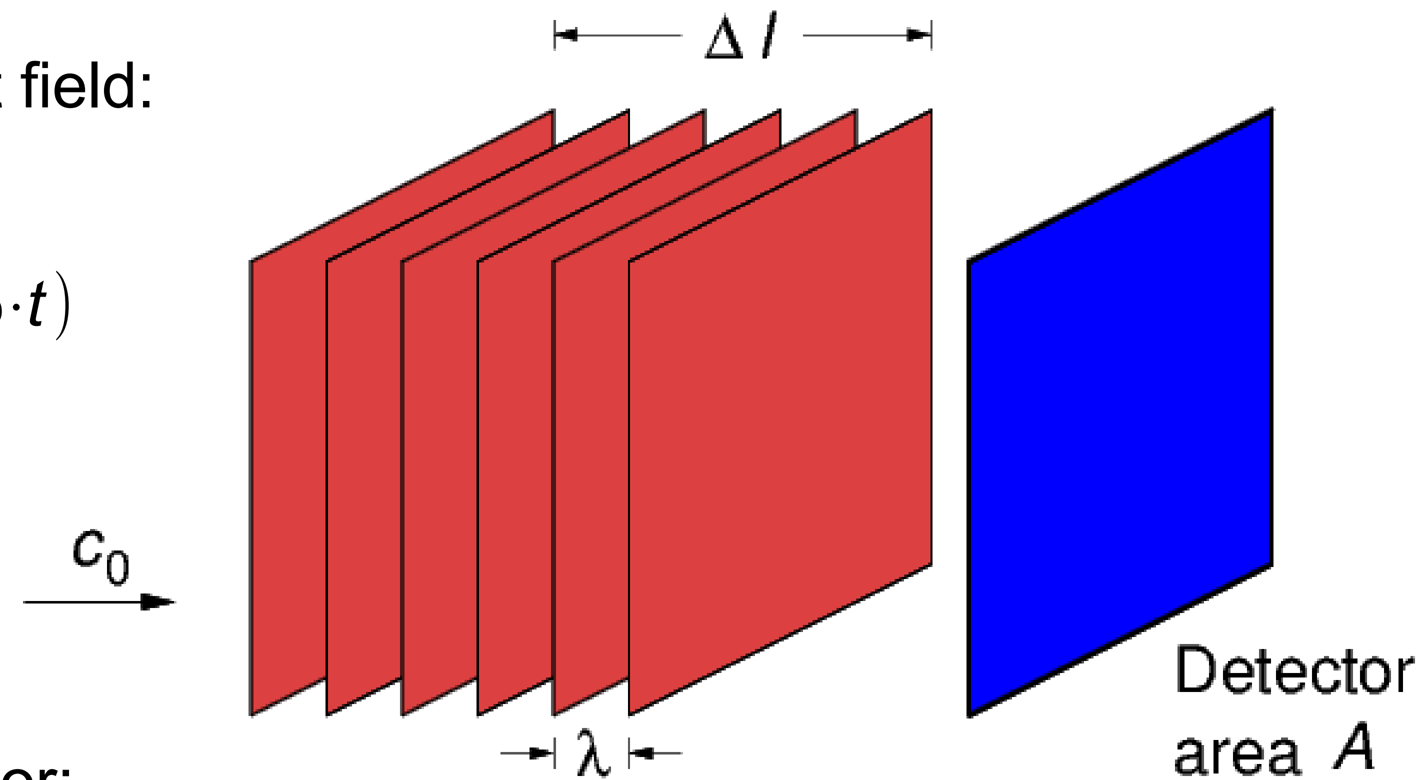
Output field:

$$\begin{aligned} E_{out} &= \frac{1}{\sqrt{2}}(E_a - E_b) \\ &= \ldots \\ &= E_0 \sin(k(L_y - L_x)) \\ &\quad \times -\sin(k(L_y + L_x) - \omega t) \end{aligned}$$

Light field and optical power

Plane wave light field:

$$E(x,t) = E_0 \cos(k \cdot x - \omega \cdot t)$$

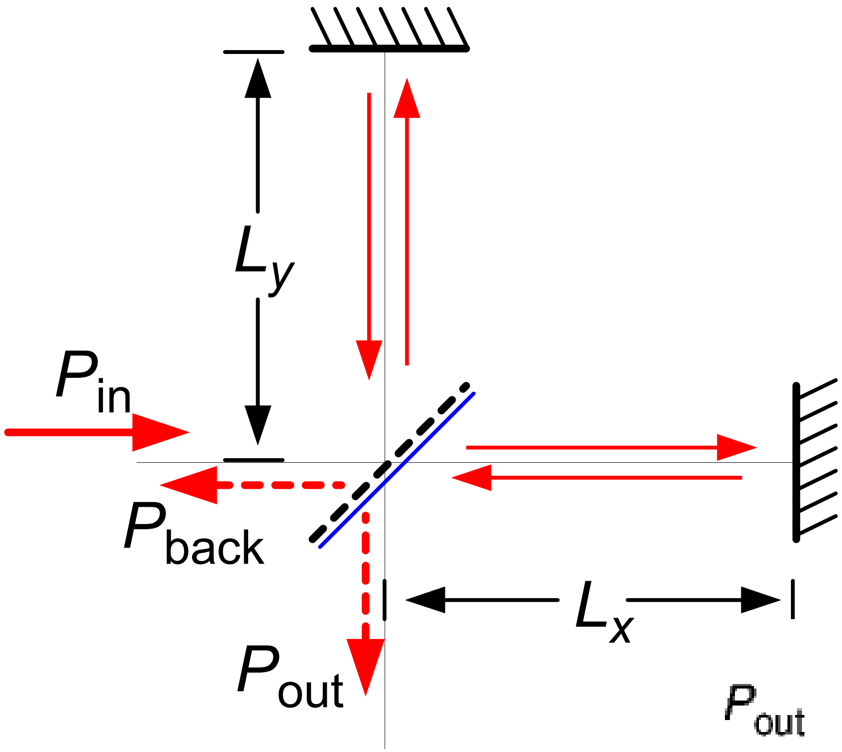


Power on detector:

$$P = \frac{\Delta \boldsymbol{E}}{\Delta t} = \frac{\Delta \boldsymbol{E}}{\Delta V} \cdot \frac{\Delta V}{\Delta t} = \frac{\Delta \boldsymbol{E}}{\Delta V} \cdot \frac{A \cdot \Delta l}{\Delta t} = \left[2 \frac{\epsilon_0}{2} \langle E^2 \rangle \right] \cdot A c_0 = \frac{\epsilon_0}{2} E_0^2 A c_0$$

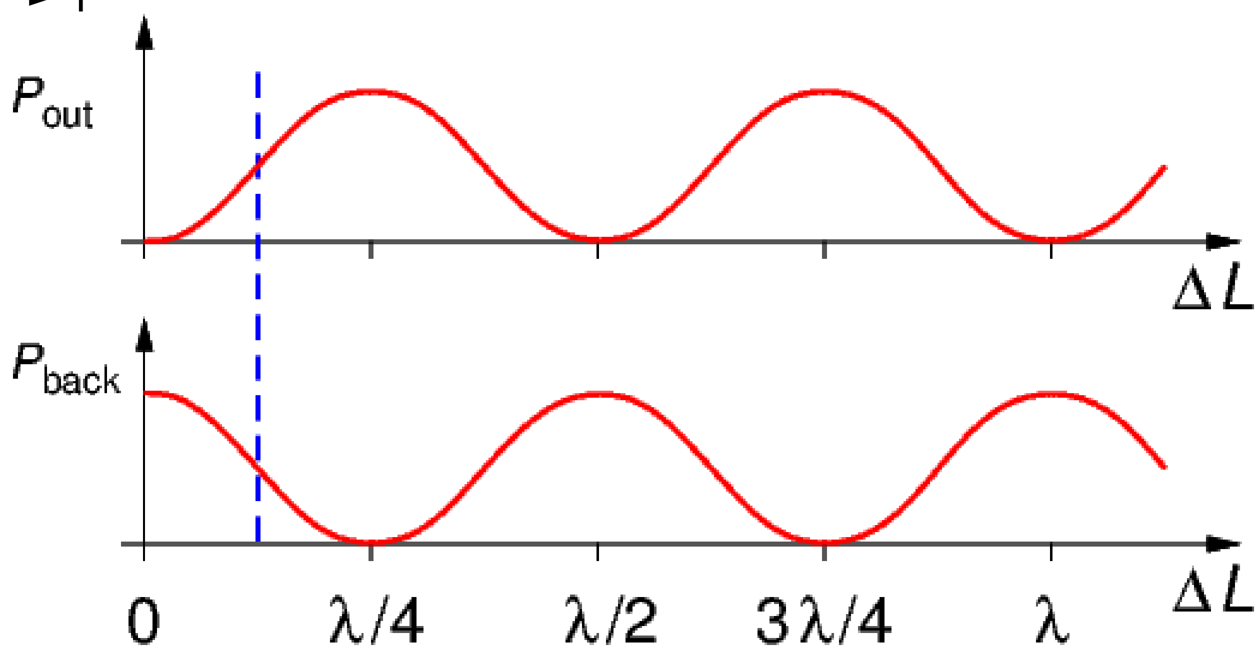
Energy per volume

Power at output of interferometer

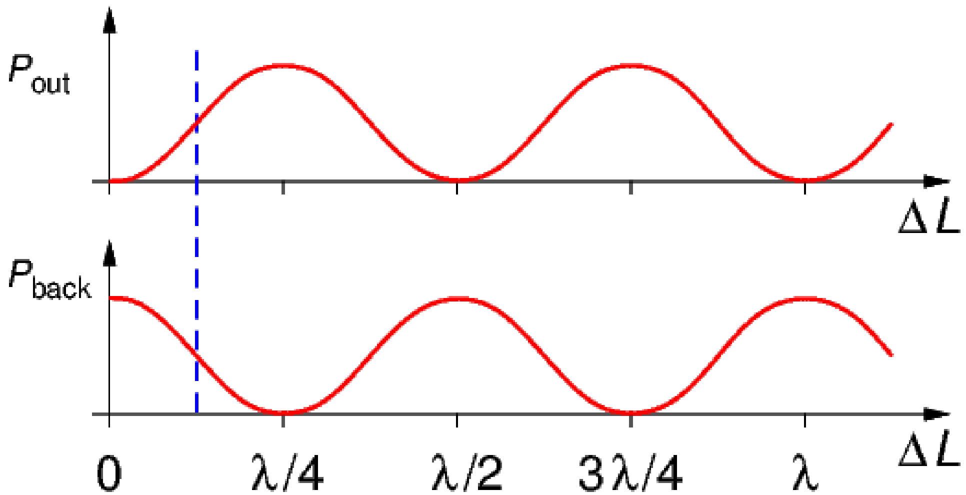


Output power:

$$P_{out} = P_{in} \sin^2(k\,\Delta L)$$
$$P_{back} = P_{in} \cos^2(k\,\Delta L)$$
with $\Delta L = L_y - L_x$



Can the demo setup detect GW?



Measure P_{out} near ΔL=λ/8:

$$\frac{\delta P_{out}}{\delta(\Delta L)} = \left.\frac{d P_{out}}{d \Delta L}\right|_{\Delta L=\lambda/8} = 2\pi \frac{P_{in}}{\lambda}$$

$$\delta(\Delta L) = \frac{\delta P_{out}}{P_{in}} \cdot \frac{\lambda}{2\pi}$$

Power resolution	$\delta P_{out} / P_{in}$	1%
Wavelength	λ	632 nm
Position resolution	$\delta(\Delta L)$	1 nm
Length	L	0.3 m
Strain resolution	$\delta\sigma = \delta(\Delta L)/ L$	$3 \cdot 10^{-9}$
GW150914 peak strain	σ	10^{-21}

Missing 13-14 orders of magnitude....

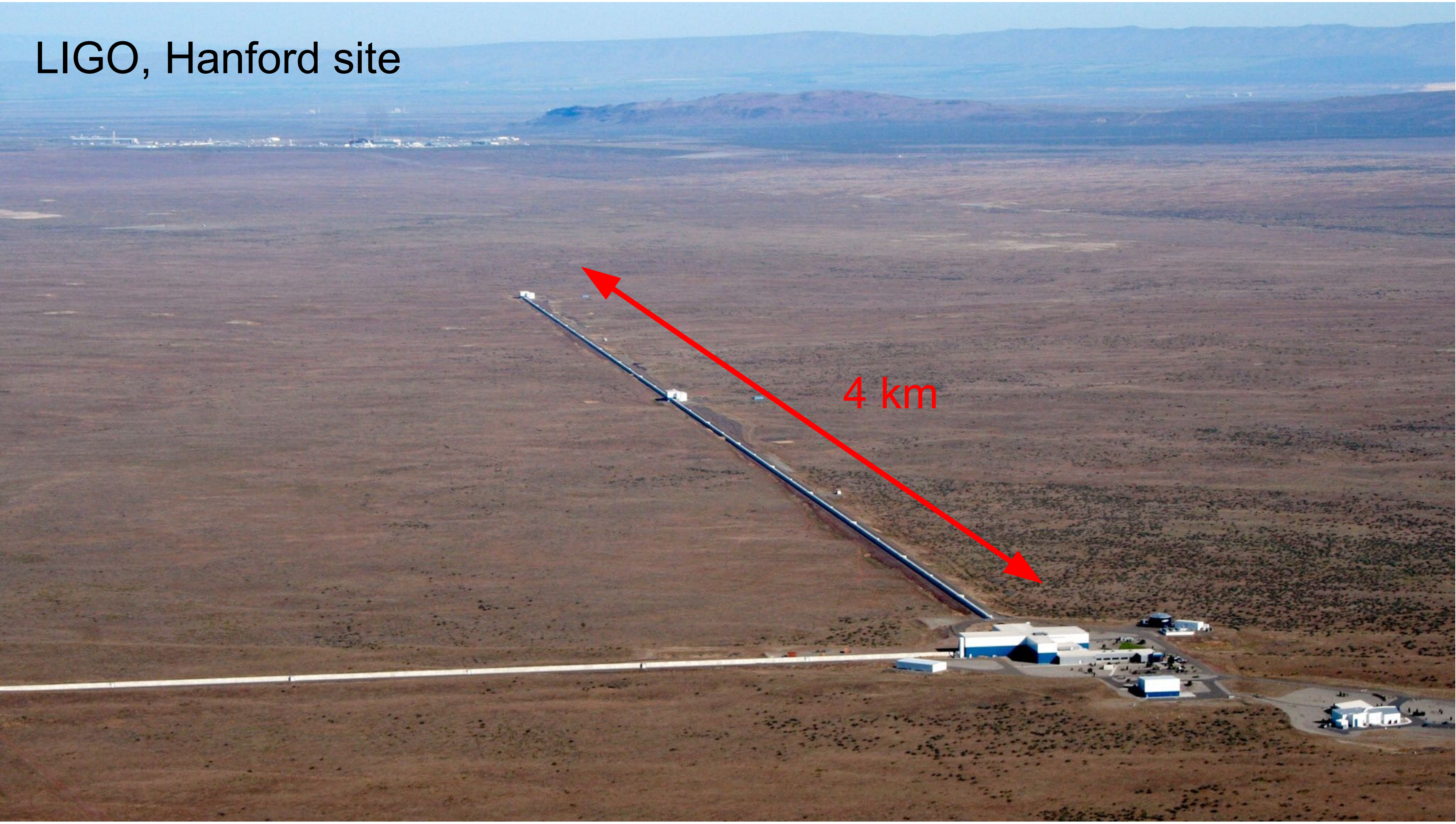
How to increase sensitivity?

Reduce strain uncertainty $\delta\sigma$:

$$\delta\sigma = \frac{\delta(\Delta L)}{L} = \underbrace{\frac{1}{L}}_{\text{arm length}} \cdot \underbrace{\left[\frac{d(P_{out}/P_{in})}{d(\Delta L)}\right]^{-1}}_{\text{interferometer responsiveness}} \cdot \underbrace{\frac{1}{P_{in}}}_{\text{power}} \cdot \underbrace{\delta P_{out}}_{\text{power uncertainty}}$$

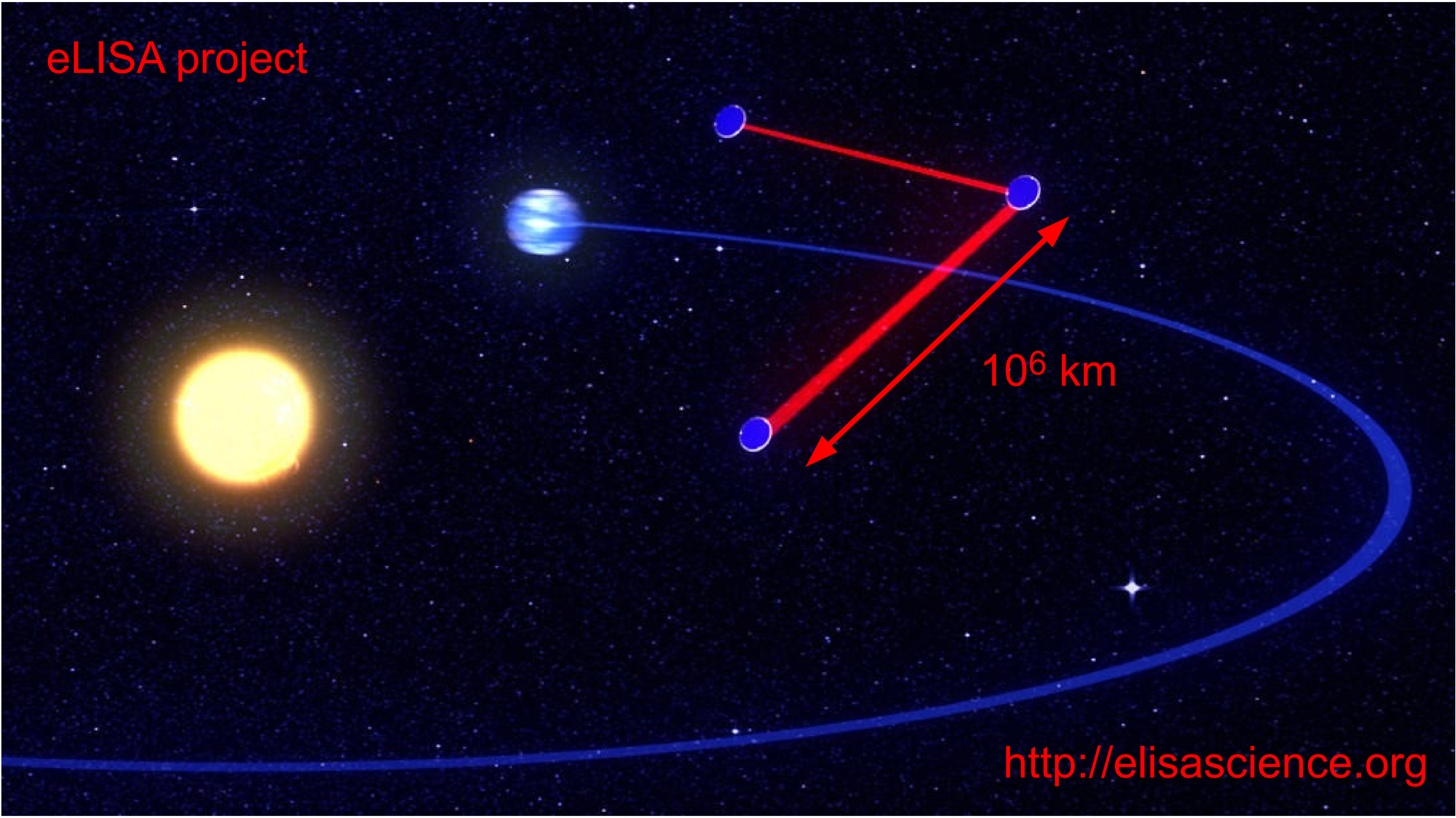
- Increase L
- Increase interferometer responsiveness
- Increase P_{in} & decrease δP_{out}
- (Decrease environmental impact)

Increase Arm Length, 1st try



0.3 m → 4 km: improve ~4 orders of magnitude in $\delta\sigma$

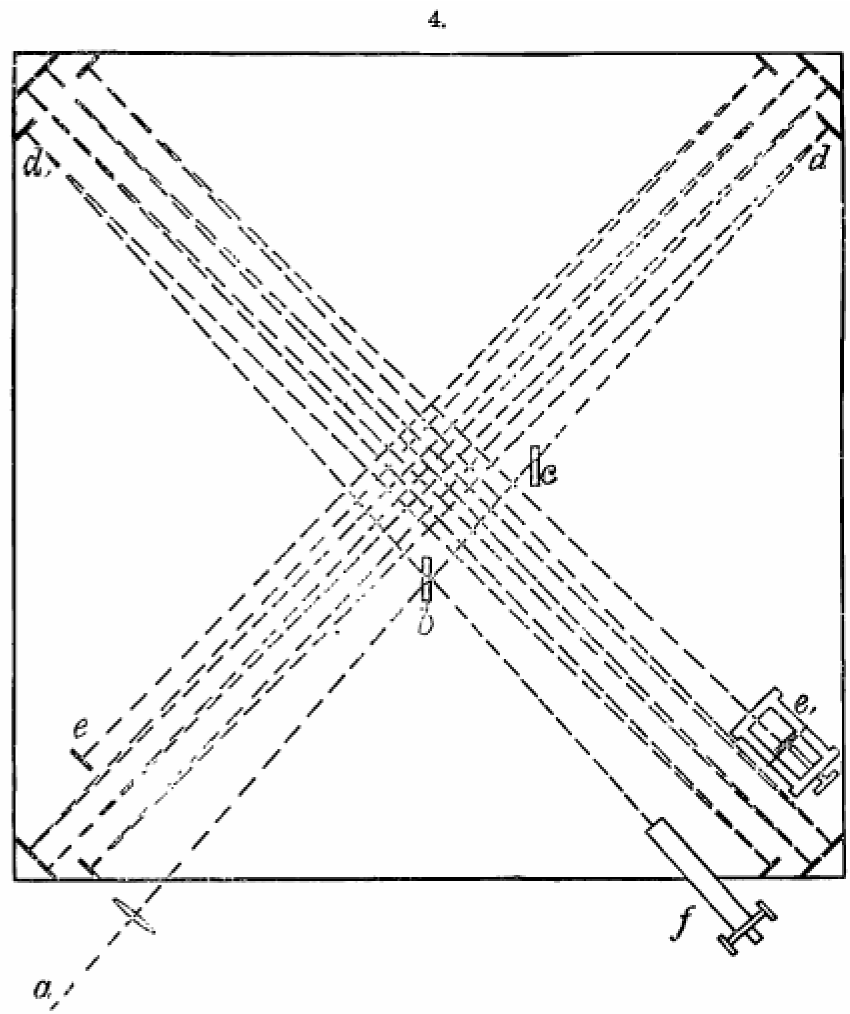
Increase Arm Length, 2nd try



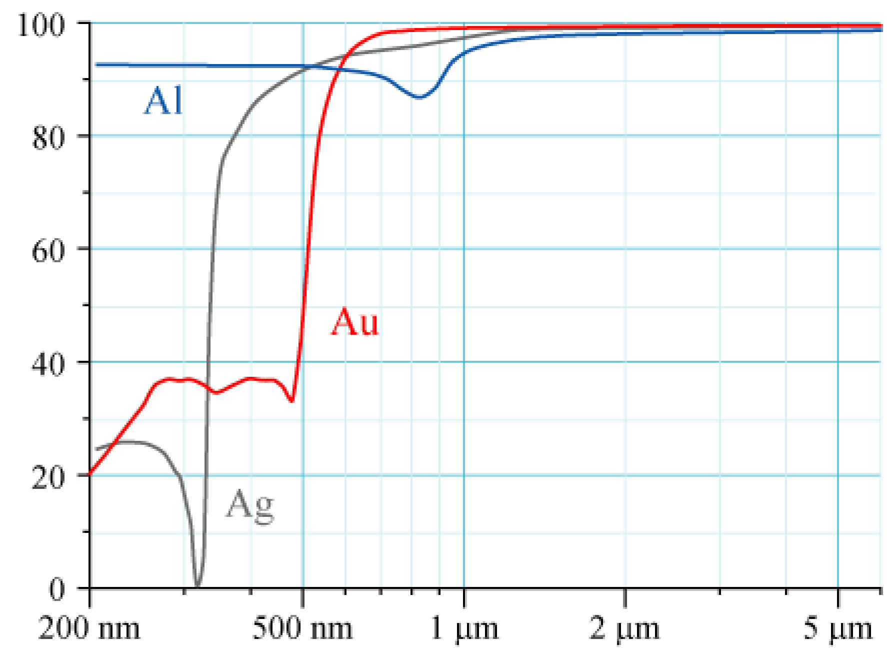
0.3m $\rightarrow 10^9$m: improve ~9-10 orders of magnitude in $\delta\sigma$

Increase response per length L

Michelson & Morley 1887:
4 round trips



Reflectivity of metal mirrors



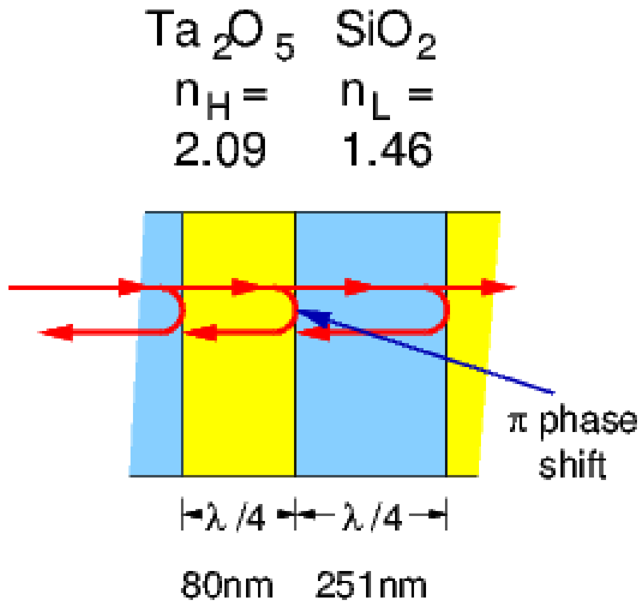
Wavelength

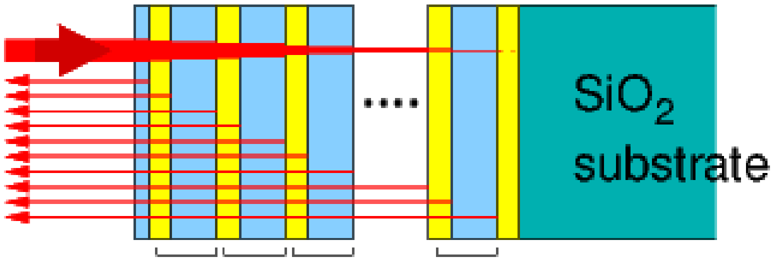
Silver: R ≈ 90% at 500 nm

Gold: R ≈ 98% at 1064 nm

Dielectric Mirrors

Interference of reflections from thin transparent films

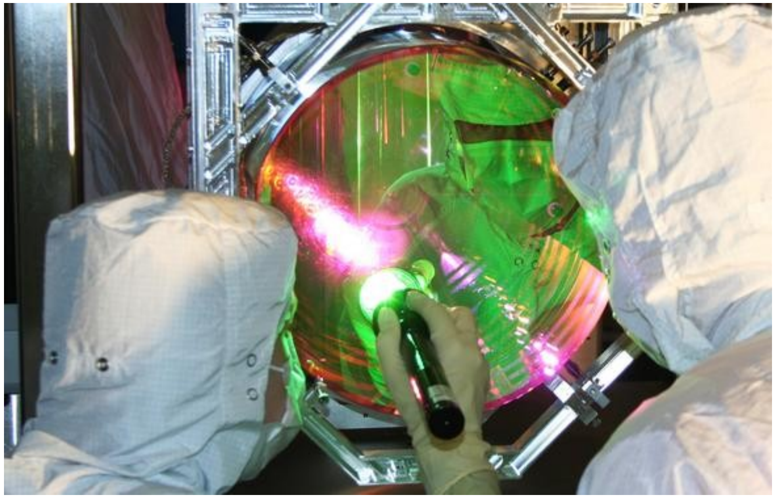




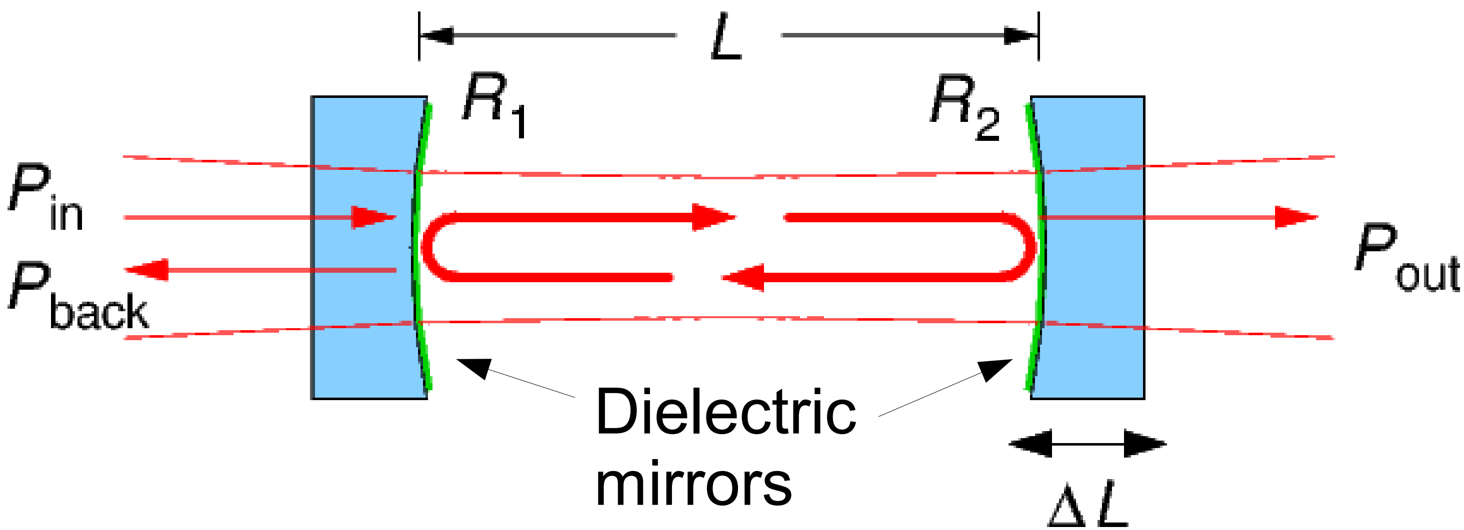
15..25 doublets

Modern optical mirrors:
R > 99.999%

for 10^3-10^4 round trips



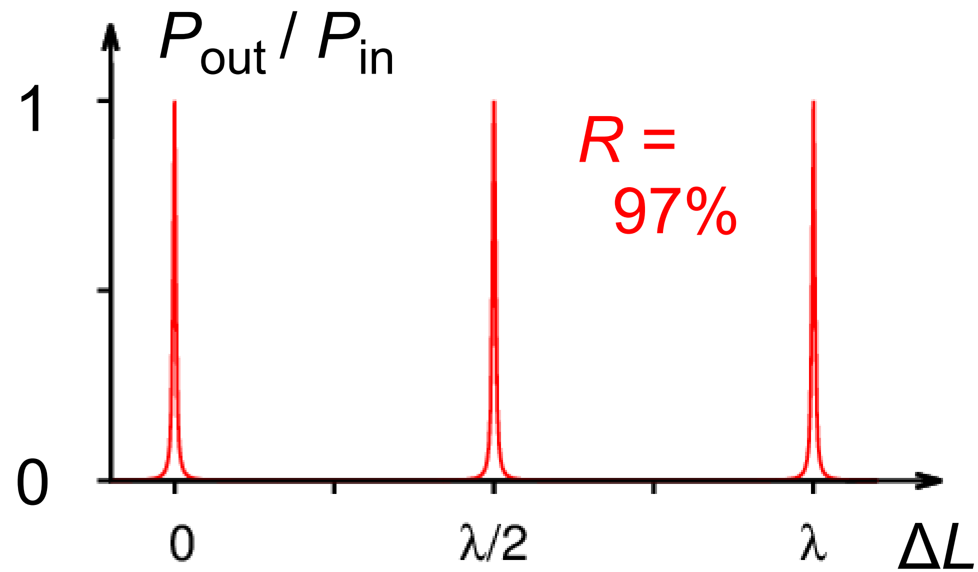
Fabry-Perot Resonator



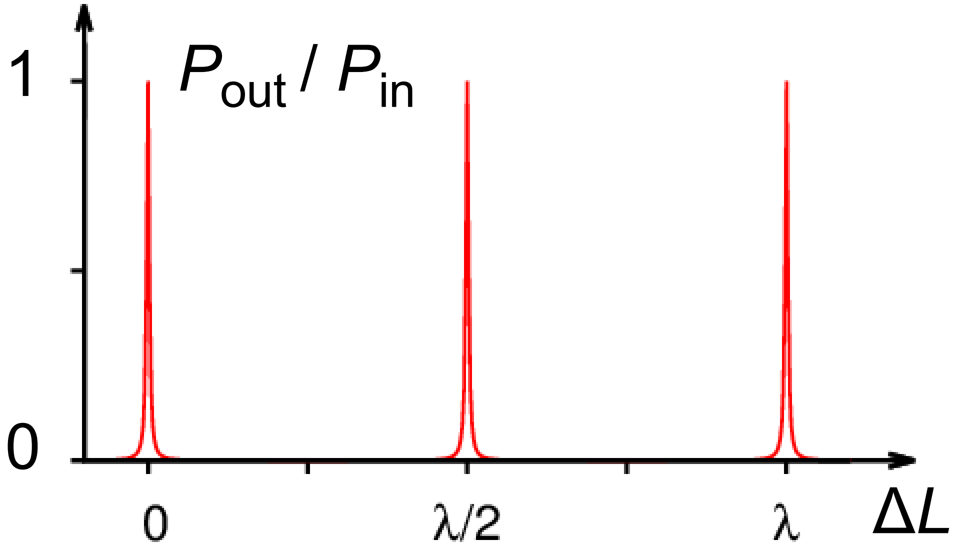
For $R_1 = R_2 = R$, no losses:

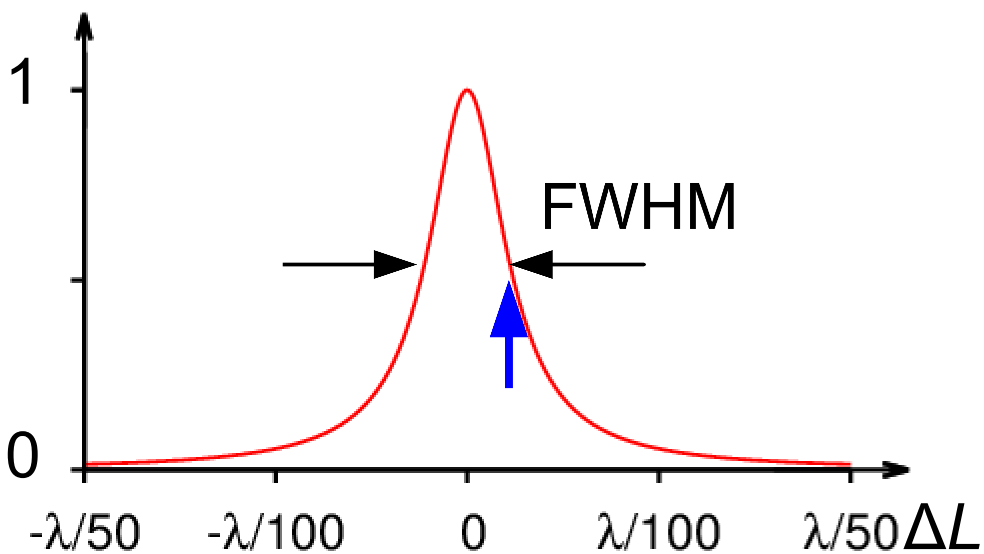
$$P_{out}(\Delta L) = \frac{P_{in}}{1 + f\sin^2\left(\frac{2\pi\Delta L}{\lambda}\right)}$$

with $f = \dfrac{4R}{(1-R)^2}$



Fabry-Perot Resonator II





Often used: finesse

$$F = \frac{\lambda/2}{FWHM} \approx \frac{\pi}{1-R}$$

$F \approx 105$ for $R = 97\%$

Fabry-Perot responsiveness:

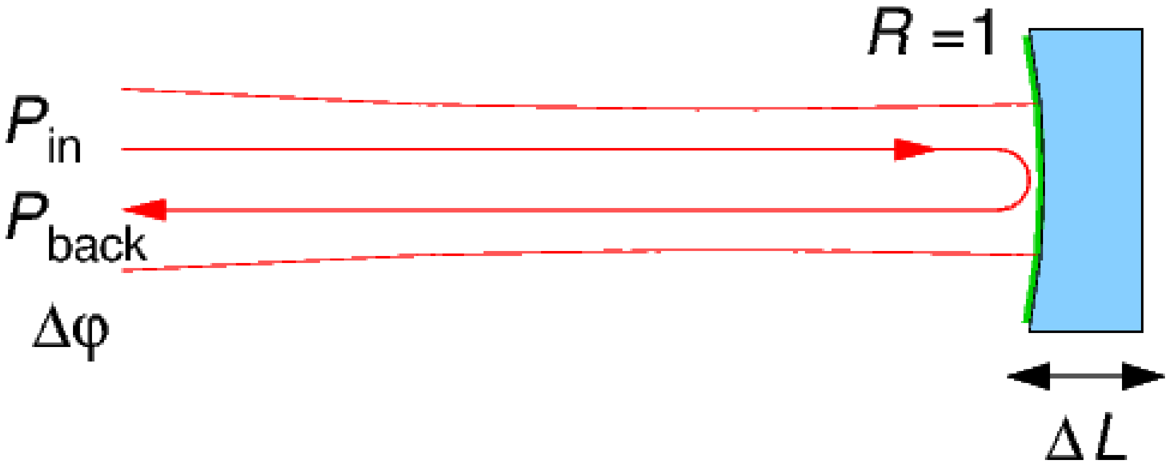
$$\left[\frac{d\left(P_{out}/P_{in}\right)}{d\left(\Delta L\right)}\right] \approx \frac{2\pi}{\lambda} \cdot \frac{1}{1-R}$$

Michelson responsiveness:

$$\left[\frac{d\left(P_{out}/P_{in}\right)}{d\left(\Delta L\right)}\right] = \frac{2\pi}{\lambda}$$

Fabry-Perot Resonator as Mirror

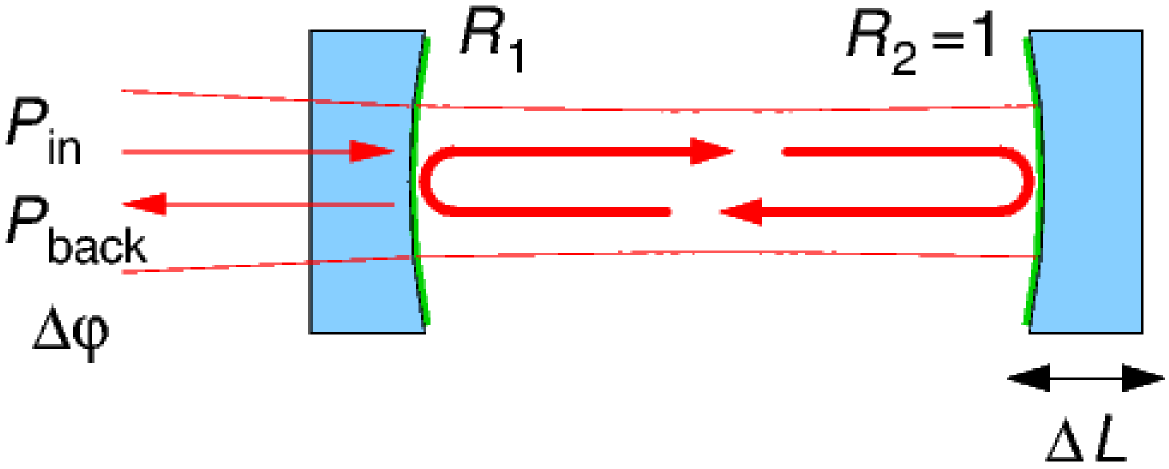
Simple mirror in Michelson interferometer:



$P_{back} = P_{in}$

$$\Delta \varphi_M = \frac{4\pi}{\lambda} \Delta L$$

Asymmetric Fabry-Perot resonator:

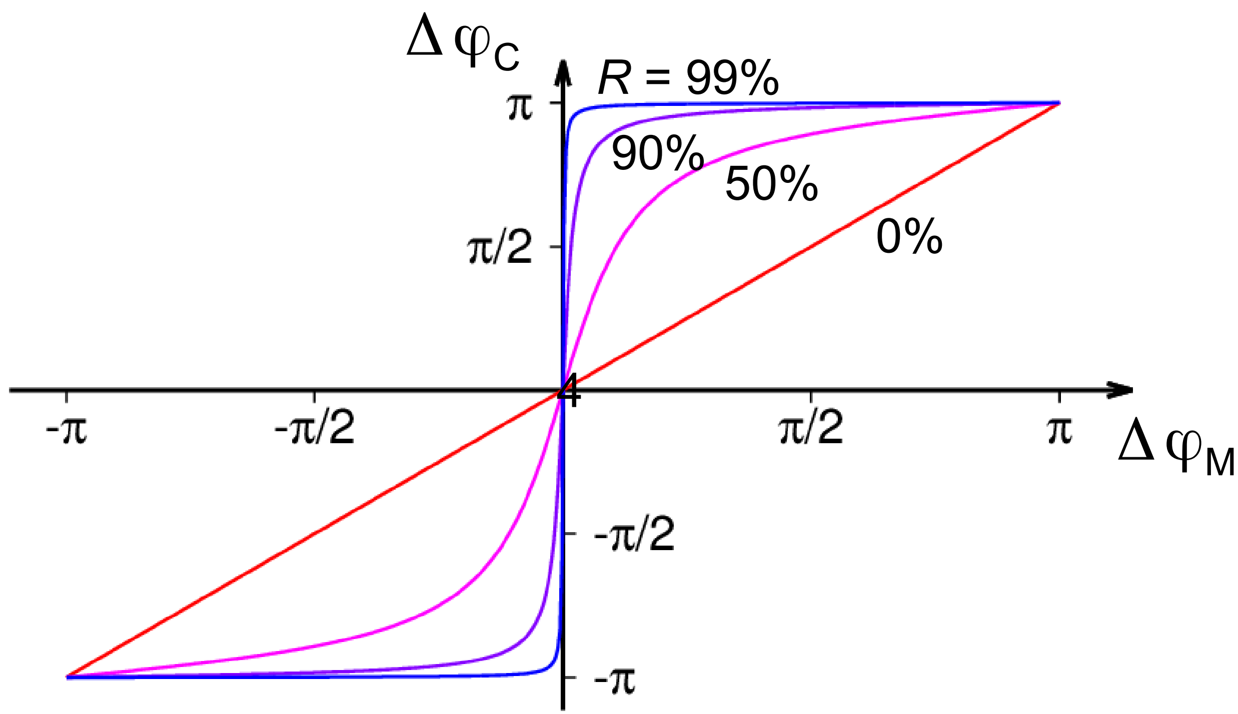


$P_{back} = P_{in}$

$$\Delta \varphi_C = ???$$

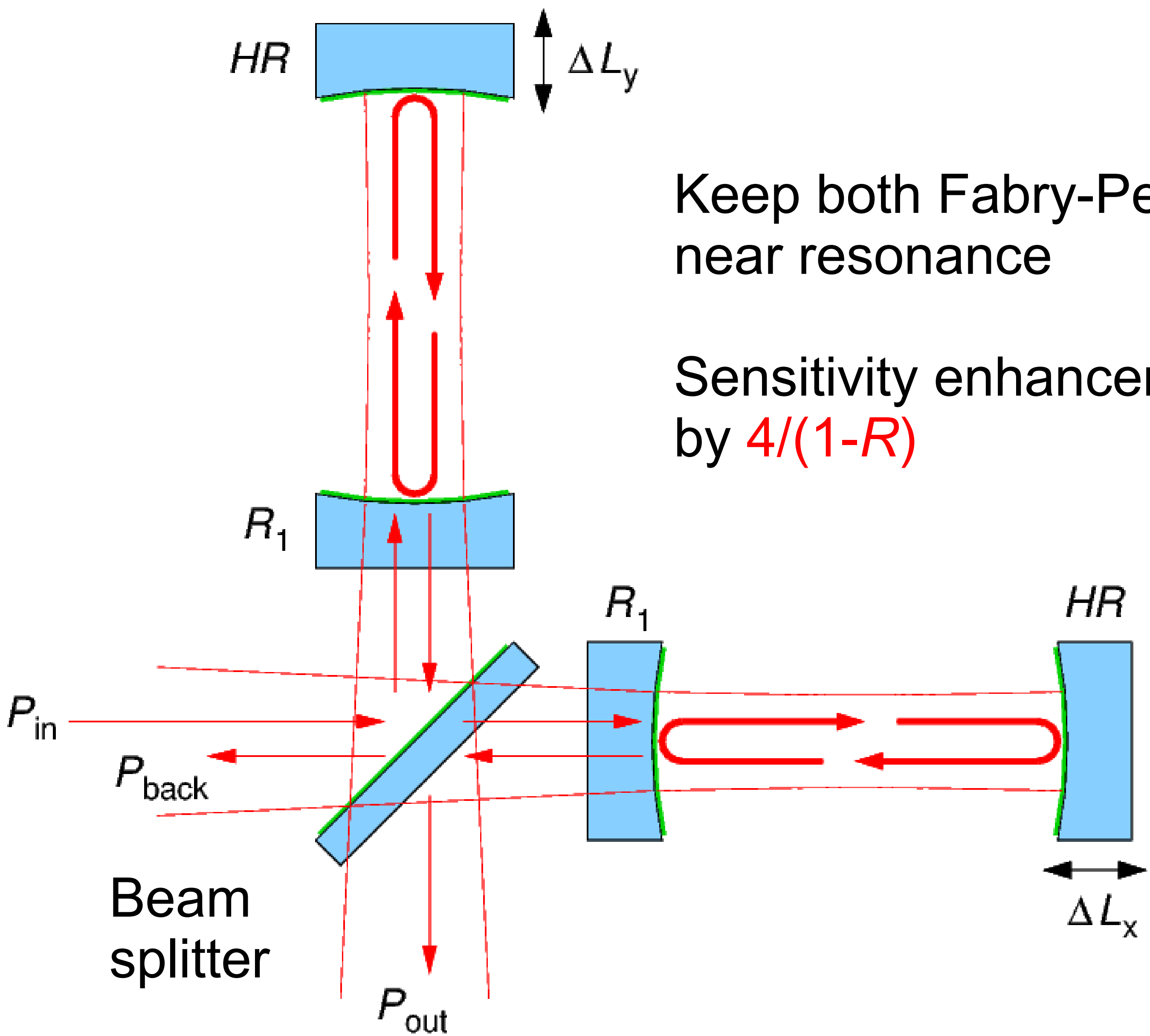
Fabry-Perot Mirror reponse

Solution: $$\tan\frac{\Delta\varphi_C}{2} = \frac{1+\sqrt{R}}{1-\sqrt{R}}\cdot\tan\frac{\Delta\varphi_M}{2}$$



Near $\Delta\varphi_C = 0$: $$\Delta\varphi_C \approx \frac{1+\sqrt{R}}{1-\sqrt{R}}\Delta\varphi_M \approx \frac{4}{1-R}\Delta\varphi_M$$

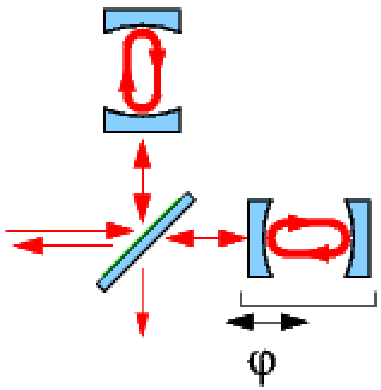
Super-Michelson interferometer



Keep both Fabry-Perots near resonance

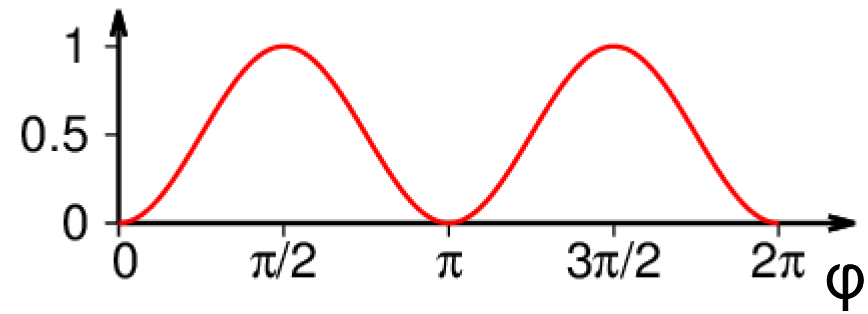
Sensitivity enhancement by 4/(1-*R*)

Michelson sensitivity sweet spot



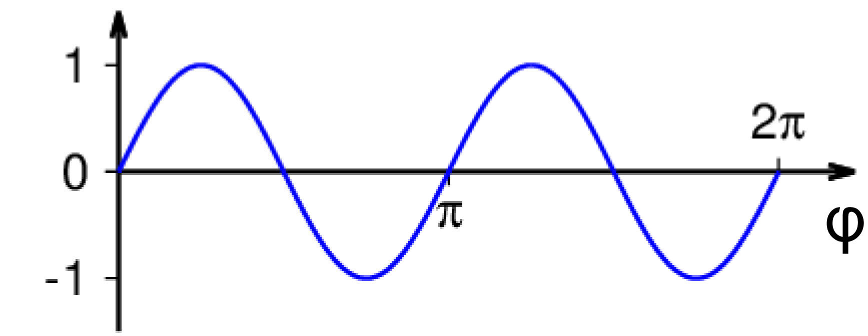
Output power:

$$P_{out}(\varphi) = P_{in} \sin^2 \varphi$$



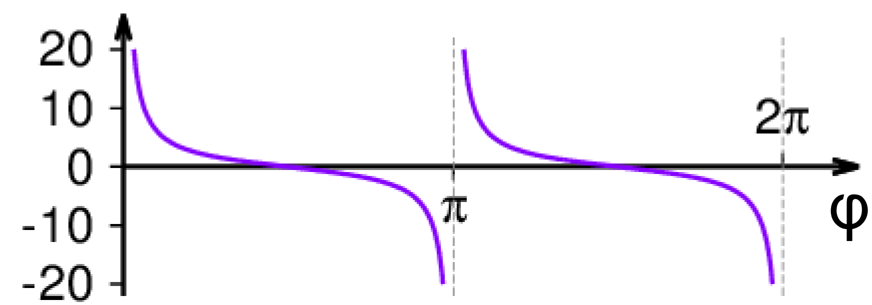
Phase sensitivity:

$$\frac{d P_{out}(\varphi)}{d\varphi} = P_{in} \sin(2\varphi)$$



Phase sensitivity per output power:

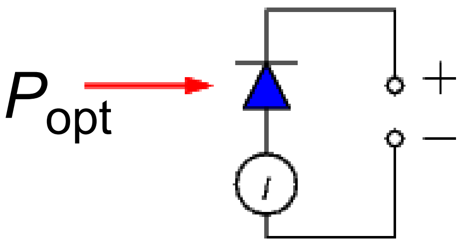
$$\frac{d P_{out}(\varphi)/d\varphi\big|_{\varphi_0}}{P_{out}} = \frac{\sin(2\varphi_0)}{\sin^2 \varphi_0} \approx \frac{2}{\varphi_0}$$



Work near dark fringe ($\varphi_0 \approx 0$), use lots of power!

Light detection

Photodiode



Amp meter

Photocurrent I is proportional to P_{opt}

Energy absorbed per time Δt :

$$\text{Energy} = P_{opt} \cdot \Delta t$$

Number of electrons per Δt :

$$n = \frac{\text{Energy}}{h \cdot f} = \frac{P_{opt} \cdot \Delta t}{h \cdot f}$$

Photocurrent:

$$I = \frac{n \cdot e}{\Delta t} = \frac{P_{opt} \cdot e}{h \cdot f}$$

Noise in Light detection

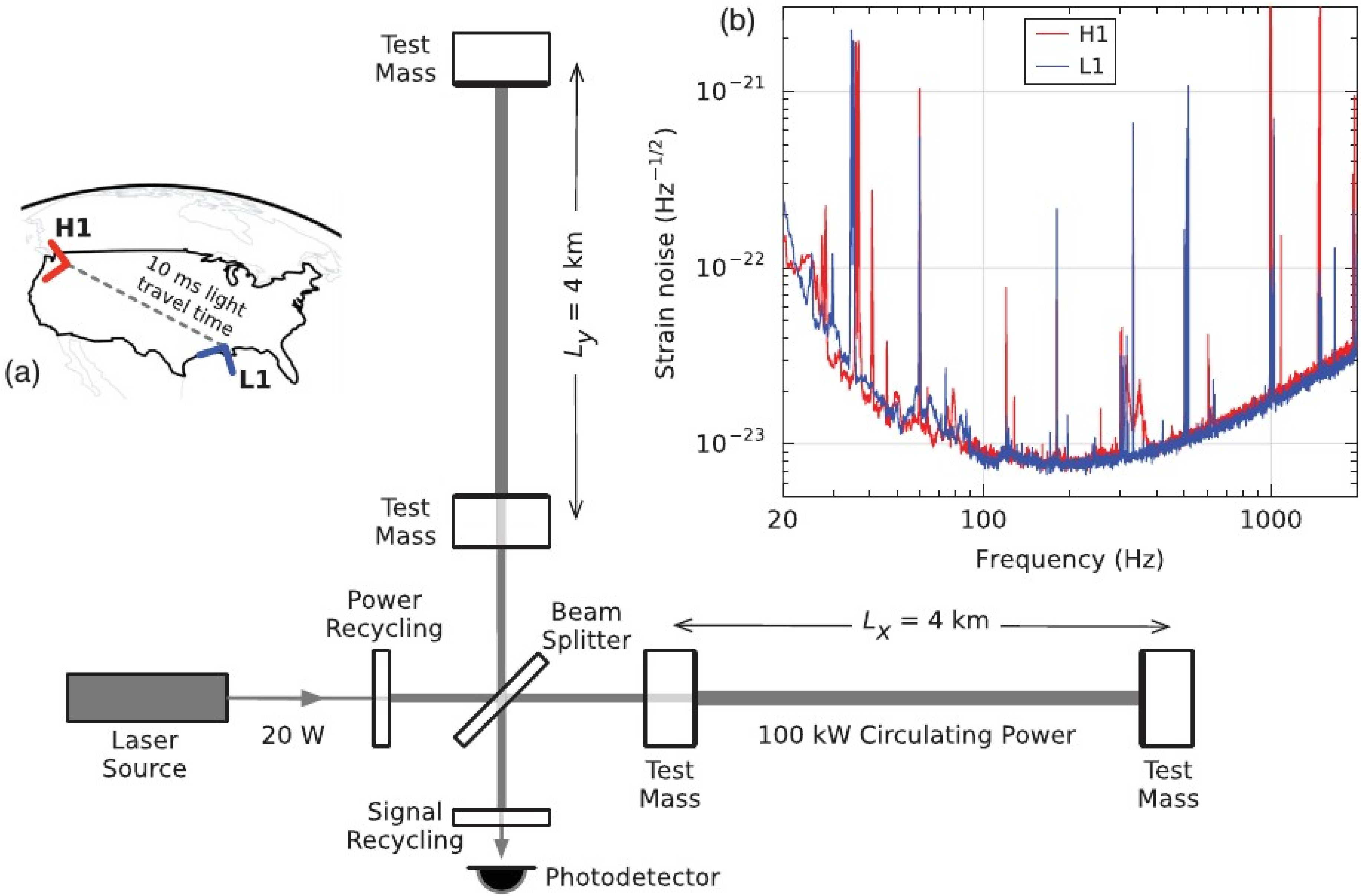
Uncertainty of electron number: $\delta n = \sqrt{n}$ Shot noise

Relative uncertainty in power; $$\frac{\delta P_{opt}}{P_{opt}} = \frac{1}{\sqrt{n}} = \sqrt{\frac{h \cdot f}{P_{opt} \cdot \Delta t}}$$

More optical power $\rightarrow$ less noise

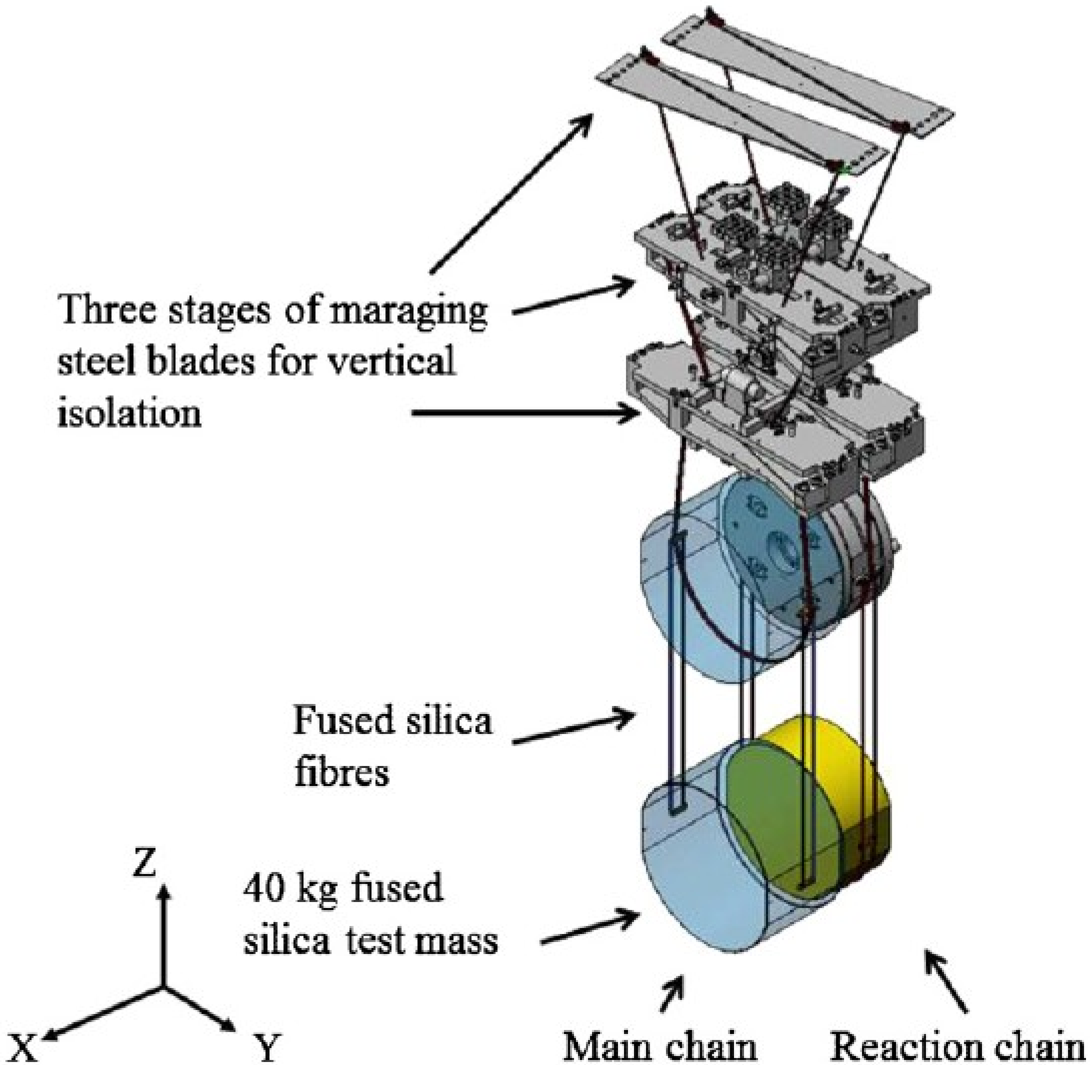
Longer measurement time $\rightarrow$ less noise

The full setup



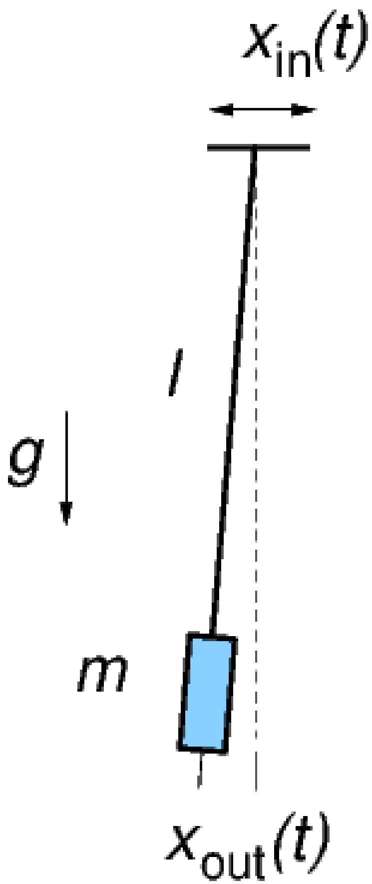
Phys. Rev. Lett. ***116****, 061102 (2016)*

Holding the mirrors "at rest"



Aston et al.,Class. Quant. Gravity ***29****, 235004 (2012)*

Pendulum as noise eater



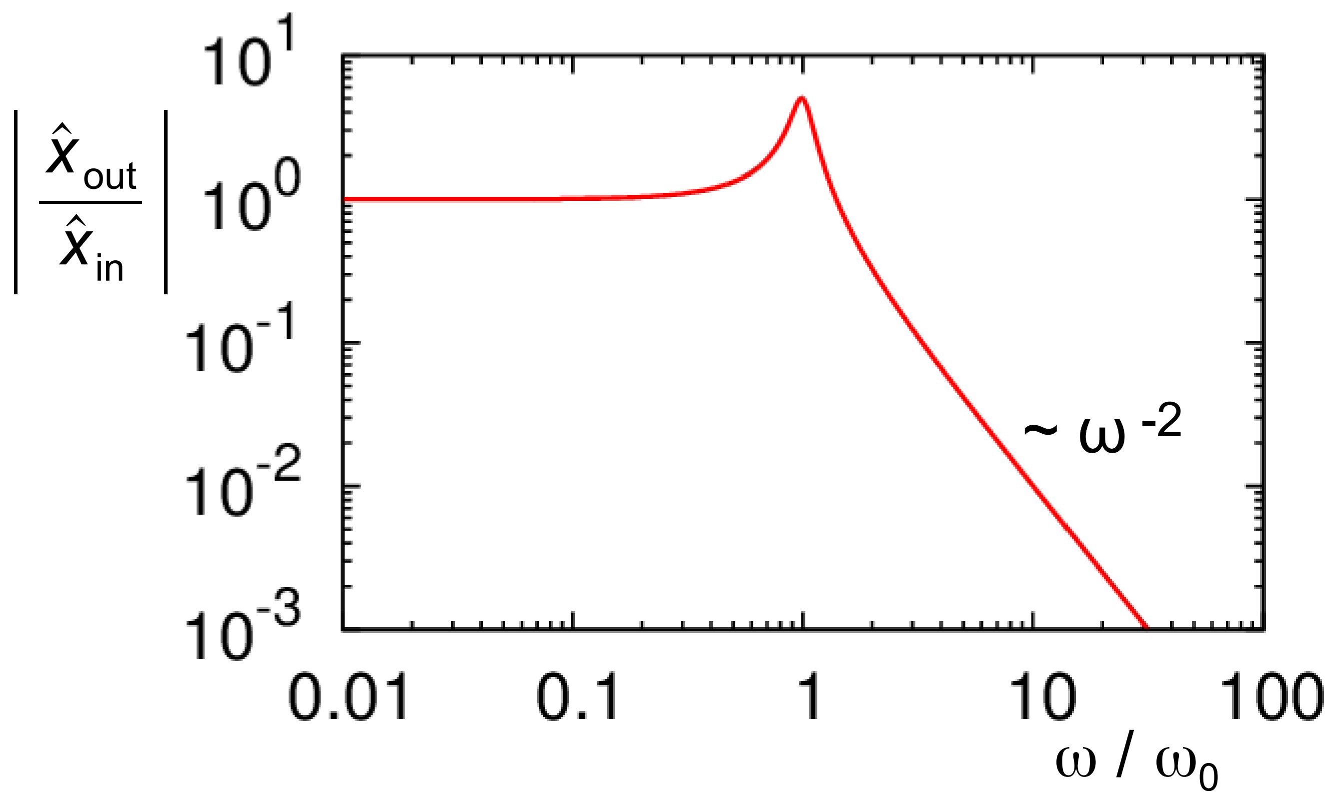
$$\omega_0 = \sqrt{\frac{g}{l}}$$

Assume harmonic shakes from suspension:

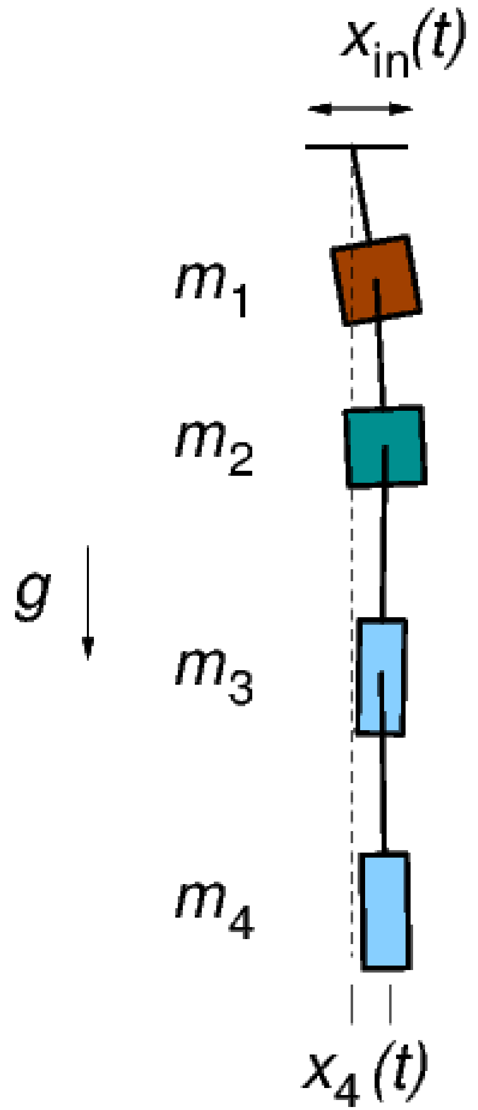
$$x_{in}(t) = \hat{x}_{in} \cos(\omega t)$$

Test mass will shake with same frequency:

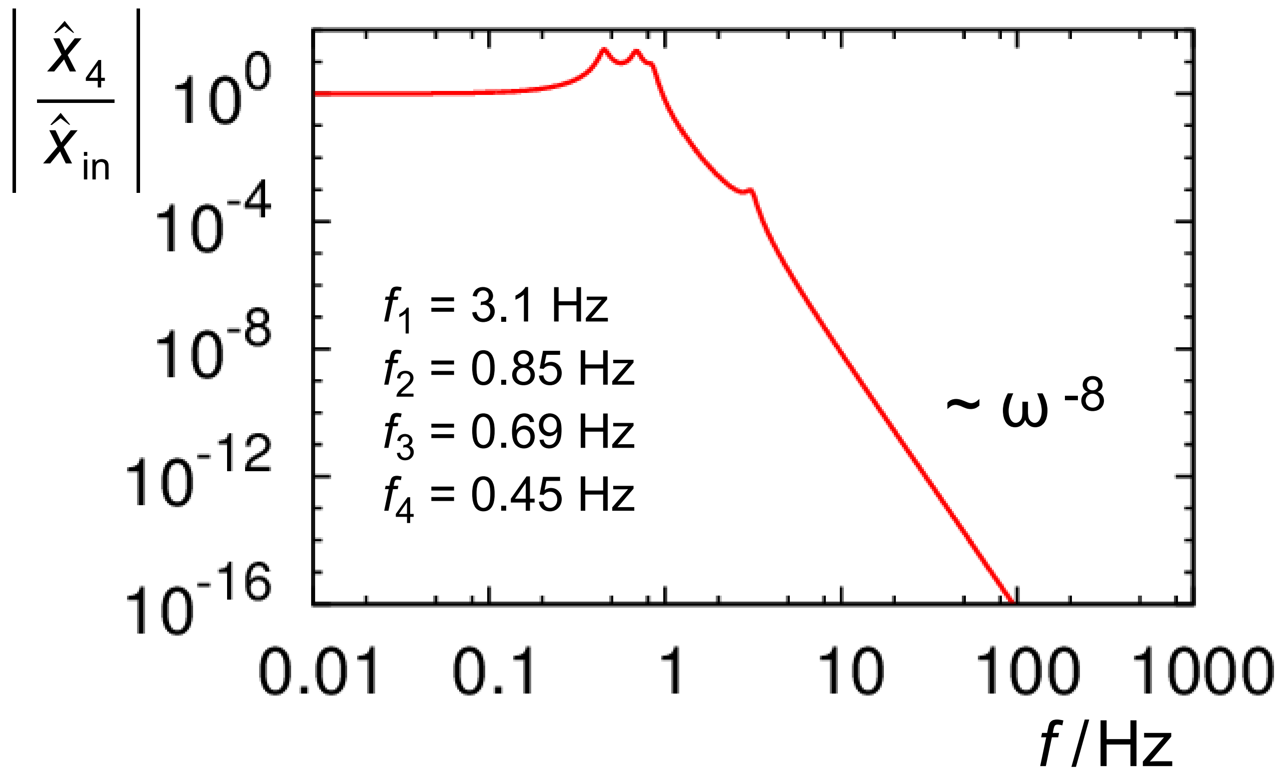
$$x_{out}(t) = \hat{x}_{out} \cos(\omega t)$$



Pendulum chain of 4

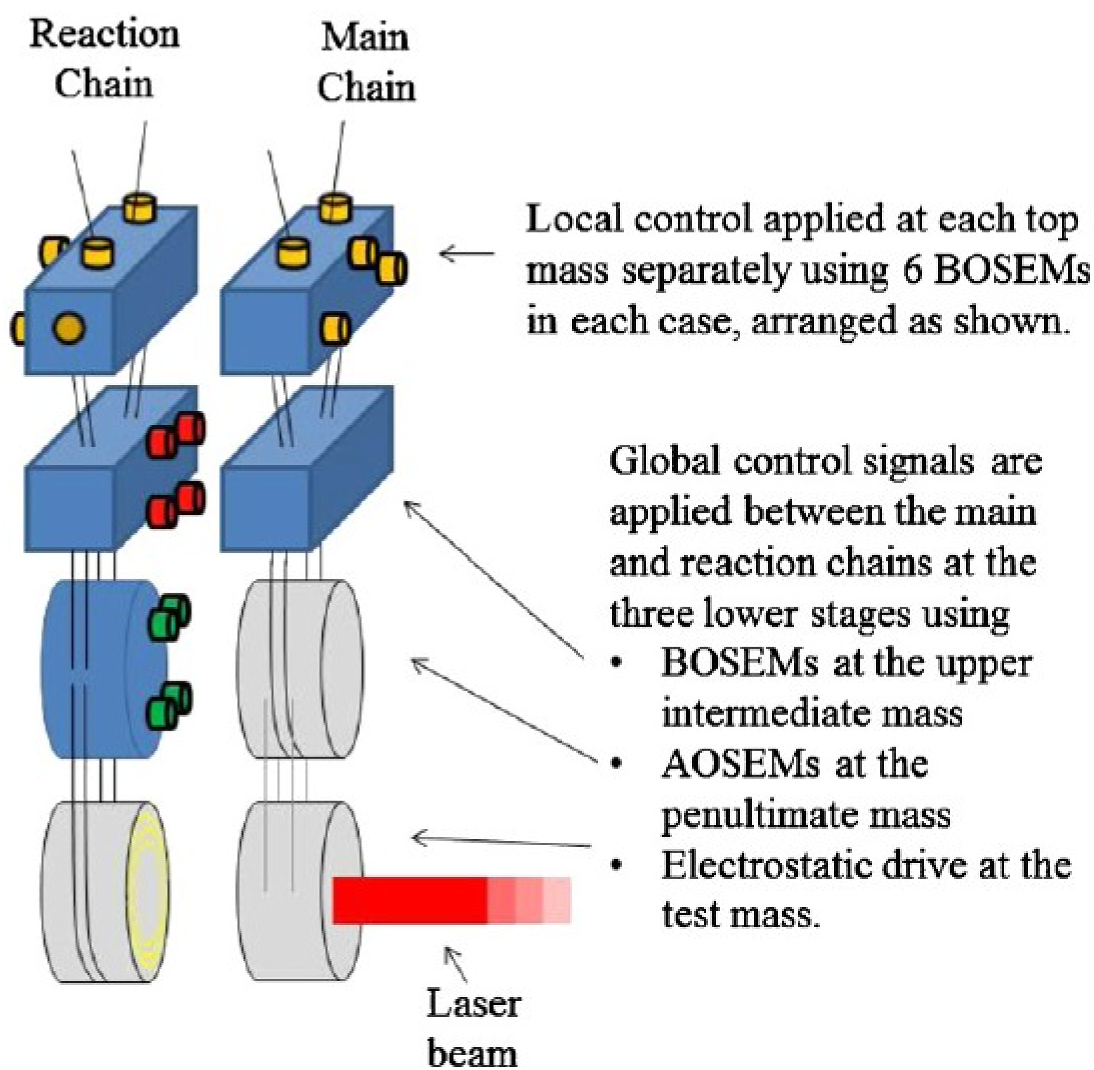


Susceptibilities get multiplied



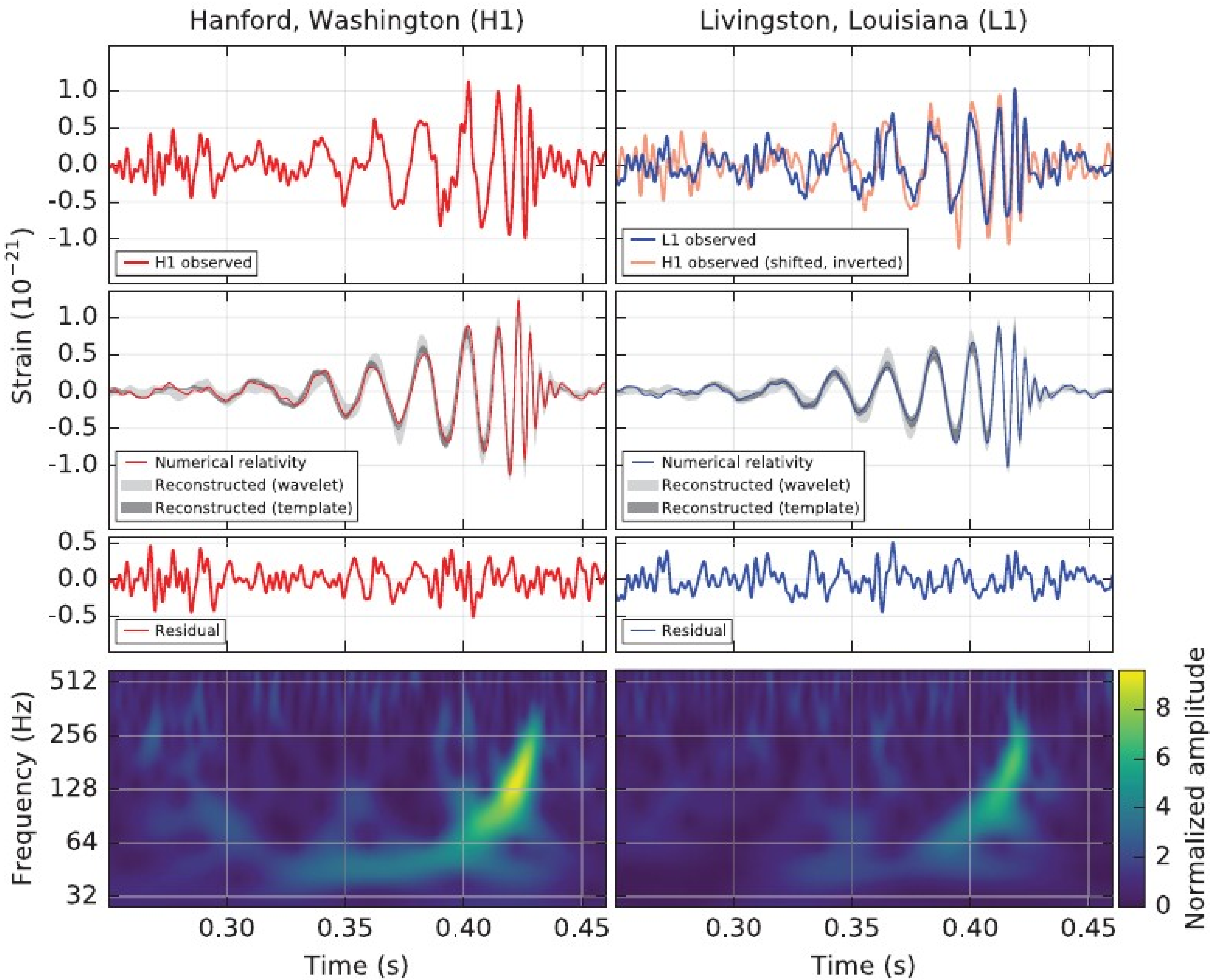
Also: avoid thermal noise in suspension

Moving the mirrors quietly...



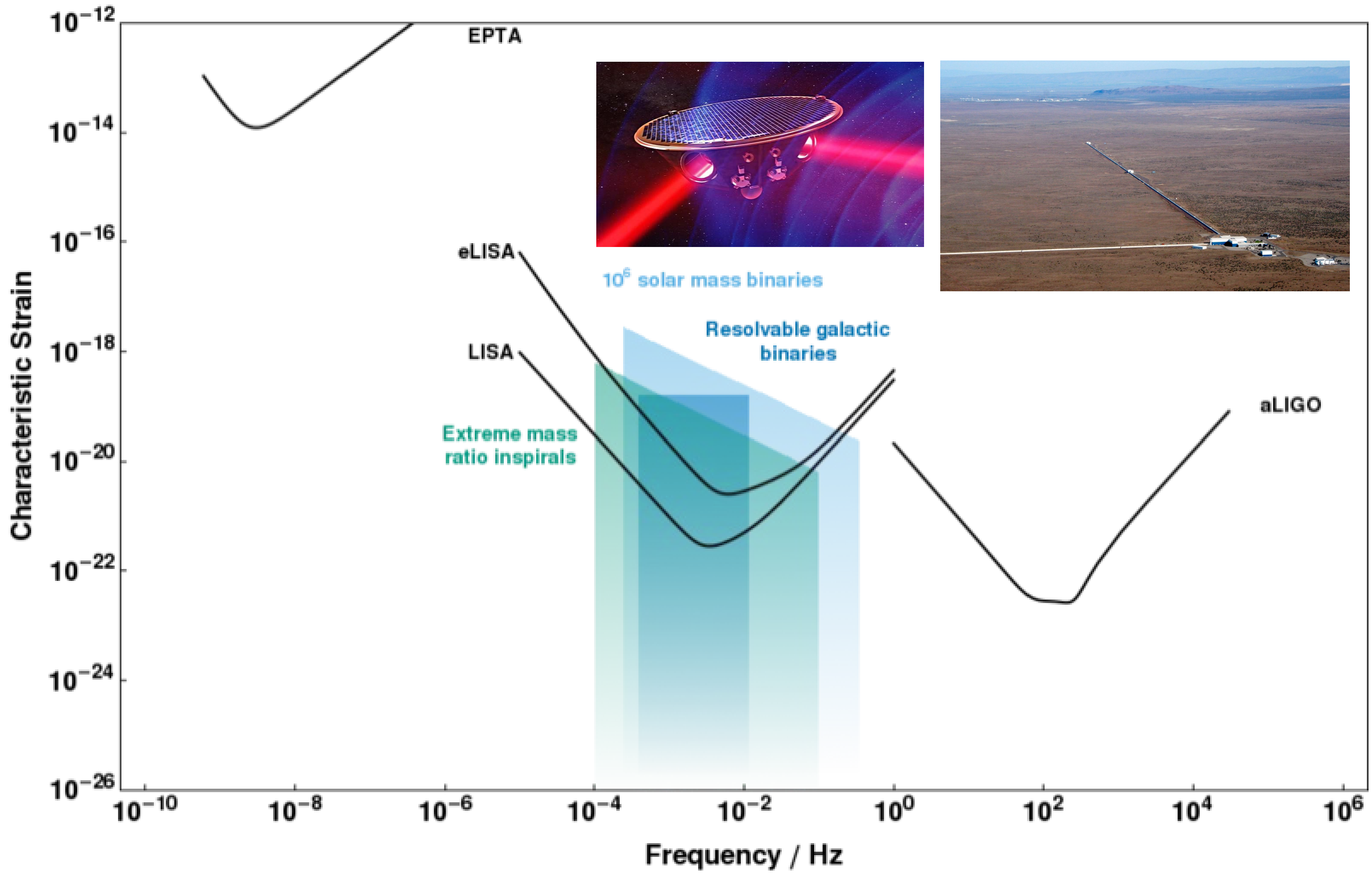
Aston et al.,Class. Quant. Gravity ***29****, 235004 (2012)*

The first result...



Where from here?

Different gravitational wave telescopes....



Souce: eLISA consortium